第 34 卷第 5 期
2017 年 5 月

控 制 理 论 与 应 用
Control Theory & Applications

Vol. 34 No. 5
May 2017

DOI: 10.7641/CTA.2017.16029

通讯信息约束下具有全局稳定性的分布式系统预测控制

郑 毅, 李少远[†], 魏永松
(上海交通大学 自动化系 系统控制与信息处理教育部重点实验室, 上海 200240)

摘要: 本文针对一类由状态相互耦合的子系统组成的分布式系统, 提出了一种可以处理输入约束的保证稳定性的非迭代协调分布式预测控制方法(distributed model predictive control, DMPC). 该方法中, 每个控制器在求解控制率时只与其它控制器通信一次来满足系统对通信负荷限制; 同时, 通过优化全局性能指标来提高优化性能. 另外, 该方法在优化问题中加入了一致性约束来限制关联子系统的估计状态与当前时刻更新的状态之间的偏差, 进而保证各子系统优化问题初始可行时, 后续时刻相继可行. 在此基础上, 通过加入终端约束来保证闭环系统渐进稳定. 该方法能够在使用较少的通信和计算负荷情况下, 提高系统优化性能. 即使对于强耦合系统同样能够保证优化问题的递推可行性和闭环系统的渐进稳定性. 仿真结果验证了本文所提出方法的有效性.

关键词: 大规模系统; 预测控制; 分布式预测控制; 约束控制
中图分类号: TP273 **文献标识码**: A

Global stabilizing distributed model predictive control systems with limited communication

ZHENG Yi, LI Shao-yuan[†], WEI Yong-song
(Department of Automation, Shanghai Jiao Tong University, Key Laboratory of System Control and Information Processing, Ministry of Education of China, Shanghai 200240.)

Abstract: A novel stabilized distributed model predictive control (DMPC) with input constraints and global cost optimization coordination strategy is proposed for spatially distributed coupling systems which are presented by states interacted models. The distributed controllers make decisions locally and merely communicate once a control period with each others. Cooperation is promoted by consideration of the system-wide objective by each local controller. Consistency constraints, which bound the estimation errors of the interaction sequences among subsystems, are designed to guarantee that, if an initially feasible solution can be found, subsequent feasibility of the algorithm is guaranteed at every update, and that the closed-loop system is asymptotically stable. The proposed control algorithm could reduce the communication and computation loads with improved performance of entire systems, and guarantee the recursive feasibility and the asymptotically stability even when the controlled subsystems are strong coupled. Simulation results show that the performance of the proposed DMPC is very close to that of a centralized model predictive control (MPC).

Key words: large-scale systems; model predictive control; distributed model predictive control; constrained control

1 Introduction

Many large scale and spatially distributed systems, such as power networks, smart grids[1], large scale chemical processes and hydro power plants[2], motivate the development of distributed control framework. The distributed model predictive control (DMPC), which controls each subsystem by a separate local model predictive control (MPC) has been more and more popular[3] since it not only inherits MPC's ability of explicitly accommodating constraints[4–10], but also possesses the advantages of the distributed framework of fault-tolerance and less computation[11–12].

The performance of a DMPC is, in most cases, not as good as that of a centralized MPC[11–13]. And a large communication loads may destroy the real-time control of DMPC although the information of the whole system is usually available to all subsystems in the most used industrial automation systems. Thus, how to design a stabilized DMPC which could improve the global performance of the closed-loop system with limited local computation and communication loads has been a very important topic issued from industries.

Many DMPC algorithms have appeared in the literature for different types of systems and for different problems in the design of DMPC, e.g. design of DMPC for nonlinear systems[14], uncertain sys-

Received: 15 February 2016; revised: 10 May 2017.
[†]Corresponding author. E-mail: syli@sjtu.edu.cn; Tel.: +86 21-34204012.
Recommended by Associate Editor WANG Wei.
Supported by National Natural Science Foundation of China (61233004, 61590924, 61673723).

tems[15–16], and networked systems with time delay[17], development of distributed optimization algorithms[18–19], and design of cooperative strategies for improving performance of DMPC[20–21], as well as the design of control structure[22]. Among them, several coordination strategies focus on studying how to improve the system-wide performance. The earliest and most adopted one is that each local MPC uses the inputs sequences of its neighbors to estimate the interactions among subsystems, then minimizes its own cost[23], we call it as local cost optimization based DMPC (LCO–DMPC). References [24] and [25] give two design methods for guaranteeing the stability. To guarantee the recursive feasibility, the algorithm uses a consistent constraints to limits the error between the presumed sequences of upstream neighbors, which are calculated based on the solution in the previous time instant, and the predictive states calculated by the corresponding subsystem in the current time instant. Then the stability is ensured by adding additional stabilization constraints, and judiciously integrating designs of the bound of the error[24], and the terminal constraint set. Furthermore, an iterative version of LCO–DMPC is developed in [26], by which the Nash Optimality of the closed-loop system can be achieved. Another commonly used coordination strategy, called impact-region cost optimization based DMPC, is proposed by [27–28], where each local MPC takes not only it's own performance but also that of the subsystems it directly impacts on into account in its optimization index. In addition, references [12] and [28] give another kind of cooperative algorithms, called cooperative DMPC, where each local MPC optimizes the cost over the entire system to improve the global performance of closed-loop system, and uses iterative algorithm to make the presumed states converge to the predictive states, then guarantees the stability by integrating terminal constraints. The Pareto Optimality of the closed-loop system can be obtained by this method. There are also several other strategies. e.g. reference [30] develops a dual decomposition based DMPC which uses Lagrange multipliers in order to relax the coupling between different agents. These multipliers can be seen as prices in a market mechanism, by means of which an agreement between the solutions of the different sub-problems is achieved. Reference [31] gives a sensitivity based DMPC to improve the robustness of the system. In [32], a comparison of parallel versus serial schemes is presented. The application areas of all these approaches are complementary. Each method possesses its own strengths and weaknesses. The practitioner, using knowledge and experience, must choose the control algorithm that is more appropriate for the problem at hand.

Consider that the cooperative DMPC can significant improve the global performance of entire system when the global information is available for each local MPC[11,20], and the iterative algorithms dramatically increase network communication burdens with the expansion of the scale of inputs and states of system since each subsystems exchange information with each other many times in one sampling time, the non-iterative Cooperative DMPC where each subsystems communicating once a control period may be a good strategy for providing a good global performance with limited local computation and communication loads.

Control design that takes state and/or input constraints into account, whether or not under the MPC framework, is an important and challenging problem. For the coordination strategy used here, there is no convergence condition can be used comparing to [12, 29]. And comparing to [24], excepted that there are errors between the presumed state/input sequences and predictive state sequences of upstream neighbors, the predictive state sequences of all subsystems calculated by current subsystem may not equal to those calculated by the others themselves, these error are hard to estimated, which makes it difficult to construct a feasible solution in the current time instant. In addition, the method in [24] uses an additional stability constraints to guarantee the asymptotically stability, which will further effect the optimization performance of the closed-system. To remove these constraints is also a problem. As a result, the existing methods for the design of stabilizing DMPC[12,24−25,28] are hard to be directly adopted to develop a stabilized global cost optimization based DMPC which communication once in a control period for spatial distributed coupling systems. All these make it difficult to design a stabilizing DMPC with limited local cooperation.

In this paper, a novel DMPC design method is proposed, where each local MPC optimizes the cost of the whole system and communicates with each other once a control period. The constrains which limit the errors between the optimal inputs sequences calculated at the previous time instant and the optimal inputs sequences calculated at the current time instant to within a prescribed bound, are designed and included in the optimization problem of each local MPC, which guarantee the recursive feasibility of proposed method. These inputs constraints combing with dual mode predictive control[25,33–34] strategy also guarantee the asymptotically stabilizing of the resulting closed-loop system without any additional stability constraints except the terminal cost and terminal constraints set. The contributed of this method are

- improve the performance of entire closed-loop system with fast computation speed and limited communication loads;
- guarantee the recursive feasibility and asymptotically stability even the interactions among subsystems is very strong;
- give a DMPC design method for the systems which are presented by state interacted models.

The remainder of this paper is organized as follows. Section 2 describes the problem to be solved in this paper. Section 3 presents the design of the proposed stabilized DMPC. The recursive feasibility and the stability are analyzed in Section 4. Section 5 presents the simulation results to demonstrate the effectiveness of the proposed DMPC. Finally, a brief conclusion to the paper is drawn in Section 6.

2 Problem formulation

Consider a spatially distributed system, as illustrated in Fig. 1, which is composed of many physically partitioned interacted subsystems, and each subsystem is controlled by a local controller which in turn is able to exchange information with other local controllers.

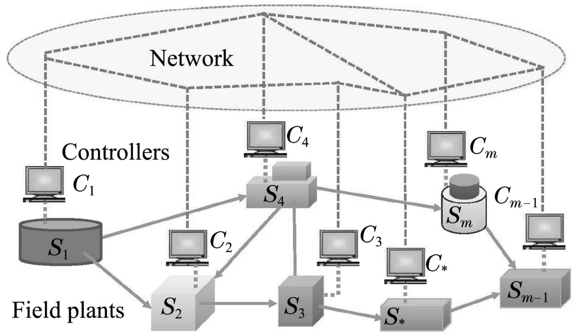

Fig. 1 An illustration of the structure of distributed system and distributed control framework

Without losing of generality, suppose that the whole system is composed of m discrete-time linear subsystems $i \in \mathcal{P}$, $\mathcal{P} = \{1, \cdots, m\}$. Let the subsystems interact with each other through their states. Then, subsystem $\mathcal{S}_i$ can be expressed as

$$\begin{cases} x_{i,k+1} = A_{ii}x_{i,k} + B_{ii}u_{i,k} + \sum\limits_{j\in\mathcal{P}_{+i}} A_{ij}x_{j,k}, \\ y_{i,k} = C_{ii}x_{i,k}, \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}^{n_{xi}}$, $u_i \in \mathcal{U}_i \subset \mathbb{R}^{n_{ui}}$ and $y_i \in \mathbb{R}^{n_{yi}}$ are respectively the local state, input and output vectors, and $\mathcal{U}_i$ is the feasible set of the input u_i, which is used to bound the input according to the physical constraints on the actuators, the control requirements or the characteristics of the plant. A non-zero matrix A_{ij}, indicates that $\mathcal{S}_i$ is affected by $\mathcal{S}_j$, $j \in \mathcal{P}$ and subsystem $\mathcal{S}_j$ is said to be an upstream system of $\mathcal{S}_i$, $\mathcal{S}_i$ is downstream system of $\mathcal{S}_j$. Let $\mathcal{P}_{+i}$ denote the set of the subscripts of the upstream systems of $\mathcal{S}_i$, that is, $j \in \mathcal{P}_{+i}$, and set $\mathcal{P}_{-i}$ be the set of the subscripts of the downstream systems of $\mathcal{S}_i$. In addition, set $\mathcal{P}_i = \{j | j \in \mathcal{P}, \text{ and } j \neq i\}$. In the concatenated vector form, the system dynamics can be written as

$$\begin{cases} x_{k+1} = Ax_k + Bu_k, \\ y_k = Cx_k, \end{cases} \tag{2}$$

where $x = [x_1^{\mathrm{T}}\ x_2^{\mathrm{T}}\ \cdots\ x_m^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n_{\mathrm{x}}}$, $u = [u_1^{\mathrm{T}}\ u_2^{\mathrm{T}}\ \cdots\ u_m^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n_{\mathrm{u}}}$ and $y = [y_1^{\mathrm{T}}\ y_2^{\mathrm{T}}\ \cdots\ y_m^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n_{\mathrm{y}}}$ are respectively the concatenated state, control input and output vectors of the overall system $\mathcal{S}$, and A, B and C are constant matrices of appropriate dimensions. Also, $u \in \mathcal{U} = \mathcal{U}_1 \times \mathcal{U}_2 \times \cdots \times \mathcal{U}_m$ and $\mathcal{U}$ contain a neighborhood of the origin.

The control objective is to stabilize the overall system $\mathcal{S}$ in an DMPC framework with limited communication resources. Meanwhile, the performance of closed-loop system should be as close as possible to the performance of the closed-system under control of a centralized MPC.

3 DMPC with limited local cooperation

In this Section, m separate optimal control problems, one for each subsystem, and the DMPC with limited local cooperation (LLC–DMPC) which communicates once a control period is defined. In every distributed optimal control problem, the same constant prediction horizon N, $N > 1$, is used. And every distributed MPC law is updated globally synchronously. At each update, every local MPC optimizes only for its own open-loop control sequence, given the current states and the estimated inputs of the whole system.

To proceed, we need the following assumption, and we also define the necessary notation in Table 1.

Assumption 1 For every subsystem $i \in \mathcal{P}$, there exist a state feedback $u_i = K_i x$ such that the closed-loop system $x_{k+1} = A_{\mathrm{c}} x_k$ is asymptotically stable, where

$$A_{\mathrm{c}} = A + BK,$$
$$K = [K_1^{\mathrm{T}}\ K_2^{\mathrm{T}}\ \cdots\ K_m^{\mathrm{T}}]^{\mathrm{T}}.$$

Remark 1 This assumption is very loose, it only presumes that the whole subsystem is able to be stabilized by a feed-back control Kx. The control gain K can be obtained by LMI or LQR technology. Another normal assumption in the design of stabilized DMPC is that each subsystem is able to be stabilized by a decentralized control $K_i x_i$, $i \in \mathcal{P}$[24–25]. It means that the algorithms are designed for weakly coupled system and is more restricted than Assumption 1. There is no any requirements on the strength of the interactions among subsystems in Assumption 1.

Table 1 Notation

Notation	Explanation
$\mathcal{P}$	the set of the subscripts of all subsystems
$\mathcal{P}_i$	the set of the subscripts of all subsystems excluding $\mathcal{S}$ itself
$+i$	all upstream subsystem of $\mathcal{S}_i$
$-i$	all downstream subsystem of $\mathcal{S}_i$
$u_{i,k+l-1\|k}$	the optimal control sequence of $\mathcal{S}_i$ calculated by $\mathcal{C}_i$ at time k
$\hat{x}_{j,k+l\|k,i}$	the predicted state sequence of $\mathcal{S}_j$ calculated by $\mathcal{C}_i$ at time k
$\hat{x}_{k+l\|k,i}$	the predicted state sequence of all subsystems calculated by $\mathcal{C}_i$ at time k
$u^{\mathrm{f}}_{i,k+l-1\|k}$	the feasible control at time $k+l-1$ of $\mathcal{S}_i$ defined by $\mathcal{C}_i$ at time k
$x^{\mathrm{f}}_{j,k+l\|k,i}$	the predictive feasible state sequence of $\mathcal{S}_j$ defined by $\mathcal{C}_i$ at time k
$x^{\mathrm{f}}_{k+l\|k,i}$	the predictive feasible state sequence of all subsystems calculated by $\mathcal{C}_i$ at time k
$x^{\mathrm{f}}_{k+l\|k}$	the predictive feasible state sequence of all subsystems, and $x^{\mathrm{f}}_{k+l\|k}=[x^{\mathrm{f}}_{1,k+l\|k}\ x^{\mathrm{f}}_{2,k+l\|k}\ \cdots\ x^{\mathrm{f}}_{m,k+l\|k}]^{\mathrm{T}}$
$\|\|\cdot\|\|_P$	refer to the P norm, P is any positive matrix, and $\|\|z\|\|_P=\sqrt{x_k^{\mathrm{T}}Px_k}$,

As the state evolution of subsystem $\mathcal{S}_j$, $j\in\mathcal{P}_{-i}$, is affected by the optimal control decision of $\mathcal{S}_i$, and the affection on the control performance of subsystem $\mathcal{S}_j$ may be negatively sometime. Thus, the idea of global cost optimization[12,20] is adopted here, that is each local MPC takes the cost function of all subsystems into account, more specifically, the performance index is defined as

$$\bar{J}_{i,k}=\|\hat{x}_{k+N|k,i}\|_P+\sum_{l=0}^{N-1}(\|\hat{x}_{k+l|k,i}\|_Q+\|u_{i,k+l|k}\|_{R_i}),\tag{3}$$

where

$$P=P^{\mathrm{T}}>0,\ Q=Q^{\mathrm{T}}>0,$$
$$R_i=R_i^{\mathrm{T}}>0,\ R=\mathrm{diag}\{R_1,R_2,\cdots,R_m\},$$

and P is chosen to satisfy the Lyapunov equation:

$$A_{\mathrm{c}}^{\mathrm{T}}PA_{\mathrm{c}}-P=-(Q+K^{\mathrm{T}}RK).\tag{4}$$

Since every local controller updates synchronously, the control sequences $\mathcal{S}_j, j\in\mathcal{P}_i$ are unknown to subsystem $\mathcal{S}_i$. Thus, at the time instant k, presume the control sequence of $\mathcal{S}_j, j\in\mathcal{P}_i$ be the optimal control sequence calculated by $\mathcal{C}_j$ at time $k-1$ concatenated with the feedback control law, that is

$$[u_{j,k|k-1}u_{j,k+1|k-1}\cdots u_{j,k+N-2|k-1}K_j\hat{x}_{k+N-1|k-1,j}].\tag{5}$$

Then, the predictive model in the MPC for $\mathcal{S}_i$ is expressed as

$$\hat{x}_{k+l|k,i}=A^lx_k+\sum_{h=1}^{l}A^{l-h}\bar{B}_iu_{i,k+h-1|k}+\sum_{j\in\mathcal{P}_i}\sum_{h=1}^{l}A^{l-h}\bar{B}_ju_{j,k+h-1|k-1},\tag{6}$$

where, for $\forall i$ and $j\in\mathcal{P}_i$,

$$\bar{B}_i=[0^{n_{ui}\times\Sigma_{j<i}n_{xj}}\ \ B_i\ \ 0^{n_{ui}\times\Sigma_{j>i}n_{xj}}]^{\mathrm{T}}.\tag{7}$$

In addition, to enlarge the feasible region, a terminal state constraint is included in each local MPC. The terminal state constraint set should guarantee that the terminal controllers are stabilizing inside it.

From Assumption 1, in the optimization problem of each local MPC, the terminal state constraint set for $\mathcal{S}$ can be set to be

$$\Omega(\varepsilon)=\{x\in\mathbb{R}^{n_{\mathrm{x}}}|\|x\|_P\leqslant\varepsilon\},\tag{8}$$

where ε is small enough positive scalar such that Kx is in the feasible input set $\mathcal{U}\subset\mathbb{R}^{n_{\mathrm{u}}}$ for all $x\in\Omega(\varepsilon)$.

Suppose that at some time k_0, $x(k_0)\in\Omega(\varepsilon)$ for every subsystem, then stabilization can be achieved if every $\mathcal{C}_i, i\in\mathcal{P}$ employs its static feedback controller $K_ix(k)$ for all time $k\geqslant k_0$. Thus, the objective of the MPC law is to drive states of all subsystems to the set $\Omega(\varepsilon)$. In what follows, we formulate the optimization problem for each local MPC.

Problem 1 Consider subsystem $\mathcal{S}_i$. Let the update time be $k\geqslant 1$. Given x_k, and $u_{k+l|k-1}$, $l=0,1,\cdots,N-1$, find the control sequence $u_{i,k+l|k}:\{0,1,\cdots,N-1\}\to\mathcal{U}_i$ that minimizes the performance index

$$\bar{J}_{i,k}=\|\hat{x}_{k+N|k,i}\|_P+\sum_{l=0}^{N-1}(\|\hat{x}_{k+l|k,i}\|_Q+\|u_{i,k+l|k}\|_{R_i}),$$

subject to the constraints:

Equation (6)

$$\begin{cases}\sum_{h=0}^{l}\beta_{l-h}\|u_{i,k+h|k}-u_{i,k+h|k-1}\|_2\leqslant\dfrac{\gamma\kappa\alpha\varepsilon}{m-1},\\ l=1,2,\cdots,N-1,\end{cases}\tag{9}$$

$$u_{i,k+l-1|k}\in\mathcal{U}_i,\ l=1,2,\cdots,N-1,\tag{10}$$

$$\hat{x}_{k+N|k,i}\in\Omega(\alpha\varepsilon).\tag{11}$$

In the constraints above,

$$\beta_l=\max_{i\in\mathcal{P}}(\lambda_{\max}((A^l\bar{B}_i)^{\mathrm{T}}PA^l\bar{B}_i)^{\frac{1}{2}}),\tag{12}$$

where $l=0,1,\cdots,N-1$. The constant $0<\kappa<1$, $0<\alpha<0.5$ and $\gamma>0$ are design parameters, and

$$\kappa\leqslant 1-\lambda_{\max}(\sqrt{A_{\mathrm{c}}^{\mathrm{T}}A_{\mathrm{c}}}).\tag{13}$$

The consistency constraints (9) requires that each optimal manipulated variables remain close to the presumed sequence. It is a key equation in proving that $x^{\rm f}_{\cdot|k,i}$ is a feasible state sequence at each update.

Remark 2 It should be noticed that following three restrictions (or assumptions) should be considered when using the LLC–DMPC: a) there are no directly couplings through inputs among subsystems; b) there are no constraints on states and outputs, or there are other measures to limit states and outputs; c) there is no coupling constraints.

Before stating the LLC–DMPC algorithm, an assumption is made to facilitate the initialization phase.

Assumption 2 At initial time k_0, there exists a feasible control $u_{i,k_0+l} \in \mathcal{U}_i,\ l \in \{1,\cdots,N\}$, for each $\mathcal{S}_i$, such that the solution to the full system $x_{l+1+k_0} = Ax_{l+k_0} + Bu_{l+k_0}$, denoted $\hat{x}_{\cdot|k_0,i}$, satisfies $\hat{x}_{N+k_0|k_0,i} \in \Omega(\alpha\varepsilon)$ and results in a bounded cost $\bar{J}_{i,k_0}$.

Remark 3 Assumption 2 bypasses the task of actually constructing an initially feasible solution in a distributed way. In fact, one way to obtain an initially feasible solution is to solve the corresponding centralized MPC solution at the initial time instant.

The dual-mode LLC–MPC law for any $\mathcal{S}_i$, which communicates once every update, is as follows.

Algorithm 1

Step 1 Initialization at time k_0.

- Initialize $x_{k_0}, u_{k_0+l-1|k_0},\ l = 1,2,\cdots,N$.

Step 2 Update control law at time k.

- Measure $x_{i,k}$; Transmit $x_{i,k}$ and $u_{i,k+l|k}$ to all other subsystems; Receive $x_{j,k}$ and $u_{j,k+l-1|k-1},\ j \in \mathcal{P}_i$ from all $\mathcal{S}_j$;
- If $x_k \in \Omega(\varepsilon)$, then apply the terminal controller $u_{i,k} = K_i x_k$; Else
- Solve Problem 1 for $u_{i,k+l-1|k}$ and apply $u_{i,k|k}$ to $\mathcal{S}_i$;

Step 3 Update control at time $k+1$.

- Let $k+1 \to k$, repeat Step 2.

In the next section, it is shown that the LLC–MPC policy drives the state x_{k+l} to $\Omega(\varepsilon)$ in a finite number of updates and the state remains in $\Omega(\varepsilon)$ for all future time. And the analysis of the feasibility and stability of LLC–DMPC algorithm is proceeded as follows.

4 Performance analysis

4.1 Feasibility

The main result of this subsection is that, provided an initially feasible solution is available, for any $\mathcal{S}_i$ and at any time $k \geqslant 1$, $u_{i,\cdot|k} = u^{\rm f}_{i,\cdot|k}$ is a feasible control solution to Problem 1. Here, $u^{\rm f}_{\cdot|k}$ is the reminder of previous control concatenating with a feedback control, that is

$$u^{\rm f}_{i,k+l-1|k} = \begin{cases} u_{i,k+l-1|k-1}, & l = 1,\cdots,N-1, \\ K_i x^{\rm f}_{k+N-1|k,i}, & l = N, \end{cases} \tag{14}$$

and $x^{\rm f}_{k+l|k,i},\ l = 1,2,\cdots,N$, can be expressed as

$$x^{\rm f}_{k+l|k,i} = A^l x_k + \sum_{h=1}^{l} A^{l-h}\bar{B}_i u^{\rm f}_{i,k+h-1|k} + \sum_{j\in\mathcal{P}_i}\sum_{h=1}^{l} A^{l-h}\bar{B}_j u_{j,k+h-1|k-1}. \tag{15}$$

Substitute (14) into (15), we have that

$$x^{\rm f}_{k+l|k,i} = x^{\rm f}_{k+l|k,j} = x^{\rm f}_{k+l|k},\ l = 1,2,\cdots,N, \tag{16}$$

$$x^{\rm f}_{k+N|k} = A_{\rm c} x^{\rm f}_{k+N-1|k}. \tag{17}$$

The control $u^{\rm f}_{i,\cdot|k}$ is a feasible solution to Problem 1 for any $\mathcal{S}_i$ and at any update $k \geqslant 1$ refers to that the control $u^{\rm f}_{i,\cdot|k}$ satisfies equation (9) and the control constraints (10), and the corresponding state $x^{\rm f}_{k+N|k}$ satisfies the terminal state constraint (11).

To establish this feasibility result, define that the state $\hat{x}_{k+N|k-1,i}$ to be the closed-loop response of

$$\hat{x}_{k+N|k-1,i} = A_{\rm c}\hat{x}_{k+N-1|k-1,i}. \tag{18}$$

Here, the state $\hat{x}_{k+N|k-1,i}$ does not equal to the result of substituting $u_{i,k+N-1|k-1}$ into (6). It is because that $\hat{x}_{k+N|k-1,i}$ is only a middle variable used in the proof of feasibility, and do not impact on the optimization problem and stability.

In this section, Lemma 1 identifies that $\hat{x}_{k+N|k-1,i} \in \Omega(\alpha\varepsilon')$. Then Lemma 2 identifies sufficient conditions to ensure that $\|x^{\rm f}_{k+l|k} - \hat{x}_{k+l|k,i}\|_P \leqslant \gamma\kappa\alpha\varepsilon$, for every $i \in \mathcal{P}$. Lemma 3 establishes the control constraint feasibility. Finally, Theorem 1 combines the results in Lemmas 1–4 to arrive at the conclusion that, for any $i \in \mathcal{P}$, the control $u^{\rm f}_{k+l|k,i}$ is a feasible solution to Problem 1 at any update $k \geqslant 1$.

Lemma 1 If the condition (13) is satisfied, $\forall i \in \mathcal{P}$, it has

$$\hat{x}_{k+N|k-1,i} \in \Omega(\alpha\varepsilon'), \tag{19}$$

provided that $\hat{x}_{k+N-1|k-1,i} \in \Omega(\alpha\varepsilon)$ and $\varepsilon' = (1-\kappa)\varepsilon$.

Proof Consider that $\Omega(\varepsilon)$ is the ε–level set of the Lyapunov function of the closed-loop dynamics $x_{k+1} = A_{\rm c}x_k$. Therefore, under the condition (13), it has $||x_{k+N}||_P < (1-\kappa)||x_{k-N-1}||_P$, and the proof of Lemma 1 is completed.

Lemma 2 Suppose that Assumptions 1–2 hold and $x_{k_0} \in \mathcal{X}$, for any $k \geqslant 0$, if Problem 1 has a solution at every update time $0,\cdots,k-1$, then

$$\|x^{\mathrm{f}}_{k+l|k}-\hat{x}_{k+l|k,i}\|_P\leqslant\gamma\kappa\alpha\varepsilon,\tag{20}$$

where $0<\gamma<1$ is a design parameter, for $\forall i\in\mathcal{P},\ j\in\mathcal{P}_i$, and all $l\in\{1,\cdots,N\}$. In addition, $u^{\mathrm{f}}_{i,k+l-1|k}$, $l=1,2,\cdots,N-1$ satisfies the constraint (9).

Proof First we will prove (20), provided there is a solution at update time $0,1,2,\cdots,k-1$. Substitute (14) into (15) and consider that $x_k=Ax_{k-1}+\sum\limits_{i\in\mathcal{P}}\bar{B}_iu_{i,k-1|k-1}$, for any $l=1,2,\cdots,N-1$, the feasible state is given by

$$x^{\mathrm{f}}_{k+l|k}=A^{l+1}x_{k-1}+\sum_{h=0}^{l}A^{l-h}\bar{B}_iu_{i,k+h-1|k-1}+\sum_{j\in\mathcal{P}_i}\sum_{h=0}^{l}A^{l-h}\bar{B}_ju_{j,k+h-1|k-1}.\tag{21}$$

The state predicted at time $k-1$ is given by

$$\hat{x}_{k+l|k-1,i}=A^{l+1}x_{k-1}+\sum_{h=0}^{l}A^{l-h}\bar{B}_iu_{i,k+h-1|k-1}+\sum_{j\in\mathcal{P}_i}\sum_{h=0}^{l}A^{l-h}\bar{B}_ju_{j,k+h-1|k-2}.\tag{22}$$

Subtract (21) from (22), then discrepancy between the feasible state sequence and the state sequence predicted at time $k-1$ is obtained as

$$\|x^{\mathrm{f}}_{k+l|k}-\hat{x}_{k+l|k-1,i}\|_P=\|\sum_{h=0}^{l}\sum_{j\in\mathcal{P}_i}A^{l-h}\bar{B}_j(u_{j,k+h-1|k-1}-u_{j,k+h-1|k-2})\|_P.\tag{23}$$

Let $\mathcal{S}_r$ be the subsystem which maximizes

$$\sum_{h=0}^{l}\beta_{l-h}\|u_{i,k+h-1|k-1}-u_{i,k+h-1|k-2}\|_2,\ i\in\mathcal{P}.\tag{24}$$

Then, the following equation can be deduced from (23)

$$||x^{\mathrm{f}}_{k+l|k}-\hat{x}_{k+l|k-1,i}||_P\leqslant\sum_{h=0}^{l}\beta_{l-h}\|u_{r,k+h-1|k-1}-u_{r,k+h-1|k-2}\|_2.\tag{25}$$

Since there is a solution at update time $0,1,2,\cdots,k-1$,
$\forall i\in\mathcal{P}$ satisfied the constraint (9), for all $l=1,2,\cdots,N-1$, it has

$$\sum_{h=0}^{l}\beta_{l-h}\|u_{r,k+h-1|k-1}-u_{r,k+h-1|k-2}\|_2\leqslant\frac{\gamma\kappa\alpha\varepsilon}{(m-1)}.\tag{26}$$

Then, for $l=1,2,\cdots,N-1$, following equation can be deduced

$$\|x^{\mathrm{f}}_{k+l|k}-\hat{x}_{k+l|k-1,i}\|_P\leqslant\gamma\kappa\alpha\varepsilon.\tag{27}$$

Thus, (20) hold for all $l=1,2,\cdots,N-1$.

When $l=N$, from (17) and (18), it has

$$\begin{aligned}&\|x^{\mathrm{f}}_{k+N|k}-\hat{x}_{k+N|k-1,i}\|_P\leqslant\\&\lambda_{\max}(A_{\mathrm{c}}^{\mathrm{T}}A_{\mathrm{c}})\|x^{\mathrm{f}}_{k+N-1|k}-\hat{x}_{k+N-1|k-1,i}\|_P\leqslant\\&(1-\kappa)\gamma\kappa\alpha\varepsilon.\end{aligned}\tag{28}$$

Consequently, (20) hold for all $l=1,2,\cdots,N$.

In addition, from definition (14), it has $u^{\mathrm{f}}_{i,k+l-1|k}-u_{i,k+l-1|k-1}=0$. Thus $u^{\mathrm{f}}_{i,k+l-1|k}$ satisfied constraints (9) when $l=1,2,\cdots,N-1$. The proof is completed.

Lemma 3 Suppose that Assumptions 1–2 hold, $x_{k_0}\in\mathcal{X}$, and the conditions (13) and (20) are satisfied. For any $k\geqslant0$, if Problem 1 has a solution at every update time t, $t=0,\cdots,k-1$, then $u^{\mathrm{f}}_{i,k+l-1|k}\in\mathcal{U}$ for all $l=1,2,\cdots,N$, and for any $i\in\mathcal{P}$.

Proof Since Problem 1 has a feasible solution at $k-1$, and $u^{\mathrm{f}}_{i,k+l-1|k}=u_{i,k+l-1|k-1}$ for all $l\in\{1,\cdots,N-1\}$, it need only be shown that the $u^{\mathrm{f}}_{i,k+N-1|k}$ is in $\mathcal{U}$.

Since ε is chosen satisfy that $K_ix\in\mathcal{U}$ for all $i\in\mathcal{P}$ when $x\in\Omega(\varepsilon)$. Consequently, a sufficient condition for $u^{\mathrm{f}}_{i,k+N-1|k}$ is that $x^{\mathrm{f}}_{k+N-1|k}\in\Omega(\varepsilon)$.

In view of Lemma 1, Lemma 2 and $\alpha\leqslant0.5$, using the triangle inequality, it has

$$\begin{aligned}&\|x^{\mathrm{f}}_{k+N-1|k}\|_P\leqslant\\&\|x^{\mathrm{f}}_{k+N-1|k}-\hat{x}_{k+N-1|k-1}\|_P+\|\hat{x}_{k+N-1|k-1}\|_P\leqslant\\&\gamma\kappa\alpha\varepsilon+\alpha\varepsilon\leqslant\varepsilon,\end{aligned}\tag{29}$$

that is, $x^{\mathrm{f}}_{k+N-1|k}\in\Omega(\varepsilon)$, $\forall i\in\mathcal{P}$. This concludes the proof.

Lemma 4 Suppose that Assumptions 1 and 2 hold, $x_{k_0}\in\mathcal{X}$, and the conditions (13) and (20) are satisfied. For any $k\geqslant0$, if Problem 1 has a solution at every update time t, $t=0,\cdots,k-1$, then the terminal state constraint is satisfied, for any $i\in\mathcal{P}$.

Proof Since there is a solution for Problem 1 at updates $t=1,\cdots,k-1$, Lemmas 1–3 can be invoked. Using the triangle inequality, it has

$$\begin{aligned}&\|x^{\mathrm{f}}_{k+N|k}\|_P\leqslant\\&\|x^{\mathrm{f}}_{k+N|k}-\hat{x}_{k+N|k-1,i}\|_P+\|\hat{x}_{k+N|k-1,i}\|_P\leqslant\\&(1-\kappa)\gamma\kappa\alpha\varepsilon+(1-\kappa)\alpha\varepsilon\leqslant\alpha\varepsilon,\end{aligned}\tag{30}$$

for each $i\in\mathcal{P}$. This shows that the terminal state constraint is satisfied.

Theorem 1 Suppose that Assumptions 1 and 2 hold, $x_{k_0}\in\mathcal{X}$ and equations (9)–(11) are satisfied at k_0. Then, for every $i\in\mathcal{P}$, the control $u^{\mathrm{f}}_{i,\cdot|k}$ and state $x^{\mathrm{f}}_{\cdot|k}$, defined by (14)(15) and (17), is a feasible solution to Problem 1 at every update $k\geqslant1$.

Proof Suppose $u_{i,\cdot|t} = u^{\mathrm{f}}_{i,\cdot|t}$ is a feasible solution for $t = 1, \cdots, k-1$. Lemmas 1–4 can be invoked. The consistency constraints (9) are trivially satisfied, the feasibility of control constraint and the terminal state constraint is guaranteed, and the proof of Theorem 1 is completed.

4.2 Stability

By Algorithm 1, if $x_k \in \Omega(\varepsilon)$ for any $k \geqslant 0$, the terminal controllers take over and stabilize the system to the origin. Therefore, to present the asymptotically stability of the closed-loop system with proposed DMPC, it remains to show that if $x_{k_0} \in \mathcal{X}\backslash\Omega(\varepsilon)$, then by application of Algorithm 1, the closed-loop system (2) is driven to the set in finite time.

Define the non-negative function V_k for all system $\mathcal{S}$, $V_k = \sum\limits_{i=1}^{m} V_{k,i}$, and

$$V_{k,i} = \bar{J}_{i,k}. \tag{31}$$

Theorem 2 Suppose that Assumptions 1 and 2 hold, $x_{k_0} \in \mathcal{X}$, (9)–(11) are satisfied, and the following parametric conditions hold

$$\rho - \alpha(0.42 + ((N-1)\rho' + 1)\gamma\kappa) > 0, \tag{32}$$

where

$$\rho = \lambda_{\min}(P^{-\frac{1}{2}}QP^{-\frac{1}{2}})^{\frac{1}{2}}, \tag{33}$$

$$\rho' = \lambda_{\max}(P^{-\frac{1}{2}}QP^{-\frac{1}{2}})^{\frac{1}{2}}. \tag{34}$$

Then, by application of Algorithm 1, the closed-loop system (2) is asymptotically stabilized to the origin.

Proof We will show that for any $k \geqslant 0$, if $x_k \in \mathcal{X}\backslash\Omega(\varepsilon)$, then there exists a constant $\eta \in (0, \infty)$ such that $V_k \leqslant V_{k-1} - \eta$. Since the performance index of $\mathcal{S}_i$, $\forall i \in \mathcal{P}$, with the optimal solution of $u_i(\cdot|k)$ is not more than the performance index of $\mathcal{S}_i$ with the feasible solution of $u^{\mathrm{f}}_i(\cdot|k)$, it has

$$\begin{aligned}&V_{k,i} - V_{k-1,i} \leqslant \\ &-\|\hat{x}_{k-1|k-1,i}\|_Q - \|u_{i,k-1|k-1}\|_{R_i} + \\ &\sum_{l=0}^{N-2}(\|x^{\mathrm{f}}_{k+l|k}\|_Q + \|u^{\mathrm{f}}_{i,k+l|k}\|_{R_i}) + \\ &(\|x^{\mathrm{f}}_{k+N-1|k}\|_Q + \|u^{\mathrm{f}}_{i,k+N-1|k}\|_{R_i}) + \|x^{\mathrm{f}}_{k+N|k}\|_P - \\ &\sum_{l=0}^{N-2}(\|\hat{x}_{k+l|k-1,i}\|_Q + \|u_{i,k+l|k-1}\|_{R_i}) - \\ &\|\hat{x}_{k+N-1|k-1,i}\|_P.\end{aligned} \tag{35}$$

In order to describe the equation simply, we define

$$\rho = \lambda_{\min}(P^{-\frac{1}{2}}QP^{-\frac{1}{2}})^{\frac{1}{2}}, \tag{36}$$

$$\rho' = \lambda_{\max}(P^{-\frac{1}{2}}QP^{-\frac{1}{2}})^{\frac{1}{2}}. \tag{37}$$

Assuming $x_k \in \mathcal{X}\backslash\Omega(\varepsilon)$, that is $\|\hat{x}_{k-1|k-1,i}\|_P > \varepsilon$, by the definition of ρ, we can get $\|\hat{x}_{k-1|k-1,i}\|_Q \geqslant \rho\varepsilon$. By the definition of $u^{\mathrm{f}}_i(\cdot|k)$ by (14),

$$\begin{aligned}&V_{k,i} - V_{k-1,i} \leqslant \\ &-\rho\varepsilon + \sum_{l=0}^{N-2}(\|x^{\mathrm{f}}_{k+l|k}\|_Q) - \\ &\sum_{l=0}^{N-2}(\|\hat{x}_{k+l|k-1,i}\|_Q) + \\ &(\|x^{\mathrm{f}}_{k+N-1|k}\|_Q + \|u^{\mathrm{f}}_{i,k+N-1|k}\|_{R_i}) + \\ &\|x^{\mathrm{f}}_{k+N|k}\|_P - \|\hat{x}_{k+N-1|k-1,i}\|_P.\end{aligned} \tag{38}$$

Then substitute (20) and ρ' into (38)

$$\begin{aligned}&V_{k,i} - V_{k-1,i} \leqslant \\ &-\rho\varepsilon + \rho'(N-1)\gamma\kappa\alpha\varepsilon + \\ &\|x^{\mathrm{f}}_{k+N-1|k}\|_Q + \|u^{\mathrm{f}}_{i,k+N-1|k}\|_{R_i} + \\ &\|x^{\mathrm{f}}_{k+N|k}\|_P - \|\hat{x}_{k+N-1|k-1,i}\|_P.\end{aligned} \tag{39}$$

According to the Cauchy equation, it has

$$\begin{aligned}&\|x^{\mathrm{f}}_{k+N-1|k}\|_Q + \|u^{\mathrm{f}}_{i,k+N-1|k}\|_{R_i} + \|x^{\mathrm{f}}_{k+N|k}\|_P \leqslant \\ &2^{\frac{1}{2}}(\|x^{\mathrm{f}}_{k+N-1|k}\|^2_Q + \|u^{\mathrm{f}}_{i,k+N-1|k}\|^2_{R_i} + \\ &\|A_c x^{\mathrm{f}}_{k+N-1|k}\|^2_P)^{\frac{1}{2}}.\end{aligned} \tag{40}$$

Consider that $A^{\mathrm{T}}_{\mathrm{c}}PA_{\mathrm{c}} + (Q + K^{\mathrm{T}}RK) = P$, it has

$$\begin{aligned}&\|x^{\mathrm{f}}_{k+N-1|k}\|^2_Q + \|u^{\mathrm{f}}_{k+N-1|k}\|^2_R + \|A_{\mathrm{c}}x^{\mathrm{f}}_{k+N-1|k}\|^2_P = \\ &\|x^{\mathrm{f}}_{k+N-1|k}\|^2_P.\end{aligned} \tag{41}$$

Substituting (41) into (40) yields

$$\begin{aligned}&\|x^{\mathrm{f}}_{k+N-1|k}\|_Q + \|u^{\mathrm{f}}_{i,k+N-1|k}\|_{R_i} + \|x^{\mathrm{f}}_{k+N|k}\|_P \leqslant \\ &2^{\frac{1}{2}}\|x^{\mathrm{f}}_{k+N-1|k}\|_P.\end{aligned} \tag{42}$$

Considering that

$$\begin{aligned}&2^{\frac{1}{2}}||x^{\mathrm{f}}_{k+N-1|k}||_P - ||\hat{x}_{k+N-1|k-1,i}||_P \leqslant \\ &0.42||x^{\mathrm{f}}_{k+N-1|k}||_P + ||x^{\mathrm{f}}_{k+N-1|k}||_P - \\ &||\hat{x}_{k+N-1|k-1,i}||_P \leqslant \\ &0.42||x^{\mathrm{f}}_{k+N-1|k}||_P + ||x^{\mathrm{f}}_{k+N-1|k} - \\ &\hat{x}_{k+N-1|k-1,i}||_P \leqslant \\ &0.42\alpha\varepsilon + \gamma\kappa\alpha\varepsilon,\end{aligned} \tag{43}$$

and substituting (42)–(43) into (39) yields

$$\begin{aligned}&V_{k,i} - V_{k-1,i} \leqslant \\ &-\rho\varepsilon + (N-1)\rho'\gamma\kappa\alpha\varepsilon + 2^{\frac{1}{2}}\|x^{\mathrm{f}}_{k+N-1|k}\|_P - \\ &||\hat{x}_{k+N-1|k-1,i}||_P \leqslant \\ &-\rho\varepsilon + (N-1)\rho'\gamma\kappa\alpha\varepsilon + 0.42\alpha\varepsilon + \gamma\kappa\alpha\varepsilon = \\ &-\varepsilon(\rho - \alpha(0.42 + ((N-1)\rho' + 1)\gamma\kappa)).\end{aligned} \tag{44}$$

According to sufficient condition (32) in the theorem 2, which implies that

$$V_{k,i} - V_{k-1,i} < 0. \tag{45}$$

Thus, for any $k \geqslant 0$, if $x(k) \in \mathcal{X}\backslash\Omega(\varepsilon)$, there is a constant $\eta_i \in (0, \infty)$ such that $V_{k,i} \leqslant V_{k-1,i} - \eta_i$.

Further more, we have the inequality of $V_k \leqslant V_{k-1} - \eta$, where $\eta = \sum_{i=1}^{m} \eta_i$, since m is limited. From this inequality, it follows by contradiction that there exists a finite time k' such that $x(k') \in \Omega(\varepsilon)$. If this is not the case, the inequality implies $V_k \to -\infty$ as $k \to \infty$. However, $V_k \geqslant 0$, therefore, there exists a finite time k' such that $x(k') \in \Omega(\varepsilon)$. This concludes the proof.

Thus, provided an initially feasible solution could be found, subsequent feasibility of the algorithm is guaranteed at every update, and the resulting closed-loop system is asymptotically stable at the origin.

5 Simulation validation

In this section, the load-frequency control (LFC) problem in power networks is used to show effectiveness of LLC–DMPC. The purpose of LFC is to keep the power generation close to power consumption under consumption disturbances, such that the frequency is maintained close to a nominal frequency of typically 50 Hz or 60 Hz. Power systems are decomposed into subnetworks with consumption and generation capabilities. We consider a network divided into 5 subnetworks as shown in Fig. 2.

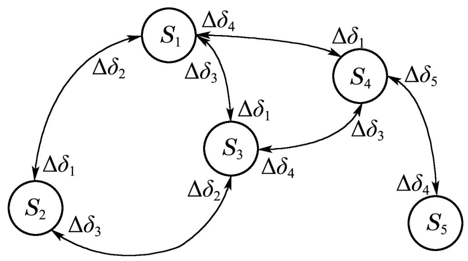

Fig. 2 The interaction relationship among subsystems

The simplified dynamics for the subnetwork models are considered, that do however include the basic elements of power injection, power consumption and power flow over power line, and that do show the basic characteristics of LFC problem. the continuous-time linearized dynamics of subnetwork i be described by the following second-order dynamics from [23].

$$\frac{\mathrm{d}\Delta\delta_i(t)}{\mathrm{d}t} = 2\pi\Delta f_i(t), \tag{46}$$

$$\frac{\mathrm{d}\Delta f_i(t)}{\mathrm{d}t} = -\frac{1}{\eta_{\mathrm{T},i}}\Delta f_i(t) + \frac{\eta_{\mathrm{K},i}}{\eta_{\mathrm{T},i}}\Delta P_{\mathrm{g},i}(t) - \frac{\eta_{\mathrm{K},i}}{\eta_{\mathrm{T},i}}\Delta P_{\mathrm{d},i}(t) + \frac{\eta_{\mathrm{K},i}}{\eta_{\mathrm{T},i}}\Big(\sum_{i\in\mathcal{P}_{+i}} \frac{\eta_{\mathrm{S},ij}}{2\pi}(\Delta\delta_j(t) - \Delta\delta_i(t))\Big), \tag{47}$$

where at time t, $\Delta\delta_i$ is the incremental phase angel deviation in rad, Δf_i is the incremental frequency deviation in Hz, $\Delta P_{\mathrm{g},i}$ is the incremental change in power generation in per unit, $\Delta P_{\mathrm{d},i}$ is a disturbance in the load in per unit; $\eta_{\mathrm{S},ij}$ is synchronizing coefficient of the line between subnetwork i and j, which value is given in Table 2.

Table 2 Parameters of the subnetworks, for $i \in \{1, \cdots, m\}$ and $j \in \mathcal{P}_{+i}$.

Constant	$\eta_{\mathrm{K},i}$	$\eta_{\mathrm{S},ij}$	$\eta_{\mathrm{S},ji}$	$\eta_{\mathrm{T},i}(s)$
value	120	0.5	0.5	20

For the purpose of comparison, the centralized MPC, LCO–DPMC and the proposed LLC–DMPC, as well as the cooperative DMPC[29] are all applied to this system. Set $\varepsilon = 0.1$, and set the control horizon of all the controllers to be $N = 10$. Set the initial presumed inputs and states, at time $k_0 = 0$, be zeros. In the centralized MPC, the local MPCs of the LCO–DMPC and cooperative DMPC, the dual mode strategy is adopted, and set the parameters, the initial states and the initial presumed inputs be the same as those used in the LLC–DMPC. Define the up and low bounds of the inputs to be 1 and -1, and up and low bounds of the input increments to be 0.2 and -0.2, respectively.

We simulate the network in MATLAB, and each local MPC is solved by ILOG CPLEX. The MATLAB solver of fmincon is also able to be used to solve each local MPC. When implementing this algorithm in Automation systems, to compile the MATLAB code can be an alternative choice if there is no solver to resolve Problem 1 on hand. When disturbances are injected in to subsystem $\mathcal{S}_1$, $\mathcal{S}_3$ and $\mathcal{S}_4$, the state responses and the inputs of the closed-loop systems under the control of the centralized MPC, cooperative DMPC, LLC–DMPC and LCO–DMPC are shown in Figs. 3–5. The shapes of the states response curves under the control of cooperative DMPC nearly equal to those under the centralized MPC. The performance of closed-loop system under the control of LLC–DMPC is very close to those under the control of centralized MPC and cooperative DMPC. Under the control of LCO–DMPC, the states of all subsystems could converge to set point, but there exists much larger variations comparing to those under the control of other three methods. The roots of the sum of square errors under the control of LCO–DMPC, LLC–DMPC, cooperative DMPC and the centralized MPC are 1.0646, 0.5171, 0.5998 and 0.4789, respectively. The total errors resulting from the LCO–DMPC is more than the twice of that resulting from the LLC–DMPC.

It can be seen from the simulations that the proposed constraint LLC–DMPC is able to steer the system states to the set point when disturbance exists if there is a feasible solution at the initial states, and the performance of the closed-loop system with LLC –DMPC is very similar to that with the centralized

MPC, and the communication and computation loads are smaller than the cooperative DMPC since each local MPC in proposed LLC–DMPC only communicates with each other and solves optimization problem once a control period.

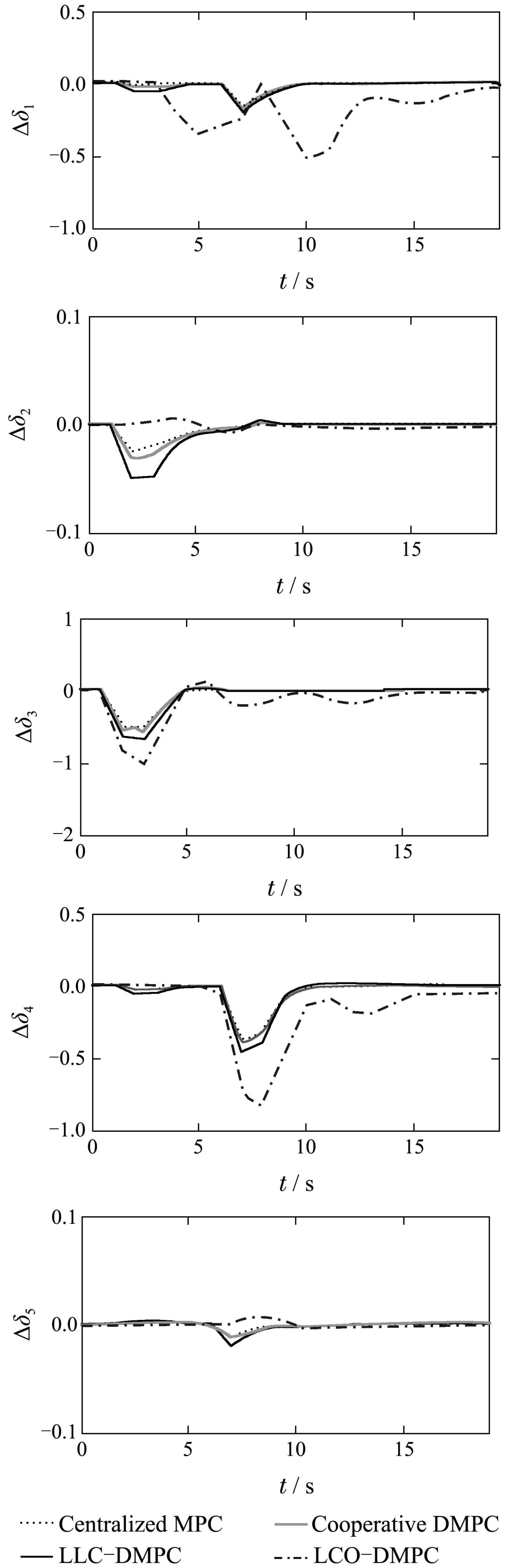

Fig. 3 The evolutions of $\Delta\delta_i,\ i \in \mathcal{P}$ under the control of centralized MPC, cooperative DMPC, LLC–DMPC and LCO–DMPC

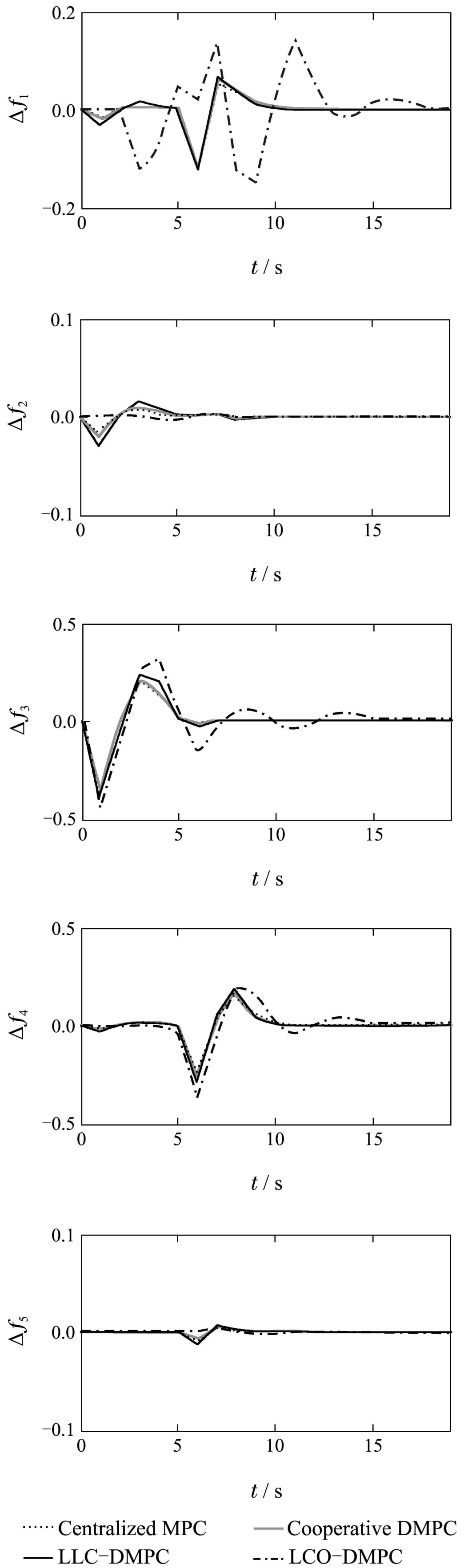

Fig. 4 The evolutions of $\Delta f_i,\ i \in \mathcal{P}$ under the control of centralized MPC, cooperative DMPC, LLC–DMPC and LCO–DMPC

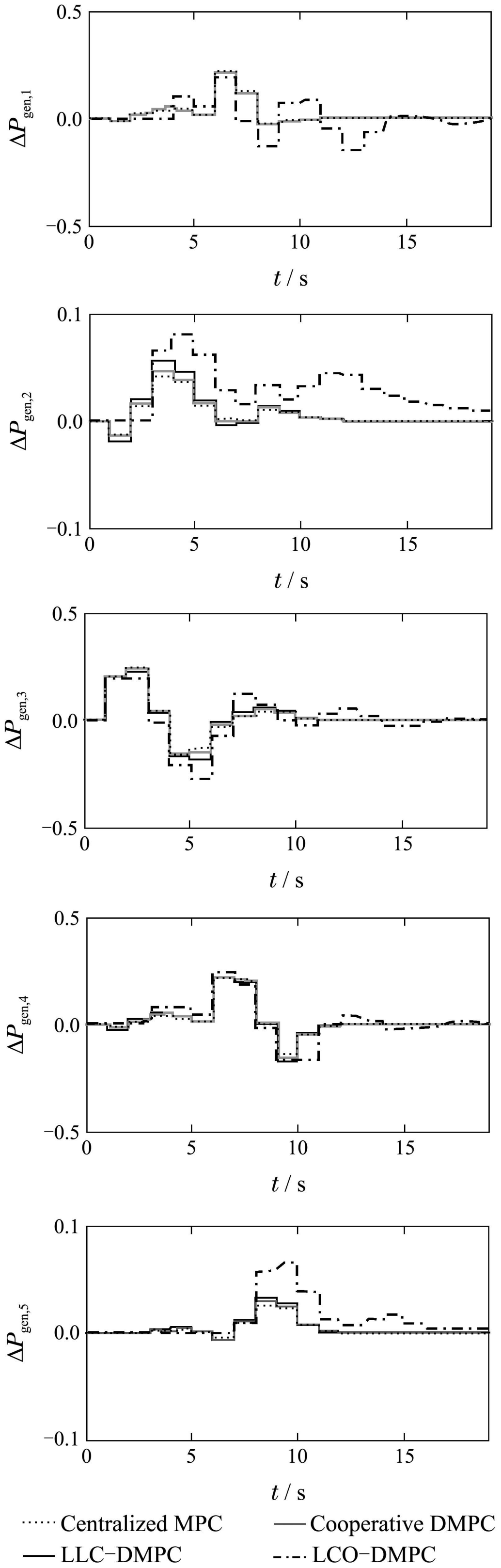

Fig. 5 The $\Delta P_{g,i},\ i \in \mathcal{P}$ under the control of centralized MPC, cooperative DMPC, LLC–DMPC and LCO–DMPC

6 Conclusions

In this paper, a stabilizing DMPC where each local MPC considers the performance of all subsystems and communicates with each other only once a sampling time, is developed for dynamically coupled spatially distributed systems subject to decoupled input constraints. If an initially feasible solution and a feed back control law $K_i x$ could be found, the subsequent feasibility of the algorithm is guaranteed at every update, and the resulting closed-loop system is asymptotically stable without any other additional assumptions. The simulations illustrate that the performance of global system under the control of proposed method is very close to that under the control of centralized MPC.

References:

[1] LING W, LIU D, LU Y, et al. IEC 61850 model expansion toward distributed fault localization, isolation, and supply restoration [J]. *IEEE Transactions on Power Delivery*, 2014, 29(3): 977 – 984.

[2] BRDYS M A. Integrated monitoring, control and security of critical infrastructure systems [J]. *Annual Reviews in Control*, 2014, 38(1): 47 – 70.

[3] CHRISTOFIDES P D, SCATTOLINI R, PENA D M D L, et al. Distributed model predictive control: a tutorial review and future research directions [J]. *Computers & Chemical Engineering*, 2013, 51(14): 21 – 41.

[4] QIN S, BADGWELL T. A survey of industrial model predictive control technology [J]. *Control Engineering Practice*, 2003, 11(7): 733 – 764.

[5] XU Q, LI Y. Micro-nanopositioning using model predictive output integral discrete sliding mode control [J]. *IEEE Transactions on Industrial Electronics*, 2012, 59(2): 1161 – 1170.

[6] LIM C S, RAHIM N, HEW W P, et al. Model predictive control of a two-motor drive with five-leg-inverter supply [J]. *IEEE Transactions on Industrial Electronics*, 2013, 60(1): 54 – 65.

[7] TARISCIOTTI L, ZANCHETTA P, WATSON A, et al. Modulated model predictive control for a seven-level cascaded H-bridge back-to-back converter [J]. *IEEE Transactions on Industrial Electronics*, 2014, 61(10): 5375 – 5383.

[8] HAN H, QIAO J. Nonlinear model-predictive control for industrial processes: an application to wastewater treatment process [J]. *IEEE Transactions on Industrial Electronics*, 2014, 61(4): 1970 – 1982.

[9] ZHENG Y, LI S, WANG X. Horizon-varying model predictive control for accelerated and controlled cooling process [J]. *IEEE Transactions on Industrial Electronics*, 2011, 58(1): 329 – 336.

[10] ZHENG Y, LI N, LI S. Hot-rolled strip laminar cooling process plant-wide temperature monitoring and control [J]. *Control Engineering Practice*, 2013, 21(1): 23 – 30.

[11] SCATTOLINI R. Architectures for distributed and hierarchical model predictive control-a review [J]. *Journal of Process Control*, 2009, 19(5): 723 – 731.

[12] VENKAT A, RAWLINGS J, WRIGHT S. Distributed model predictive control of large-scale systems [M] //*Assessment and Future Directions of Nonlinear Model Predictive Control*. Berlin: Springer, 2007: 591 – 605.

[13] WANG Q, GAO H, ALSAADI F, et al. An overview of consensus problems in constrained multi-agent coordination [J]. *Systems Science & Control Engineering: an Open Access Journal*, 2014, 2(1): 275 – 284.

[14] KIRUBAKARAN V, RADHAKRISHNAN T, SIVAKUMARAN N. Distributed multiparametric model predictive control design for a quadruple tank process [J]. *Measurement*, 2014, 47(1): 841 – 854.

[15] ZHANG L, WANG J, LI C. Distributed model predictive control for polytopic uncertain systems subject to actuator saturation [J]. *Journal of Process Control*, 2013, 23(8): 1075 – 1089.

[16] AL-GHERWI W, BUDMAN H, ELKAMEL A. A robust distributed model predictive control algorithm [J]. *Journal of Process Control*, 2011, 21(8): 1127 – 1137.

[17] LIU J, DAVID M D L P, CHRISTOFIDES P D. Distributed model predictive control of nonlinear systems subject to asynchronous and delayed measurements [J]. *Automatica*, 2010, 46(1): 52 – 61.

[18] CHENG R, FRASER J F, YIP W S. Dantzig–Wolfe decomposition and plant-wide MPC coordination [J]. *Computers & Chemical Engineering*, 2008, 32(7): 1507 – 1522.

[19] CAMPONOGARA E, SCHERER H F. Distributed optimization for model predictive control of linear dynamic networks with control-input and output constraints [J]. *IEEE Transactions on Automation Science and Engineering*, 2011, 8(1): 233 – 242.

[20] ZHENG Y, LI S, QIU H. Networked coordination-based distributed model predictive control for large-scale system [J]. *IEEE Transactions on Control Systems Technology*, 2013, 21(3): 991 – 998.

[21] NAMARA P M, NEGENBORN R R, SCHUTTER B D, et al. Weight optimisation for iterative distributed model predictive control applied to power networks [J]. *Engineering Applications of Artificial Intelligence*, 2013, 26(1): 532 – 543.

[22] AL-GHERWI W, BUDMAN H, ELKAMEL A. Selection of control structure for distributed model predictive control in the presence of model errors [J]. *Journal of Process Control*, 2010, 20(3): 270 – 284.

[23] CAMPONOGARA E, JIA D, KROGH B, et al. Distributed model predictive control [J]. *IEEE Control Systems Magazine*, 2002, 22(1): 44 – 52.

[24] DUNBAR W. Distributed receding horizon control of dynamically coupled nonlinear systems [J]. *IEEE Transactions on Automatic Control*, 2007, 52(7): 1249 – 1263.

[25] FARINA M, SCATTOLINI R. Distributed predictive control: a non-cooperative algorithm with neighbor-to-neighbor communication for linear systems [J]. *Automatica*, 2012, 48(6): 1088 – 1096.

[26] LI S, ZHANG Y, ZHU Q. Nash-optimization enhanced distributed model predictive control applied to the shell benchmark problem [J]. *Information Sciences*, 2005, 170(2/4): 329 – 349.

[27] ZHENG Y, LI S, WANG X. Distributed model predictive control for plant-wide hot-rolled strip laminar cooling process [J]. *Journal of Process Control*, 2009, 19(9): 1427 – 1437.

[28] ZHENG Y, LI S, LI N. Distributed model predictive control over network information exchange for large-scale systems [J]. *Control Engineering Practice*, 2011, 19: 757 – 769.

[29] STEWART B T, VENKAT A N, RAWLINGS J B, et al. Cooperative distributed model predictive control [J]. *Systems & Control Letters*, 2010, 59(8): 460 – 469.

[30] DOAN M D, KEVICZKY T, SCHUTTER B D. An iterative scheme for distributed model predictive control using Fenchel's duality [J]. *Journal of Process Control*, 2011, 21(5): 746 – 755.

[31] SCHEU H, MARQUARDT W. Sensitivity-based coordination in distributed model predictive control [J]. *Journal of Process Control*, 2011, 21(5): 715 – 728.

[32] NEGENBORN R R, SCHUTTER B D, HELLENDOORN J. Multi-agent model predictive control for transportation networks: serial versus parallel schemes [J]. *Engineering Applications of Artificial Intelligence*, 2008, 21(3): 353 – 366.

[33] LEE J, KWON W H, CHOI J. On stability of constrained receding horizon control with finite terminal weighting matrix [J]. *Automatica*, 1998, 34(12): 1607 – 1612.

[34] MICHALSKA H, MAYNE D. Robust receding horizon control of constrained nonlinear systems [J]. *IEEE Transactions on Automatic Control*, 1993, 38(11): 1623 – 1633.

[35] MAYNE D, RAWLINGS J, RAO C, et al. Constrained model predictive control: stability and optimality [J]. *Automatica*, 2000, 36(6): 789 – 814.

作者简介:

郑 毅 (1978–), 男, 副教授, 目前研究方向为综合能源网系统建模控制与优化及分布式预测控制, E-mail: yizheng@sjtu.edu.cn;

李少远 (1965–), 男, 教授, 目前研究方向为预测控制、自适应智能控制、模糊智能控制, E-mail: syli@sjtu.edu.cn;

魏永松 (1986–), 男, 博士研究生, 目前研究方向为分布式预测控制, E-mail: yswei@sjtu.edu.cn.