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非线性末制导系统参数灵敏度分析

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摘要: 末制导系统参数随着飞行环境及飞行条件的改变而存在摄动, 针对这一问题本文提出根据动态灵敏度来分析参数摄动对脱靶量的影响. 基于伴随法推导出与系统动态方程相同规模的伴随方程, 并通过一次伴随求解计算得到脱靶量对所有可调参数及摄动参数的动态灵敏度, 有效的提高了计算效率. 传统的直接分析法是将系统状态变量直接对参数变量进行微分, 需要对每个参数变量求解一组代数或微分方程, 对于状态变量及参数变量较多的情况效率较低. 本文基于两种方法对末制导系统的参数灵敏度进行分析, 分析结果揭示了参数摄动对脱靶量的影响程度, 较小的参数灵敏度为提高系统的鲁棒性提供了依据.

关键词: 制导系统; 脱靶量; 灵敏度分析; 伴随法

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Parameter sensitivity analysis for nonlinear terminal guidance system

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Abstract: This paper analyzes the parameter robustness for a homing terminal guidance system (TGS). The parameter robustness is reflected in assessing the miss distance performance influenced by the parameter perturbation which is described as the miss distance sensitivity with respect to the parameter. An efficient numerical method for sensitivity computation of nonlinear TGS is developed based on the adjoint method, which consists of both forward integration of the TGS and backward integration of the adjoint equation. Based on adjoint method, the sensitivity analysis of the TGS against various scenarios of target maneuvers is conducted. Analysis results are examined with the direct sensitivity analysis method, which reveal the perfect accuracy of the adjoint method. Comparing to direct method, adjoint method provides the miss distance sensitivity with respect to all parameters in a single simulation. It reveals great advantage in the calculation efficiency regarding integral index functions. By the parameter robustness analysis, with adjoint method, parameters with minimum sensitivities can be obtained to ensure the robustness of TGS.

Key words: guidance system; miss distance; sensitivity analysis; adjoint method

1 Introduction

For the performance analysis of a homing guidance loop, the miss distance is one of the key indicators of success or failure of the interceptor's mission. Parameters of a guided missile, the target maneuver and the measurement are the main miss distance influences[1]. Being such an important factor in determining the performance of a guided loop, the parameters of the guided loop are considered as constants both from the point of view of performance analysis as from the point of view of guidance loop design almost all of the research[2–3]. However, these parameters are not exactly known most of the time, due to perturbations of the terminal guidance system (TGS) objects, such as the overloading settling time and the time constant of the seeker are likely to change, as flight conditions change. These changes of the parameters have great effect on missile's control and guide precision[4]. Thus, there is a need for parameters sensitivity analysis of the TGS models. The sensitivity also reveals the influence of design parameters' changes on TGS dynamic performance[5–9]. Thus, this paper presents the study of the miss distance sensitivity with respect to the parameters. Existing studies about the performance analysis are based on linear time vary-

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ing system under the assumption that the approaching velocity between the flight vehicle and the target vehicle is constant, it is just suitable for the near head-on or tail-chase case[3, 10–11]. However, TGS is a complex nonlinear system, especially when the target with large angle maneuvers or the intercept angle is demanded; the linear time varying model can not describe the TGS precisely[2]. Thus, this paper mainly studies the miss distance sensitivity with respect to parameters based on the nonlinear model of a guide missile system.

There are three methods to solve the sensitivity problem, such as the finite difference method, the direct analysis and the adjoint methods. The finite difference method has two shortcomings, low accuracy of calculation and more computational expense, which is in proportion to the number of design variables[11]. In order to analyze the parameters sensitivity performance of a guide missile system, the direct differentiation method that differentiates a state variable with respect to parameters and solves the simultaneous equation directly is used. In [4–5, 12] the direct sensitivity system for differential-algebraic equation systems (DAEs) of index is derived. The parameters sensitivity of a guide missile performance can also be obtained by adjoint methods which is an alternative way to obtain the sensitivity. If the sensitivities with respect to a larger number of parameters need to be solved, meanwhile, the number of state variables is also very large, the direct sensitivity approach then will be intractable. However, the adjoint sensitivity analysis approach gives the information through solving adjoint equations once without many procedures analysis. In [4, 13] the adjoint sensitivity system for DAEs of index is derived and some of its fundamental properties are investigated. Thus, the direct sensitivity analysis approach and the adjoint variable sensitivity analysis approach are all considered to solve the sensitivity problems in this paper. And in the following sections, some of the issues for the numerical solution and the effectiveness of the adjoint method are addressed. To compare with these two methods, their own advantages and disadvantages are also given in the end of the paper.

The outline of this study is as follows. In Section 2 the nonlinear model of TGS and performance index are established. In Section 3 the sensitivities are evaluated accurately. In Section 4, the numerical results of miss distance sensitivity with respect to the parameters between two methods are obtained. Conclusions and perspective for future work are given in Section 5.

2 Models of the terminal guidance system

The homing guidance systems are considered in this paper. This section briefly describes the basic subsystems of a missile's guidance system. A typical guidance system contains three main parts: the seeker, the guidance law and the autopilot system[14–15]. The principal frame of the missile guidance system is shown in Fig.1.

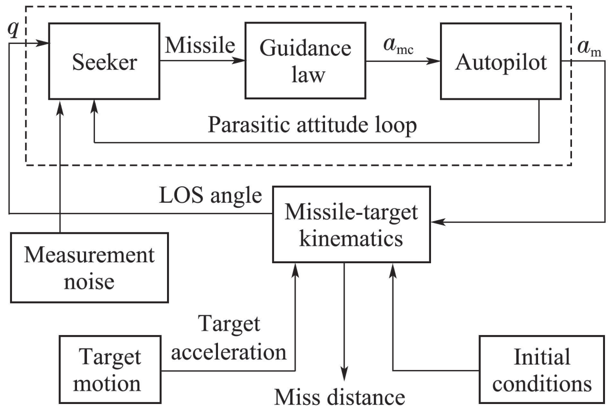

Fig. 1 Major subsystems of a missile guidance system

The missile-target kinematics determines the relative geometry of the intercept, and provides the line-of-sight (LOS) angle for the seeker. The seeker provides the measurement of target motion required to mechanize the guidance law, and measures the LOS rate and the closing velocity. For a missile, the inputs are target location and the measurement noises. The missile is guided by a certain guidance law to track the target. In this paper, the guidance law takes the LOS rate information and produces guidance acceleration commands for the autopilot system. The autopilot system is a closed-loop system inside the main guidance subsystem that ensures the missile achieves accelerations as commands and maintains stability. It takes these commands and produces achieved acceleration and angular rates.

Consider a missile-target engagement under the following assumptions: the force due to gravity is ignored, the missile and the target are assumed as mass particles, the engagement is confined to the line-of-sight (LOS) plane. The geometry of the engagement is depicted in Fig.2.

In Fig.2, R represents the relative range between the flight vehicle and the target along the LOS, γ represents the trajectory inclination angle, q represents the LOS angle, a_m and a_t represent the achieved missile acceleration and the target acceleration normal to the LOS, v_m and v_t represent the velocity of missile and target.

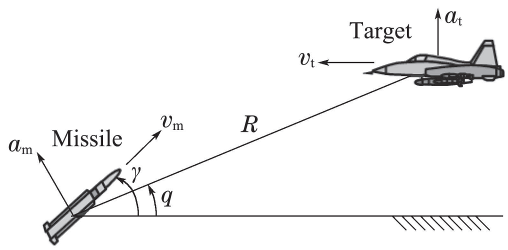

Fig. 2 Missile-target intercept geometry

The relative kinematics relationship between the missile and the target in the LOS coordinate can be described as

$$\begin{cases} \ddot{R} = R\dot{q}^2, \\ \ddot{q} = -2\dfrac{\dot{R}}{R}\dot{q} + \dfrac{a_t - a_m}{R}, \end{cases} \tag{1}$$

where $\dot{q}$ represents the LOS angle rate, $\dot{R}$ represents the missile-target closing velocity.

For simplicity, the dynamic of the autopilot system and the seeker are assumed as the following first order system.

$$\tilde{\dot{q}} = \frac{1}{T_s s+1}\dot{q},\ a_m = \frac{1}{Ts+1}a_{mc}, \tag{2}$$

where a_{mc} denotes the commanded missile lateral acceleration which is produced by the guidance law system, $\tilde{\dot{q}}$ denotes the LOS angle rate measured by the seeker, T_s denotes the time constant of the seeker, T denotes the overload settling time reflecting the closed-loop system dynamic of the flight vehicle and its autopilot. These two time constants parameters are two of several factors which effect the accuracy of the missile, and they are not exactly known. Thus these two parameters have perturbations for the guidance system.

Augmented proportional navigation guidance law is a proportional navigation with an extra term to account for the maneuvering target, it is given by

$$a_{mc} = -N\dot{R}\tilde{\dot{q}} + K\tilde{a}_t, \tag{3}$$

where N represents the effective navigation ratio. K represents the modified coefficient to compensation the target maneuver, when $K = 0$, this guidance law is the true proportional navigation (TPN). The value of target maneuver cannot be exactly measured. Therefore, target maneuver estimating is required. Supposing that the estimation formula is

$$\tilde{a}_t = \frac{1}{\tau s+1}a_t, \tag{4}$$

where $\tilde{a}_t$ is the estimation value of the real target maneuver a_t, τ is the time constant of the estimation process which exists perturbation.

Combining Eqs.(1)–(4), suppose the state vector is $x = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6]^T = [R\ \dot{R}\ \dot{q}\ \tilde{\dot{q}}\ a_m\ a_t]^T$, then the state space description of TGS is obtained.

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_1x_3^2, \\ \dot{x}_3 = -\dfrac{2x_2x_3}{x_1} - \dfrac{x_5}{x_1} + \dfrac{a_t}{x_1}, \\ \dot{x}_4 = -\dfrac{1}{T_s}x_4 + \dfrac{1}{T_s}x_3, \\ \dot{x}_5 = -\dfrac{1}{T}x_5 - \dfrac{N}{T}x_2x_4 + \dfrac{K}{T}x_6, \\ \dot{x}_6 = -\dfrac{1}{\tau}x_6 + \dfrac{1}{\tau}a_t. \end{cases} \tag{5}$$

For simplicity, the missile DAEs description depending on parameters is

$$\begin{cases} F(\dot{x}, x, t, \alpha) = 0, \\ x(0) = x_0, \end{cases} \tag{6}$$

or

$$\begin{cases} \dot{x} = f(x, t, \alpha), \\ x(0) = x_0, \end{cases} \tag{7}$$

where $\alpha = [T\ \ NT_s\tau]^T$ is the parameter vector. N is the design parameter, T, T_s and τ are the system parameters. The general approach in studying the performance of guidance loops affected by parameters in all previous studies is to assume that parameters have fixed values. This paper studies the performance estimation problem when there are parameter perturbations in the missile system.

In this paper, the index function is given below

$$G'(x, \alpha, t) = \int_0^t g(x, t, \alpha)\mathrm{d}t + S(x, t, \alpha)|_0, \tag{8}$$

where $g(x, t, \alpha) = x_2$, $S(x, t, \alpha)|_0 = x_1|_0$. The point of closest approach of the missile and target is known as the miss distance. The definition of miss distance is

$$\mathrm{ZEM} = R_{\min} = R|_{t_e}, \tag{9}$$

where t_e is the terminal time when the approach of the missile and target is closest. Then the index function is the miss distance of the TGS. Thus, the miss distance index is

$$G(x, \alpha, t_e) = \int_0^{t_e} g(x, t, \alpha)\mathrm{d}t + S(x, t, \alpha)|_0. \tag{10}$$

The parameters sensitivities problems take the following form: find $\dfrac{\mathrm{d}G}{\mathrm{d}\alpha_i}$ at terminal time t_e, for $i = 1, \cdots, m$. The solution requires the simultaneous solution of the original DAE system with the m sensitivity systems obtained by differentiating the original DAE with respect to each parameter in turn. For large systems this may look like a lot of work, if m is relatively small, by exploiting the fact that the sensitivity systems are linear and all share the same Jacobian matrices with the original system.

Some problems require the sensitivities with respect to a large number of parameters. For these problems, particularly if the number of state variables is also very large, the direct sensitivity approach is intractable. These problems can often be handled more efficiently by the adjoint method[16]. In the adjoint approach, we are interested in calculating the sensitivity $\frac{\mathrm{d}G}{\mathrm{d}\alpha}$ of the index function denoted by Eq.(10). The cost in computing the sensitivity $\frac{\mathrm{d}G}{\mathrm{d}\alpha}$ via the adjoint approach is the calculation of the intermediate adjoint variable. While the direct sensitivity analysis approach is best suited to investigate sensitivities with respect to a small number of parameters, the adjoint sensitivities analysis approach is best suited to find the sensitivity with respect to a large number of parameters.

The purpose of the sensitivity analysis for TGS is

1) The effects of parameter perturbations on the system state variables and the output can be obtained. Decide which parameters are more sensitive to the state variables and the output of TGS.

2) As the future application of the sensitivity function, in the design demand, find a set of parameters corresponding to the minimum sensitivity function based on the relationship between the sensitivity and the parameters. So that under the consideration of design requirements, the system well robust to the variation of parameters.

3 The algorithms of performance index

To analyze the miss distance sensitivity with respect to parameters, the direct method and the adjoint method these two methods are given to solve this sensitivity problem in this section.

3.1 Direct sensitivity analysis

He sensitivity problem usually takes the form: find $\frac{\mathrm{d}G}{\mathrm{d}\alpha}$, where α is a vector of parameters. First, the state variable sensitivity with respect to α is

$$\beta_i = (\frac{\partial x}{\partial \alpha})_n,\ i = 1, 2, \cdots, n, \tag{11}$$

where $\beta_1 = \frac{\partial x}{\partial T}, \beta_2 = \frac{\partial x}{\partial N}, \beta_3 = \frac{\partial x}{\partial T_\mathrm{s}}, \beta_4 = \frac{\partial x}{\partial \tau}$.

The sensitivity differential equation of TGS nonlinear description and the definition of sensitivity is given by the[17].

$$\dot{\beta}_i = (\frac{\partial f}{\partial x_i}) \cdot \beta_i + (\frac{\partial f}{\partial \alpha}),\ i = 1, \cdots, n, \tag{12}$$

where $(\frac{\partial f}{\partial x_i})$ is the Jacobian matrix.

The initial condition of sensitivity β is

$$\beta(t_0) = \frac{\partial x_0}{\partial \alpha} - \dot{x}_0 \frac{\partial t_0}{\partial \alpha}, \tag{13}$$

where x_0 is the initial condition of system (5).

Thus, the miss distance sensitivity with respect to model parameter α is

$$\begin{aligned}\frac{\mathrm{d}G}{\mathrm{d}\alpha} = &\int_0^{t_\mathrm{e}} (g_x x_\alpha + g_\alpha)\mathrm{d}t + (S_x x_\alpha + S_\alpha)|_0 = \\ &\int_0^{t_\mathrm{e}} (g_x \beta_2 + g_\alpha)\mathrm{d}t + (S_x \beta_1 + S_\alpha)|_0 = \\ &\beta_1(t_\mathrm{e}).\end{aligned} \tag{14}$$

Combining Eq.(5), Eq.(12) and the sensitivity differential equation (14), the miss distance sensitivity with respect to α will be obtained.

By solving the nonlinear differential equation of TGS and the sensitivity differential equation, the miss distance sensitivity with respect to parameter α can be obtained. Two approaches can be taken for this direct method, namely, discrete approach and continuous approach. The discrete approach approximates the index function by a discrete nonlinear system and then differentiates the discrete system with respect to the parameters[12]. The continuous approach differentiates the index function with respect to the parameters first and then discretizes the sensitivity to compute the approximate sensitivities[17]. This continuous approach is much simpler than that from the discrete approach.

The direct method has disadvantage of heavy computing burden for large systems, thus, the direct sensitivity analysis method has been recognized as a costly approach.

3.2 Adjoint sensitivity analysis

For convenience, this section presents a short account of the adjoint method. Some problems require the sensitivity with respect to a large number of parameters. For these problems, if the system is very complex, the direct sensitivity approach is intractable. These problems can often be solved more efficiently by the adjoint method[16, 18]. Thus, we have interested in calculating the sensitivity of the miss distance index function denoted by Eq.(10) with respect to α by this approach. The sensitivity of $G(x, \alpha)$ with respect to the parameter

α is given by Eq.(14). To derive the adjoint sensitivity, we introduce the adjoint variable λ. First, a theorem to solve the adjoint sensitivity analysis problem is given.

Theorem 1 The miss distance sensitivity with respect to the parameter α is described as the Eq.(14). If there is existing the adjoint system

$$\begin{cases} -g_x + \lambda^* F_x - (\lambda^* F_{\dot{x}})^{\mathrm{T}} = 0, \\ \lambda(t_{\mathrm{e}}) = 0, \end{cases} \tag{15}$$

then the sensitivity of TGS index function can be described as

$$\frac{\mathrm{d}G}{\mathrm{d}\alpha} = \int_0^{t_{\mathrm{e}}} (g_\alpha - \lambda^* F_\alpha)\mathrm{d}t. \tag{16}$$

Proof The adjoint variable $\lambda \in \mathbb{R}^n$ is introduced. The differential algebraic equation multiply the transpose the adjoint variable, integral it in the interval $[0, t_{\mathrm{e}}]$, then the description is

$$\int_0^{t_{\mathrm{e}}} \lambda^* F(\dot{x}, x, \alpha)\mathrm{d}t = 0. \tag{17}$$

Since $F(\dot{x}, x, \alpha) = 0$, and $\int_0^{t_{\mathrm{e}}} \lambda^* F(\dot{x}, x, \alpha)\mathrm{d}t = 0$. Eq.(10) minus Eq.(17), the derived function satisfies

$$I(x, \alpha) = G(x, \alpha) - \int_0^{t_{\mathrm{e}}} \lambda^* F(\dot{x}, x, \alpha)\mathrm{d}t. \tag{18}$$

Thus, the sensitivity of index function $G(x, \alpha)$ with respect to parameter α can be obtained by the sensitivity of $I(x, \alpha)$ with respect to parameter α. We have the following form of the sensitivity equations.

$$\begin{aligned} \frac{\mathrm{d}I}{\mathrm{d}\alpha} = &\int_0^{t_{\mathrm{e}}} (g_x x_\alpha + g_\alpha)\mathrm{d}t + (S_x x_\alpha + S_\alpha)|_0 - \\ &\int_0^{t_{\mathrm{e}}} \lambda^* (F_{\dot{x}} \dot{x}_\alpha + F_x x_\alpha + F_\alpha)\mathrm{d}t, \end{aligned} \tag{19}$$

where subscripts $\dot{x}$, x and α on functions such as g, F and S are used to denote partial derivatives. The subscript α on state variables such as $\dot{x}_\alpha$, x_α are used to denote partial derivatives to α.

By integration by parts, we have

$$\begin{aligned} &\int_0^{t_{\mathrm{e}}} \lambda^* F_{\dot{x}} \dot{x}_\alpha \mathrm{d}t = \\ &\lambda^* F_{\dot{x}} x_\alpha|_0^{t_{\mathrm{e}}} - \int_0^{t_{\mathrm{e}}} (\lambda^* F_{\dot{x}})^{\mathrm{T}} x_\alpha \mathrm{d}t. \end{aligned} \tag{20}$$

Thus, the sensitivity equation of Eq.(19) becomes

$$\begin{aligned} &\frac{\mathrm{d}G}{\mathrm{d}\alpha} = \frac{\mathrm{d}I}{\mathrm{d}\alpha} = \\ &\int_0^{t_{\mathrm{e}}} (g_\alpha - \lambda^* F_\alpha)\mathrm{d}t - \int_0^{t_{\mathrm{e}}} [-g_x + \\ &\lambda^* F_x - (\lambda^* F_{\dot{x}})^{\mathrm{T}}] x_\alpha \mathrm{d}t - \lambda^* F_{\dot{x}} x_\alpha|_0^{t_{\mathrm{e}}}. \end{aligned} \tag{21}$$

Define the adjoint system is

$$\begin{cases} -g_x + \lambda^* F_x - (\lambda^* F_{\dot{x}})^{\mathrm{T}} = 0, \\ \lambda(t_{\mathrm{e}}) = 0. \end{cases} \tag{22}$$

Thus, the sensitivity of TGS can be described as

$$\frac{\mathrm{d}G}{\mathrm{d}\alpha} = \int_0^{t_{\mathrm{e}}} (g_\alpha - \lambda^* F_\alpha)\mathrm{d}t. \tag{23}$$

Prove up.

By Theorem 1, a is derivated to solve the adjoint sensitivity analysis of the terminal guidance system for simple.

Corollary 1 If the adjoint system is Eq.(14). Then the sensitivity $\frac{\mathrm{d}G}{\mathrm{d}\alpha}$ of TGS subscribes as the Eq.(5) is the initial value $-\xi^*(0)$ of the following description:

$$\begin{cases} \dot{\xi} = g_\alpha^* - F_\alpha^* \lambda, \\ \xi(t_{\mathrm{e}}) = 0. \end{cases} \tag{24}$$

Proof In order to avoid saving intermediate λ values just for the evaluation of the integral in Eq.(23), we extend the backward problem with the following equations. Based on Theorem 1 and the sensitivity description Eq.(14) of TGS, introducing the adjoint variable ξ, supposing

$$\begin{cases} \dot{\xi} = g_\alpha^* - F_\alpha^* \lambda, \\ \xi(t_{\mathrm{e}}) = 0. \end{cases} \tag{25}$$

Integrating the adjoint system description of Eq.(25) once yields

$$\xi(t_{\mathrm{e}}) - \xi(0) = \int_0^{t_{\mathrm{e}}} (g_\alpha^* - F_\alpha^* \lambda)\mathrm{d}t. \tag{26}$$

For $\xi(t_{\mathrm{e}}) = 0$, the Eq.(26) simplified as

$$\xi(0) = -\int_0^{t_{\mathrm{e}}} (g_\alpha^* - F_\alpha^* \lambda)\mathrm{d}t. \tag{27}$$

By Eq.(23)

$$(\frac{\mathrm{d}G}{\mathrm{d}\alpha})^* = \int_0^{t_{\mathrm{e}}} (g_\alpha^* - F_\alpha^* \lambda)\mathrm{d}t. \tag{28}$$

Similarly, the value of $\frac{\mathrm{d}G}{\mathrm{d}\alpha}$ in Eq.(21) can be obtained as $\frac{\mathrm{d}G}{\mathrm{d}\alpha} = -\xi^*(0)$.

Prove up.

The adjoint sensitivity analysis method consists of three major steps. First, the original nonlinear missile system forward to a specific output time t_{e} is solved. Second, the consistent initial conditions for the adjoint system are computed at time t_{e}, and the consistent initial conditions must satisfy the boundary conditions. Finally, the adjoint system backward to the start point is solved and sensitivities are calculated.

The adjoint state equation has to be solved only once to obtain the sensitivity with respect to parameters.

4 Simulations

Recalling the system Eq.(5), the solution of miss distance sensitivity with respect to α is computed by the direct sensitivity analysis approach and the adjoint method in this section. And some simulation results are presented as well.

We consider that the homing guidance system is guided by the augmented proportional navigation guidance law. The parameters of a typical missile terminal guidance loop and the control system at the terminal phase are presented in Tabel 1. The initial simulation conditions are presented in Table 2.

Table 1 The parameters of TGS

Sign	Value	Description
N	3.5	The effective navigation ratio
K	0.5	The correction factor
T	0.3	The time constant of autopilot
T_s	0.03	The time constant of seeker
τ	0.3	The time constant of target estimation

Table 2 The initial condition of TGS

Symbol	Value	Unit
x_{10}	8000	m
x_{20}	−1500	m/s
x_{30}	0.02	rad/s
x_{40}	0	rad/s
x_{50}	0	g
x_{60}	0	g

The Jacobian matrix of Eq.(5) is

$$\frac{\partial f}{\partial x}=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ x_3^2 & 0 & 2x_1x_3 & 0 & 0 & 0 \\ \dfrac{2x_2x_3+x_5-a_t}{x_1^2} & -\dfrac{2x_3}{x_1} & -\dfrac{2x_2}{x_1} & 0 & -\dfrac{1}{x_1} & 0 \\ 0 & 0 & \dfrac{1}{T_s} & -\dfrac{1}{T_s} & 0 & 0 \\ 0 & -\dfrac{Nx_4}{T} & 0 & -\dfrac{Nx_2}{T} & -\dfrac{1}{T} & \dfrac{K}{T} \\ 0 & 0 & 0 & 0 & 0 & -\dfrac{1}{\tau} \end{bmatrix}. \tag{29}$$

Differentiate Eq.(5) with respect to parameter variables α is

$$\frac{\partial f}{\partial \alpha}=\begin{bmatrix} 0 & 0 & 0 & 0 & \dfrac{x_5+Nx_4x_2-Kx_6}{T^2} & 0 \\ 0 & 0 & 0 & \dfrac{x_4-x_3}{T_s^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\dfrac{x_2x_4}{T} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{x_6-a_t}{\tau^2} \end{bmatrix}. \tag{30}$$

The index function with respect to parameter vector α and state variables x are

$$\frac{\partial g}{\partial \alpha}=g_\alpha=[0]_{3\times5},\ \frac{\partial g}{\partial x}=g_x=[0\ \ 1\ \ 0\ \ 0\ \ 0].$$

The results of sensitivity we interested are shown in the following Figs.3–6.

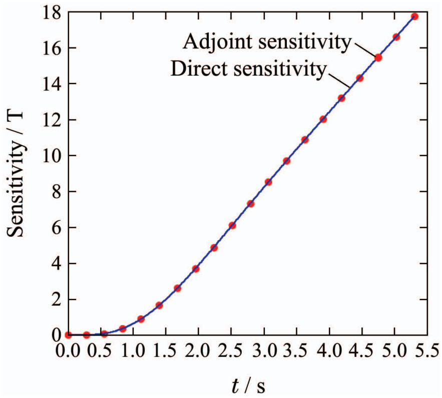

Fig. 3 Miss distance sensitivity to T

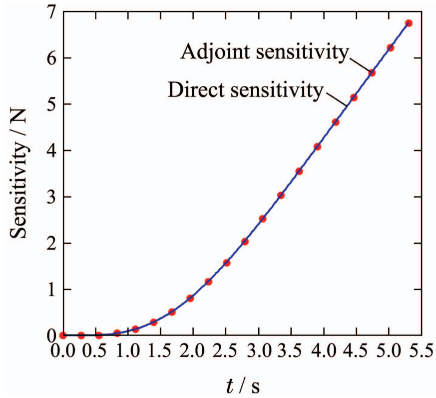

Fig. 4 Miss distance sensitivity to N

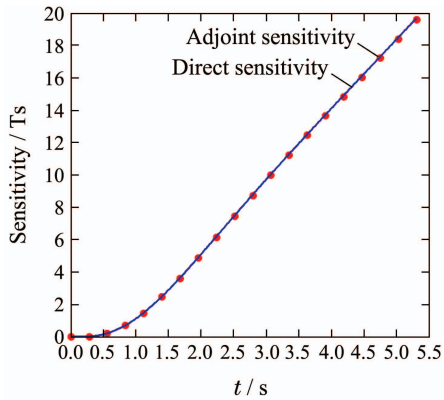

Fig. 5 Miss distance sensitivity to T_s

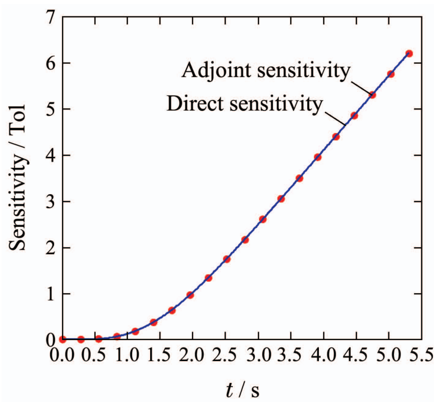

Fig. 6 Miss distance sensitivity to τ

The sensitivity increases with the decrease of the relative range between the flight vehicle and the target. As shown in the above figures, it can be seen that the results by the two methods are the same. Simulation results indicate that adjoint method is more effective and efficiency.

5 Summary and conclusions

In this paper, the direct method and adjoint method are used for calculation of higher-order sensitivity coefficients in a nonlinear TGS model. These two methods can successfully solve the miss distance sensitivity of TGS respect to parameter vector and get the same results. The adjoint state equation has to be solved only once. Thus, the adjoint method is more effective and efficiency, and is advantageous over the direct method for applications with a large number of sensitivity parameters. As the increases of parameters' value and the as the decrease of the relative range, the miss distance is more sensitive to the parameter. The first-order sensitivity model works well in the parameter sensitivity analysis of TGS. The second-order sensitivity with respect to parameters will be studied in the future research.

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