DOI: 10.7641/CTA.2014.13072

随机非线性和部分转移概率未知时马尔科夫系统的$\mathbf{H}_\infty$控制

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摘要: 研究上述系统时: 1) 利用了非线性的概率分布信息; 2) 利用了转移概率中已知部分和未知部分的关系. 利用李雅普诺夫泛函方法和线性矩阵不等式方法, 本文得到了使得系统随机稳定的充分条件并得到了相应的反馈控制增益. 文中最后给出的例子表明了所建立模型和分析方法的有效性.

关键词: 随机非线性; 部分转移概率未知; 马尔科夫跳变系统

中图分类号: TP273 **文献标识码**: A

$\mathbf{H}_\infty$ control for Markovian jump systems with incomplete transition probabilities and probabilistic nonlinearities

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Abstract: In considering the above mentioned systems we have made use of: 1) the information of the nonlinear probability distribution; 2) the relationship between the known part and the unknown part of the transition probabilities. By using the linear matrix inequality technique, we obtained the sufficient conditions of stochastic stability for the systems and feedback gains for the H1 controller based on the Lyapunov function method. A numerical example shows the effectiveness of the proposed modeling and the design approach.

Key words: probabilistic nonlinearity; partially unknown transition probabilities; Markovian jump systems

1 Introduction

Markovian jump systems (MJSs) are a class of multi-modal systems in which the transitions among different modes are governed by a Markov chain[1]. The investigations of MJSs have been absorbed considerable attention in recent years, see [2–10] and the references therein. In most of the studies, complete knowledge of the transition probabilities (TPs) is required as a prerequisite for analysis and synthesis of the MJSs. However, in some practical systems, the TPs of MJSs may be not measurable or the measurement is inaccurate[8,11]. Assuming that the TPs have uncertainties, the authors investigated the problems of stability analysis of linear MJSs in [12]. By assuming the TPs varying in some intervals, the authors in [13] studied H_∞ control for discrete MJSs. When the TPs are completely unknown, the authors in [11, 14] considered the control synthesis of MJSs.

On the other hand, because of modeling error or outside disturbance, nonlinearities exist in many practical systems. In recent years, stability analysis and control synthesis for system with nonlinearities have been paid considerable attention[10,15–19]. However, in most of the researches, the researchers only utilized the bounds information of the nonlinearities, such as upper bound, lower bound or sector bounds. Up to now, few consideration has been paid to the inner variation information of the nonlinearities between their bounds. In [20–21], probabilistic time-delay are investigated, wherein the delay is segmented into two (or more) parts and the probability distribution of the delay falling into each part is used in the modeling and analysis of the networked control systems, which can reduce the conservatism greatly. Motivated by the method in [20–21], we considered the inner variation information of the nonlinearities, which is called probabilistic nonlinearities in this paper.

The inner variation information is an important character to study the nonlinearities, which can help us have a deep understanding on them. For example, for two nonlinear functions with the same lower and upper bounds, most values of the first nonlinear function are close to its lower bound, in another word, the probability of the first nonlinear function taking small values is very large. While the probability of the second nonlinear function taking small values is very small. Obviously, these two nonlinear functions are different and we prefer the first one because small values happens with a large probability. However, if we

Received 14 May 2013; revised 11 October 2013.
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This work was supported by the National Natural Science Foundation of China (61273114, 61273115); the Natural Science Foundation of Jiangsu Province of China (BK2012847, BK2012469).

only use the bounds information, we can not distinguish them from each other, because their inner distribution information is not considered. In fact, the existing methods (only use the bounds information) are under an assumption that the nonlinearities can reach their upper bound with arbitrary probability. In fact, the value distribution of the nonlinearities can be measured through simple statistical method, which provides a possibility to utilizing these information in the nonlinearity's modeling and analysis. However, to the best of the authors' knowledge, stability analysis and control synthesis for MJSs with partially unknown TPs and probabilistic nonlinearity have not been considered in the published literature, which motivates the current study.

The main contribution of this paper is the utilization of the nonlinearity's probabilistic information, which can help us have a deep understanding of the inner variation of the nonlinearity. By using these information, new nonlinearity model is proposed, which is more general than some existing nonlinearities, such as nonlinearity satisfying Lipschitz condition, nonlinearity less than an upper bound or sector bounded nonlinearities. By using the Lyapunov function method, stochastic stability conditions and controller design method are obtained. The illustrated example can show that, by using the probabilistic information of the nonlinearity, less conservative results can be achieved.

2 System modeling and description

$\{r_k, k \geqslant 0\}$ is the Markov chain taking values in the set $\Omega = \{1, 2, \cdots, N\}$ and

$$\mathrm{P}(r_{k+1} = j | r_k = i) = \pi_{ij},$$

where the TPs $\pi_{ij} \geqslant 0$, $\forall i, j \in \Omega$, and for $i \in \Omega$, $\sum\limits_{j=1}^{N} \pi_{ij} = 1$.

In this paper, the TPs are assumed to be partially known. For example, for MJSs with 3 modes, the TP matrix is

$$\pi = \begin{bmatrix} \pi_{11} & ? & ? \\ \pi_{21} & \pi_{22} & \pi_{23} \\ ? & \pi_{32} & ? \end{bmatrix}, \tag{1}$$

where '?' represents the completely unknown TPs. Respecting for the completely known and unknown TPs, the set Ω can be divided into two subsets

$$\Omega = \Omega_k^i \cup \Omega_{uk}^i,$$

where Ω_k^i is the set of known TPs and Ω_{uk}^i is the set of unknown TPs. Define

$$\Omega_k^i = \{j : \text{if } \pi_{ij} \text{ is known }\} \triangleq \{k_1, k_2, \cdots, k_m\}, \tag{2}$$

$$\Omega_{uk}^i = \{j : \text{if } \pi_{ij} \text{ is unknown }\} \triangleq \{k_{m+1}, \cdots, k_N\}, \tag{3}$$

where m is the number of the TPs in the i^{th} row, $\{k_1, k_2, \cdots, k_m\}$ and $\{k_{m+1}, k_{m+2}, \cdots, k_N\}$ are the subsets of Ω and satisfying $\{k_1, k_2, \cdots, k_m\} \cup \{k_{m+1}, k_{m+2}, \cdots, k_N\} = \{1, 2, \cdots, N\}$. For example, for $i = 1$ in (1), $\Omega_k^1 = \{k_1\} = \{1\}$ and $\Omega_{uk}^1 = \{k_2, k_3\} = \{2, 3\}$.

Consider the following MJSs with partially unknown TPs and nonlinearity

$$x_{k+1} = A(r_k)x_k + B(r_k)u_k + h(r_k, x_k) + E(r_k)\omega_k, \tag{4}$$

$$z_k = C(r_k)x_k + D(r_k)\omega_k, \tag{5}$$

where $x_k \in \mathbb{R}^n, u_k \in \mathbb{R}^m$ and $\omega_k \in l_2[0, \infty)$ are, respectively, the state, control input and output disturbance, $z_k \in \mathbb{R}^l$ is the controlled output. A_i, B_i, E_i, C_i and D_i are matrices with appropriate dimensions. For known constant matrix M_i, the nonlinearities satisfy the following condition

$$\|h(i, x_k)\|_2 \leqslant \|M_i x_k\|_2. \tag{6}$$

In this study, the probability information of the nonlinearity occurring among different bounds are firstly utilized. For this purpose, a Bernoulli distributed variable is defined. For known matrix N_i and M_i satisfying $\mathrm{tr}(M_i) \geqslant \mathrm{tr}(N_i)$,

$$\alpha_1(k) = \begin{cases} 1, & \|h(i, x_k)\|_2 \leqslant \|N_i x_k\|_2, \\ 0, & \|N_i x_k\|_2 < \|h(i, x_k)\|_2 \leqslant \|M_i x_k\|_2, \end{cases}$$

$$\alpha_2(k) = 1 - \alpha_1(k),$$

the expectations of $\alpha_i(k)$ are

$$\mathrm{P}\{\alpha_1(k) = 1\} = \bar{\alpha}_1, \mathrm{P}\{\alpha_1(k) = 0\} = 1 - \bar{\alpha}_1,$$
$$\mathrm{P}\{\alpha_2(k) = 1\} = \bar{\alpha}_2 = 1 - \bar{\alpha}_1.$$

Based on the above definitions, a small and large nonlinearities are defined as

$$h_1(i, x_k) = \begin{cases} h(i, x_k), & \alpha_1(k) = 1, \\ 0, & \alpha_1(k) = 0, \end{cases} \tag{7}$$

$$h_2(i, x_k) = \begin{cases} h(i, x_k), & \alpha_2(k) = 1, \\ N_i x_k, & \alpha_2(k) = 0. \end{cases} \tag{8}$$

Then $h(i, x_k)$ can be expressed as

$$h(i, x_k) = \sum_{j=1}^{2} \alpha_j(k) h_j(i, x_k). \tag{9}$$

From above definitions, the values of small nonlinearity $h_1(i, x_k)$ are below the bound $\|N_i x_k\|_2$ and the values of $h_2(i, x_k)$ are between the bounds $\|N_i x_k\|_2$ and $\|M_i x_k\|_2$. $\bar{\alpha}_1$ and $\bar{\alpha}_2$ are probabilities of $h(i, x_k)$ smaller and larger than $\|N_i x_k\|_2$ respectively.

By using above modeling method (9), the whole variable range of $h(i, x_k)$ is divided into two subinterval and the value distribution of the nonlinearity is used. It should be noted that the variable range of the nonlinearity can be divided more finer if we know more inner variable information, thus a more complexity and finer model similar to (9) can be obtained. Consequently, the analysis method will become more complexity and less conservative results can be obtained.

Remark 1 In the existing methods dealing with the nonlinearities, only the bounds information is utilized, which can be seen special cases of the proposed model. The nonlinearities used in [10, 22] can be seen as special cases of the proposed model by setting $\alpha_1(k) \equiv 0$ and $N_i \equiv 0$; the sector bounded nonlinearities investigated in [16–17] can be seen as special cases by setting $\alpha_1(k) \equiv 0$; and the random occurring nonlinearities studied in [4, 23–24] can be seen as special cases of the proposed model by setting $h_1(i, x_k) \equiv 0$.

The controller in (4) is

$$u_k = K(r_k)x_k, \tag{10}$$

where $K_i(r_k = i \in \Omega)$ is the feedback gain to be designed. Replacing (10) into (4),

$$x_{k+1} = (A_i + B_iK_i)\,x_k + E_i\omega_k + \sum_{l=1}^{2} \alpha_l(k)h_l(i, x_k), \tag{11}$$

$$z_k = C_ix_k + D_i\omega_k. \tag{12}$$

3 Main results

The purpose of this section is to design the H_∞ controller to stabilize the nonlinear MJSs with partially unknown TPs.

Theorem 1 System (11)−(12) is stochastically stable if there exist matrices variables $X_i > 0,\ T_i > 0$ and scalars $\varepsilon_i > 0$ such that

$$\begin{bmatrix} \Pi_{11} & * & * & * & * \\ \mathcal{C}_i & -I & * & * & * \\ \sqrt{2}\mathcal{W}_i\mathcal{A}_i & 0 & -\mathcal{X}_i & * & * \\ \Pi_{14} & 0 & 0 & \Pi_{44} & * \\ \Pi_{15} & 0 & 0 & 0 & \Pi_{55} \end{bmatrix} < 0, \tag{13}$$

$$T_i < \varepsilon_i I, \tag{14}$$

where

$\Pi = \mathrm{diag}\{-X_i^{-1}, -\gamma^2 I, -\bar{\alpha}_1T_i, -\bar{\alpha}_2T_i\}$,

$\mathcal{C}_i = [C_i\ \ D_i\ \ 0\ \ 0],\ \mathcal{W}_i = [\mathcal{W}_i^k\ \ \mathcal{W}_i^{uk}]$,

$\mathcal{W}_i^{uk} = [\sqrt{1-\pi_i^k}I\ \ \cdots\ \ \sqrt{1-\pi_i^k}I]^{\mathrm{T}}$,

$\mathcal{W}_i^k = [\sqrt{\pi_{i,k_1}}I\ \ \cdots\ \ \sqrt{\pi_{i,k_m}}I]^{\mathrm{T}}$,

$\mathcal{A}_i = [A_i + B_iK_i\ \ B_{\omega i}\ \ 0\ \ 0]$,

$\mathcal{X}_i = \mathrm{diag}\{X_{k_1}, X_{k_2}, \cdots, X_{k_m}, X_{k_{m+1}}, \cdots, X_{k_N}\}$,

$$\Pi_{14} = \begin{bmatrix} 0 & 0 & \sqrt{2\bar{\alpha}_1}\mathcal{W}_i & 0 \\ 0 & 0 & 0 & \sqrt{2\bar{\alpha}_2}\mathcal{W}_i \end{bmatrix},$$

$$\Pi_{15} = \begin{bmatrix} \sqrt{\alpha_1}\varepsilon_iN_i^{\mathrm{T}} & 0 & 0 & 0 \\ 0 & \sqrt{\alpha_2}\varepsilon_iM_i^{\mathrm{T}} & 0 & 0 \end{bmatrix},$$

$\Pi_{44} = \mathrm{diag}\{-\mathcal{X}_i, -\mathcal{X}_i\},\ \Pi_{55} = \mathrm{diag}\{-\varepsilon_iI, -\varepsilon_iI\}$.

Proof Construct the Lyapunov function as

$$V(k, x_k) = x_k^{\mathrm{T}}\mathrm{P}(r_k)x_k \tag{15}$$

for $r_k = i$ and $r_{k+1} = j$, we obtain

$$\begin{aligned}
&\mathrm{E}\{\Delta V(k, x_k) + z_k^{\mathrm{T}}z_k - \gamma^2\omega_k^{\mathrm{T}}\omega_k\} = \\
&\mathrm{E}\{x_{k+1}^{\mathrm{T}}(\sum_{j\in\Omega_k^i}\pi_{ij}P_j + \sum_{j\in\Omega_{uk}^i}\pi_{ij}P_j)x_{k+1} - \\
&x_k^{\mathrm{T}}P_ix_k + z_k^{\mathrm{T}}z_k - \gamma^2\omega_k^{\mathrm{T}}\omega_k\} \leqslant \\
&\mathrm{E}\{x_{k+1}^{\mathrm{T}}(\sum_{j\in\Omega_k^i}\pi_{ij}P_j + (1-\pi_i^k)\sum_{j\in\Omega_{uk}^i}P_j)x_{k+1} - \\
&x_k^{\mathrm{T}}P_ix_k + z_k^{\mathrm{T}}z_k - \gamma^2\omega_k^{\mathrm{T}}\omega_k\} = \\
&\mathrm{E}\{x_{k+1}^{\mathrm{T}}\bar{P}_ix_{k+1} - x_k^{\mathrm{T}}P_ix_k + z_k^{\mathrm{T}}z_k - \gamma^2\omega_k^{\mathrm{T}}\omega_k\},
\end{aligned} \tag{16}$$

where $\pi_i^k = \sum_{j\in\Omega_k^i}\pi_{ij}$, and

$$\bar{P}_i = \sum_{j\in\Omega_k^i}\pi_{ij}P_j + (1-\pi_i^k)\sum_{j\in\Omega_{uk}^i}P_j,$$

$$\begin{aligned}
&\mathrm{E}\{x_{k+1}^{\mathrm{T}}\bar{P}_ix_{k+1}\} = \\
&\mathrm{E}\{\zeta_k^{\mathrm{T}}\mathcal{A}_i^{\mathrm{T}}\bar{P}_i\mathcal{A}_i\zeta_k + 2\zeta_k^{\mathrm{T}}\mathcal{A}_i^{\mathrm{T}}\bar{P}_i\sum_{l=1}^{2}\bar{\alpha}_lh_l(i, x_k) + \\
&\sum_{l=1}^{2}\bar{\alpha}_lh_l^{\mathrm{T}}(i, x_k)\bar{P}_ih_l(i, x_k)\},
\end{aligned} \tag{17}$$

where $\zeta_k^{\mathrm{T}} = [x_k^{\mathrm{T}}\ \ \omega_k^{\mathrm{T}}\ \ h_1^{\mathrm{T}}(i, x_k)\ \ h_2^{\mathrm{T}}(i, x_k)]$. From the definitions of $h_1(i, x_k)$ and $h_2(i, x_k)$ and recalling (14)

$$\bar{\alpha}_1\varepsilon_ix_k^{\mathrm{T}}N_i^{\mathrm{T}}N_ix_k - \bar{\alpha}_1h_1^{\mathrm{T}}(i, x_k)T_ih_1(i, x_k) > 0, \tag{18}$$

$$\bar{\alpha}_2\varepsilon_ix_k^{\mathrm{T}}M_i^{\mathrm{T}}M_ix_k - \bar{\alpha}_2h_2^{\mathrm{T}}(i, x_k)T_ih_2(i, x_k) > 0. \tag{19}$$

Replacing (17)−(19) into (16):

$$\begin{aligned}
&\mathrm{E}\{\Delta V(k, x_k) + z_k^{\mathrm{T}}z_k - \gamma^2\omega_k^{\mathrm{T}}\omega_k\} \leqslant \\
&\mathrm{E}\{\zeta_k^{\mathrm{T}}\{\Pi + \mathcal{A}_i^{\mathrm{T}}\bar{P}_i\mathcal{A}_i + \mathcal{C}_i^{\mathrm{T}}\mathcal{C}_i\}\zeta_k\},
\end{aligned}$$

where $\bar{P}_i$ has the following form

$$\bar{P}_i = \mathcal{W}_i(\mathcal{X}_i)^{-1}\mathcal{W}_i^{\mathrm{T}}. \tag{20}$$

By using Schur complements and (13)−(14), the following inequality can be obtained:

$$\mathrm{E}\{\Delta V(k, x_k) + z_k^{\mathrm{T}}z_k - \gamma^2\omega_k^{\mathrm{T}}\omega_k\} \leqslant -\delta\|x_k\|^2,$$

where $\delta = \inf\{\lambda_{\min}(\Pi + 2\mathcal{A}_i^{\mathrm{T}}\bar{P}_i\mathcal{A}_i + \mathcal{C}_i^{\mathrm{T}}\mathcal{C}_i)\}$. When $\omega_k = 0$, for any $L \to \infty$

$$\mathrm{E}\{\sum_{k=0}^{L}\|x_k\|^2\}^2\} \leqslant \frac{1}{\delta}\mathrm{E}\{V(0, x_0) - V(L+1, x_{L+1})\} \leqslant \infty,$$

where x_0 is the initial condition of x_k. For $\omega_k \neq 0$ and under zero condition, the following can be obtained

$$\mathrm{E}\{\sum_{k=0}^{\infty}\|z_k\|z_k\|^2\} \leqslant \gamma^2\mathrm{E}\{\sum_{k=0}^{\infty}\|\omega_k\|^2\}.$$

This completes the proof.

Remark 2 It should be pointed out that the proposed method can be applied into the MJSs with completely known and completely unknown TPs. When the TPs are completely known, set $\bar{P}_i = \sum_{j\in\Omega}\pi_{ij}P_j$ and $\mathcal{W}_i = [\sqrt{\pi_{i1}}I\ \sqrt{\pi_{i2}}I \cdots \sqrt{\pi_{iN}}I]$ in Theorem 1, then the MJSs reduce to the systems considered in [3]. When the TPs are completely unknown, set $\bar{P}_i = P_j, \mathcal{W}_i = [I\ \ I\ \ \cdots\ \ I]$ in Theorem 1. Therefore, the cases of completely known TPs and completely unknown TPs are special cases of the proposed model.

Based on Theorem 1, the following theorem provides a way to design the feedback gains K_i.

Theorem 2 For given contacts $\varepsilon_i > 0$ and $\gamma > 0$, system (11)−(12) is stochastic stability with the controller feedback gains $K_i = Y_iX_i^{-1}$ if there exist matrices variables $T_i > 0,\ X_i > 0,\ Y_i$ such that

$$\begin{bmatrix} \hat{\Pi}_{11} & * & * & * & * \\ \hat{\mathcal{C}}_i & -I & * & * & * \\ \sqrt{2}\mathcal{W}_i\hat{\mathcal{A}}_i & 0 & -\mathcal{X}_i & * & * \\ \Pi_{14} & 0 & 0 & \Pi_{44} & * \\ \hat{\Pi}_{15} & 0 & 0 & 0 & \Pi_{55} \end{bmatrix} < 0, \tag{21}$$

$$T_i < \varepsilon_iI, \tag{22}$$

where

$\hat{\Pi} = \mathrm{diag}\{-X_i, -\gamma^2I, -\bar{\alpha}_1T_i, -\bar{\alpha}_2T_i\}$,

$\hat{\mathcal{C}}_i = [C_iX_i\ \ D_i\ \ 0\ \ 0]$,

$\hat{\mathcal{A}}_i = [A_iX_i + B_iY_i\ \ B_{\omega i}\ \ 0\ \ 0]$,

$$\hat{\Pi}_{15} = \begin{bmatrix} \sqrt{\bar{\alpha}_1}\varepsilon_iN_i^{\mathrm{T}}X_i & 0 & 0 & 0 \\ 0 & \sqrt{\bar{\alpha}_2}\varepsilon_iM_i^{\mathrm{T}} & 0 & 0 \end{bmatrix}.$$

Proof Pre- and post-multiply (13) with $\mathrm{diag}\{X_i, I, \cdots, I\}$ and its transpose and define $Y_i = K_i X_i^{-1}$, Eq.(21) can be obtained.

Remark 3 When studying the Markovian jump systems with partially unknown TPs, the authors in [11, 14] neglected the following information: in every row of the TP matrix, the sum of the known TPs and unknown TPs equals to 1, that is, $\sum\limits_{j\in\Omega_{uk}^i} \pi_{ij} = 1 - \sum\limits_{j\in\Omega_k^i} \pi_{ij}$.

Remark 4 By using the inner variation information of the nonlinearities, less conservative results can be obtained than those only considering the lower or upper bounds, which can be shown from numerical example. To simplify the analysis, time delay is not considered in the systems and only classical Lyapunov function method is utilized. It should be noted that the proposed modeling and analysis method can also be applied to system with time delay.

In order to make a comparison, a corollary is proposed based on Theorem 2 when neglecting the relation $\sum\limits_{j\in\Omega_{uk}^i} \pi_{ij} = 1 - \sum\limits_{j\in\Omega_k^i} \pi_{ij}$.

Corollary 1 System (11)−(12) is stochastic stability if there exist matrices variables $T_i > 0,\ X_i > 0,\ Y_i$ such that

$$\begin{bmatrix} \hat{\Pi}_{11} & * & * & * & * \\ \hat{\mathcal{C}}_i & -I & * & * & * \\ \sqrt{2}\tilde{\mathcal{W}}_i\hat{\mathcal{A}}_i & 0 & -\mathcal{X}_i & * & * \\ \tilde{\Pi}_{14} & 0 & 0 & \Pi_{44} & * \\ \hat{\Pi}_{15} & 0 & 0 & 0 & \Pi_{55} \end{bmatrix} < 0, \quad (23)$$

$$T_i < \varepsilon_i I, \quad (24)$$

where

$$\tilde{\Pi}_{14} = \begin{bmatrix} \sqrt{2\bar{\alpha}_1}\tilde{\mathcal{W}}_i X \\ \sqrt{2\bar{\alpha}_2}\tilde{\mathcal{W}}_i X \end{bmatrix},$$

$$\tilde{\mathcal{W}}_i = \begin{cases} \left[\sqrt{\pi_{i,k_1}}I \ \cdots \ \sqrt{\pi_{i,k_m}}I\right], & \text{if } j \in \Omega_k^i, \\ \left[\sqrt{1-\pi_i^k}I \ \cdots \ \sqrt{1-\pi_i^k}I\right], & \text{if } j \in \Omega_{uk}^i. \end{cases}$$

4 Simulation example

Example 1 Consider the system (11)−(12) with four modes, the parameters are

$$A_1 = \begin{bmatrix} 0.72 & -0.40 \\ 0.81 & 0.81 \end{bmatrix},\ A_2 = \begin{bmatrix} 0.18 & -0.26 \\ 0.81 & 0.13 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -0.53 & -0.81 \\ 0.81 & 0.47 \end{bmatrix},\ A_4 = \begin{bmatrix} 1.07 & -0.18 \\ 0.81 & 0.29 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -0.3 \\ 1.6 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.5 \\ 0.8 \end{bmatrix},\ B_3 = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix},$$

$$B_4 = \begin{bmatrix} -0.7 \\ 0.2 \end{bmatrix},\ B_{\omega 1} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},\ C_1 = [0.2\ \ 0.5],$$

$$B_{\omega 2} = \begin{bmatrix} 0.2 \\ -0.1 \end{bmatrix},\ B_{\omega 3} = \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix},\ B_{\omega 4} = \begin{bmatrix} 0.1 \\ 0.4 \end{bmatrix},$$

$$C_2[-0.3\ \ 0.7],\ C_3 = [0.2\ \ -0.5],$$

$$C_4 = [0.2\ \ 0.1],\ D_1 = D_2 = D_3 = D_4 = 0.1.$$

The TP matrix is

$$\pi = \begin{bmatrix} 0.3 & 0.2 & 0.1 & 0.4 \\ ? & 0.2 & 0.3 & ? \\ ? & 0.4 & ? & 0.3 \\ 0.2 & 0.2 & 0.1 & 0.5 \end{bmatrix}$$

where the TPs $\pi_{21}, \pi_{24}, \pi_{31}, \pi_{33}$ are completely unknown. The following two cases are discussed: in Case 1, there is no nonlinearity, through comparing the results obtained from Theorem 2 and Corollary 1, it can be found that by using the relation between known and unknown TPs, less conservative results can be obtained. Case 2 shows that by using the probabilistic information of the nonlinearity, the system performance can be greatly improved.

Case 1 For $N_i = M_i = \mathrm{diag}\{0, 0\}$, that is, there is no nonlinear disturbance in the system. For $\varepsilon_i = 80(i = 1, 2, 3, 4)$, using Theorem 2 and Corollary 1, the minimum H_∞ performance γ is obtained as $\gamma_{\min} = 0.68$ (Theorem 2) and $\gamma_{\min} = 1.15$ (Corollary 1). Notice that the only difference of Theorem 2 and Corollary 1 is that the relation between the known and unknown TPs is used in Theorem 2, i.e., $\pi_{21} + \pi_{24} = 1 - \pi_{22} - \pi_{23} = 0.5$, $\pi_{31} + \pi_{33} = 1 - \pi_{32} - \pi_{34} = 0.3$. However, in Corollary 1 and the literature [11, 14], this relation is neglected. It can be concluded that using this information can lead to a better control performance.

Case 2 In this case, we will investigate the effect of using the inner variation information on the system performance. For $\varepsilon_i = 80(i = 1, 2, 3, 4)$ and the parameters

$$N_i = \mathrm{diag}\{0.05, 0.05\}, M_i = \mathrm{diag}\{0.16, 0.16\}.$$

Firstly, if we do not use the inner variation information, that is, set $\bar{\alpha}_1 = 0, \bar{\alpha}_2 = 1$ and $N_i = \mathrm{diag}\{0, 0\}$, using Theorem 2, the minimum H_∞ performance index is obtained as $\gamma_{\min} = 10.20$, if we use Corollary 1, no feasible solution can be found. By using the inner variation information, the obtained $\gamma_{\min}$ from Theorem 2 and Corollary 1 are shown in Table 1.

Table 1 The obtained $\gamma_{\min}$ by using Theorem 2 and Corollary 1 for different α_1

$\bar{\alpha}_1$	1	0.8	0.6	0.4	0.2	0
Theorem 2	1.22	1.84	2.30	2.73	3.77	10.20
Corollary 1	2.21	3.39	4.25	5.17	7.49	No solution

From the results in Table 1, it can be concluded that when the inner variation information of the nonlinearities is known, a better control performance can be obtained, and the improvement become more obviously when the probability of the small nonlinearities happens. Compared the results obtained from Theorem 2 and Corollary 1 in Table 1, it can also be found that the relation between known and unknown TPs can help us improve the system control performance.

5 Conclusion

This paper studies the H_∞ control for nonlinear Markovian jump systems with partially unknown transition probabilities. New character of the nonlinearities, the inner variation information, is firstly utilized in this paper, which makes the considered nonlinearities more general. Using this new character, new kind of nonlinear Markovian jump system model is built. By using the Lyapunov function method and linear matrix inequalities technique, sufficient conditions for the stochastic stability of the proposed system model can be obtained. The controller feedback gain can be solved out by using the LMI toolbox in Matlab. The simulation example has demonstrated the effectiveness and advantages of the utilization of the new characters and proposed method.

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