The Discrete-Time Controller Design Based on Sliding Mode for Manipulators*

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Abstract: In this paper, the design approach of the sliding mode tracking controller with boundary layer in [1] is developed in discrete-time systems for the computer control of robots. This paper also deals with the effect of sampling interval on system performance, proposes a new compensation method for further improving the tracking precision, and a method for sampling interval selection is given. Finally, the effectiveness and good performance of the proposed design approach is examined through the six degrees of freedom Arm PUMA560 with digital simulations.

Keywords: sampling systems; robots; the variable structure control

1 Introduction

Recently, considerable attention has been paid on the variable structure control for discrete-time systems because of the calculating devices being used as the switching elements in Variable Structure Systems (VSS). Although partial work[1~4] on this field has been done, open problems still focus on the following questions: 1) the selection of reaching condition for a fast reaching to the sliding surface; 2) the elimination of chattering; 3) the effect of sampling interval on system performance and so on. In this paper the design approach of the sliding mode tracking controller is developed in discrete-time systems in order to study the computer control of a continuous plant with bounded parameter variations and external disturbances. The results obtained include a comprehensive consideration of the effect of sampling interval on system performance and further improvement of tracking performance. And a better design performance is illustrated by comparing with the design approach in [1] with digital simulations of PUMA 560 manipulator.

2 VSS Control of Discrete-Time Systems

2.1 Reaching Condition and Equivalent Control

Consider the following discrete-time state equation related to the SISO system $x^{(n)}=f(X,t)+b(X,t)u(t)+d(t)$, considered in [1]

$$X(k+1)=A(k,T)X(k)+G(k)u(k)+W(k), \tag{2.1}$$

where $X=(x,\dot{x},\cdots,x^{(n-1)})^{\mathrm{T}}$ is the state and u is the control input.

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In discrete-time systems, a sliding surface in state space is defined as

$$s(k) = C(X(k) - X_d(k)), \tag{2.2}$$

where $X_d(k) = (x_d(k), \dot{x}_d(k), \cdots, x_d^{(n-1)}(k))^T$, $x_d(k)$ is the desired trajectory to be tracked, $C = (c_1, \cdots, c_n)$ is the n-dimensional row vector.

Similar to [1], if the desired bandwidth of system is λ, then a matching relation must be satisfied as

$$c_i = \begin{bmatrix} n-1 \\ i-1 \end{bmatrix} \lambda^{n-i}, \quad (i = 1, \cdots, n). \tag{2.3}$$

Since the discrete-time implementation of the sliding mode, a special sliding mode—quasisliding mode is caused, which have been studied in [2]~[4], etc. In order to assure the existence of a quasisliding motion and convergence of the state trajectory onto the sliding surface, a sufficient and necesary condition must be satisfied as

$$-s(k)\mathrm{sgn}(s(k)) < s(k+1) < s(k)\mathrm{sgn}(s(k)), \tag{2.4}$$

where sgn $(\cdot)$ is the sign function. And Eq. (2.4) can be derived easily from the sufficient and necessary condition defined in Eq. (7) by Sarpturk [2].

In sliding mode, the state satisfies the equations

$$s(k) = s(k+1) = s(k+2)\cdots. \tag{2.5}$$

From Eq. (2.1) and Eq. (2.5), an equivalent control is obtained as

$$u_{eq}(k) = -[C(A(k,T) - I)X(k) + C\hat{W}(k) + C(X_d(k) - X_d(k+1))]/C\hat{G}(k), \tag{2.6}$$

where $\hat{G}(k)$ and $\hat{W}(k)$ are the estimations of $G(k)$ and $W(k)$.

2.2 VSS Control Strategy

In this paper, the following type control law is considered

$$u(k) = u_{eq}(k) - \frac{1}{C\hat{G}(k)} P(k)\mathrm{sat}\left(\frac{s(k)}{\Phi(k)}\right), \tag{2.7}$$

where $P(k)$ is the vibration coefficient, and sat$(\cdot)$ is defined by

$$|y| \leqslant 1 \Rightarrow \mathrm{sat}(y) = y; \quad |y| > 1 \Rightarrow \mathrm{sat}(y) = \mathrm{sgn}(y).$$

Substituting Eq. (2.7), Eq. (2.6) into $s(k+1)$, yields

$$s(k+1) = s(k) - \frac{CG(k)}{C\hat{G}(k)} P(k)\mathrm{sat}\left(\frac{s(k)}{\Phi(k)}\right) + E(k, X(k)), \tag{2.8}$$

where

$$E(k, X(k)) = C(G(k) - \hat{G}(k))u_{eq}(k) + C(W(k) - \hat{W}(k)) \tag{2.9}$$

represents the system parameter and disturbance uncertainty.

First, assume that the boundary layer $\Phi(k)$ is time-invariant. And the control laws are discussed from the following two cases:

1° $s(k)$ is outside the boundary layer

Here, the state trajectory is expected to enter into the boundary layer in one sampling interval by control, i.e.

$$-\Phi(k) < s(k+1) = s(k) - \frac{CG(k)}{C\hat{G}(k)} P(k)\mathrm{sat}\left(\frac{s(k)}{\Phi(k)}\right) + E(k, X(k)) < \Phi(k). \tag{2.10}$$

Because of $|s(k)|>\Phi(k)$, Eq. (2. 10) is a stronger condition than Eq. (2. 4), Eq. (2. 10) gives

$$s(k)-\Phi(k)+E(k,X(k))<\frac{CG(k)}{C\hat{G}(k)}P(k)\mathrm{sat}\left(\frac{s(k)}{\Phi(k)}\right)$$
$$<s(k)+\Phi(k)+E(k,X(k)). \tag{2. 11}$$

If $CG(k)$ is estimated by $C\hat{G}(k)$ with a gain margin β, and $|E(k,X(k))|\leqslant D(k)$, $D(k)$ can be estimated by

$$D(k)=\max|C(G(k)-\hat{G}(k))||u_{eq}(k)|+\max|C(W(k)-\hat{W}(k))|, \tag{2. 12}$$

then Eq. (2. 11) is satisfied if

$$\beta(|s(k)|-\Phi(k)+D(k))<P(k)<(|s(k)|+\Phi(k)-D(k))/\beta. \tag{2. 13}$$

2° $s(k)$ is inside the boundary layer

Eq. (2. 8) can be written by

$$s(k+1)=rs(k)+E(k,X(k)), \tag{2. 14}$$

where

$$r=1-\frac{CG(k)P(k)}{C\hat{G}(k)\Phi(k)}. \tag{2. 15}$$

Eq. (2. 14) represents a one-order digital filter with lowpass filteringproperty as $0<r<1$. Similar to [1], a matching condition must besatisfied so as to avoid exciting the unmodelled high frequency dynamics, shown as follows

$$r_{\min}\geqslant-\cos(\lambda T)-\sqrt{(2-\cos\lambda T)^2-1}+2,\quad \lambda T\leqslant\pi, \tag{2. 16}$$

where $r_{\min}$ is the minimum of r, related to the biggest bandwidth of the digital filter. And Eq. (2. 15) gives

$$\frac{CG(k)P(k)}{C\hat{G}(k)\Phi(k)}=1-r\leqslant1-r_{\min}\leqslant r_1, \tag{2. 17}$$

where

$$r_1=\cos(\lambda T)+\sqrt{(2-\cos\lambda T)^2-1}-1. \tag{2. 18}$$

A corresponding balance condition of the variable structure control indiscrete-time systems is derived easily from Eq. (2. 17)

$$P(k)\leqslant r_1\Phi(k)/\beta. \tag{2. 19}$$

Next, we discuss what a control is required to keep the state trajectorymoving inside boundary layer.

Eq. (2. 14) represents a one-order linear function of $s(k)$ correspondingto the sampling instant kT, shown in Fig. 1. In Fig. 1, $r_{\min}<r<r_{\max}<1$, and $D_{\max}=\max|E(k,X(k))|$.

From Fig. 1, we know if $s(k+1)$ is inside the boundary layer with $|s(k)|=\Phi(k)$, all the $s(k+1)$ will be assured inside the boundary layerwith $|s(k)|<\Phi(k)$. So the required control can be obtained by

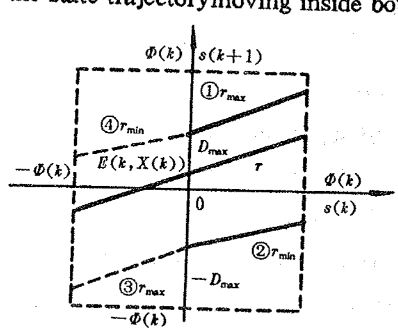

Fig. 1 Relations among $s(k+1)$, $s(k)$ and $\Phi(k)$

substituting $|s(k)| = \pm\Phi(k)$ into Eq. (2.13)

$$\beta D(k) < P(k) < (2\Phi(k) - D(k))/\beta. \tag{2.20}$$

Considering Eq. (2.14) and Fig. 1, a constrained condition must be imposed on $\Phi(k)$ for the existence of such a control as

$$\Phi(k) > D_{\max}/(1 - r_{\max}), \tag{2.21}$$

which means $r_1\Phi(k) < 2\Phi(k) - D(k)$, then a control can be obtained from Eq. (2.19) and Eq. (2.20) as

$$\beta D(k) < P(k) \leqslant r_1\Phi(k)/\beta. \tag{2.22}$$

Eq. (2.22) represents a control by which $s(k+1)$ is kept moving inside the boundary layer.

If the boundary layer can vary with the system parameter uncertainty, then

$$-\Phi(k+1) \leqslant s(k+1) \leqslant \Phi(k+1). \tag{2.23}$$

In this case, the desired dynamic balance equation satisfying Eq. (2.23) can be obtained easily as

for
$$P_d(k, X_d(k)) \leqslant r_1\Phi(k)/\beta,$$
$$\Delta\Phi(k+1) = P_d(k, X_d(k))/\beta - r_1\Phi(k)/\beta^2, \tag{2.24}$$

for
$$P_d(k, X_d(k)) > r_1\Phi(k)/\beta,$$
$$\Delta\Phi(k+1) = \beta P_d(k, X_d(k)) - r_1\Phi(k), \tag{2.25}$$

$$\Phi(0) = \frac{\beta P(0, X_d(0))}{r_2},$$

where

$$P_d(k) = \frac{CG(k)}{C\hat{G}(k)} |E(k, X(k))|_{x(k)=x_d(k)}. \tag{2.26}$$

2.3 Effect of Sampling Interval on System Performance

If $\Delta\Phi(k+1) \approx 0$, an approximate relationship of the tracking precision $\varepsilon(k)$ can be derived as

$$\varepsilon(k) = \beta D_o(k, X_d(k))\left(1 + \sum_{i=1}^{n-1} a_i(\lambda T)^i\right)T/r_1\lambda^{n-1}, \tag{2.27}$$

where $D_o(k, X_d(k))$ is the system parameter uncertainty corresponding to $T \to 0$, and

$$a_i = \begin{bmatrix} n-1 \\ i-1 \end{bmatrix} / (n-i)! \qquad (i = 1, \cdots, n). \tag{2.28}$$

(2.27) shows that as $T \to 0$, $r_1 = \lambda T$, it is identical to the case in continuous-time systems. However, as T is getting bigger, such that $r_1 < \lambda T$, the matching condition in VSS control of continuous-time systems is not satisfactory here any more.

Fig. 1 shows that two factors affect the tracking precision. One is the slope of straight line, r, related to the bandwidth of digital filter, which affects the dynamic tracking precision. The bigger the bandwidth is, the better the dynamic tracking precision is. Another factor is the parameter uncertainty, $E(k, X(k))$, assume that the variation of $E(k, X(k))$ is slow with respect to sampling interval, Fig. 1 shows that once the slope of straight line is smaller than certain value, the tracking precision is mainly determined by $E(k, X(k))$. From Eq. (2.9), $E(k, X(k))$ is approximately proportional to T, so the decrease of sampling interval can improve the dynamic tracking precision. Theory and experiment analysis shows that in the fixed sampling interval, r_1 must be more than 0.15 for a good tracking precision, and as $r_1 \geqslant 0.4$, the tracking precision improvement is smaller, therefore a constraint of r_1 is

$$0.15 < r_1 < 0.4. \tag{2.29}$$

2.4 Disturbance Compensation Scheme

The system tracking precision can be further improved by estimating $E(k, X(k))$. If the disturbance dynamics are assumed to be slow with respect to the sampling interval, then disturbance compensation equations are as follows

$$u_P(k) = s(k+1) - s(k) + P(k)\mathrm{sat}\left(\frac{s(k)}{\Phi(k)}\right), \tag{2.30}$$

$$E(k+1, X(k+1)) = u_P(k) - E(k, X(k)), \tag{2.31}$$

$$u_{eq}(k+1) = \hat{u}_{eq}(k+1) - K_2\{E(k+1, X(k+1)) + K_1 E(k, X(k))\}/(1+K_1), \tag{2.32}$$

where $\hat{u}_{eq}(k+1)$ is determined by Eq. (2.6), and $K_2 = (0.7 \sim 0.9)$ or $(1.1 \sim 1.3)/C\hat{G}(k)$, for $K_1 > 0.6$, $f < f_s/20$.

2.5 The Choice of Design Parameter and Sampling Interval

Parameter λ is selected according to the following constraints

$$\lambda \leqslant \lambda_R = 2\pi u_R/3; \quad \lambda \leqslant \lambda_A = 1/(3T_A), \tag{2.33}$$

where u_R is the frequency of the lowest unmodelled structural resonant mode, T_A is the largest unmodelled time-delay in the actuator.

The sampling interval is selected according to following constraints

$$\omega_s = (5 \sim 15)\omega_b; \quad u_u T < \pi; \quad 0.15 < r_1 < 0.4, \tag{2.34}$$

where ω_b is the bandwidth of the closed system, u_u is the frequency of the largest unmodelled high-frequency dynamics.

3 An Application Example

In this section, a six degrees of freedom Arm PUMA560 is used as an application example for design and simulations, the effectiveness of the proposed control algorithms for manipulators is examined by comparing with the design method for continuous-time systems in [1]. In order to evaluate the design results of the control algorithms, the following indexes are defined to represent the tracking precision for positions, orientations and the whole control torque for all links.

$$J_1 = \int_0^\infty (|x - x_d| + |y - y_d| + |z - z_d|)dt, \tag{3.1}$$

$$J_2 = \int_0^\infty (|Pox - Pox_d| + |Poy - Poy_d| + |Poz - Poz_d|)dt, \tag{3.2}$$

$$J_3 = \int_0^\infty \sum_{i=1}^{6} |\tau_i| dt, \tag{3.3}$$

where x, y, z, Pox, Poy, Poz are the practical positions and orientations of the manipulator in the task based coordinate system; x_d, y_d, z_d, Pox_d, Poy_d, Poz_d are the desired positions and orientations, $\tau_i (i=1,\cdots,6)$ are the control torques for each link.

Simulations are carried out in software package for robot systems at the first stage developed by the department of computer science and technology of Tsinghua university, with a integral step 0.001s., a sampling interval $T=0.01$s. and design parameter $\lambda=20$. The desired trajectory is a regular hexagon in X-Y plane with the center (0.3 0.4 0.6 3.0 0 $-$3.0)

Table 1 are the indexes and the corresponding values designed by the following design algorithms with no payload and payload 3kg. Fig. 2 ~ Fig. 3 are the simulation results corresponding to the design algorithms, where ① the designed result of the design algorithm for continuous-time systems in [1]; ② the designed result of design algorithm for discrete-time systems; ③ the designed result of design algorithm for discrete-time systems with disturbance compensation scheme. ⓪the desired trajectory.

Table 1 Simulation results

approaches \ conditions \ values \ indexes		J_1	J_2	J_3
The design approach for continuous-time systems	payload(0kg)	5.78235E-2	0.154381	115.566
	payload(3kg)	7.45568E-2	0.200216	134.047
The design approach for discrete-time systems	payload(0kg)	5.25325E-2	0.144356	114.775
	payload(3kg)	6.54592E-2	0.184395	132.646
The discrete approach with disturbance compensation	payload(0kg)	3.69054E-3	6.62904E-2	113.124
	payload(3kg)	4.16448E-3	7.9785E-2	130.274

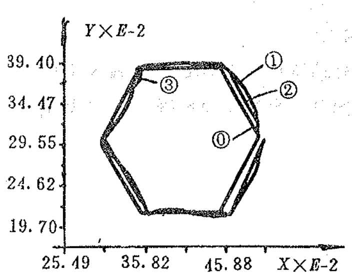

Fig. 2 Responses following the desired trajectory

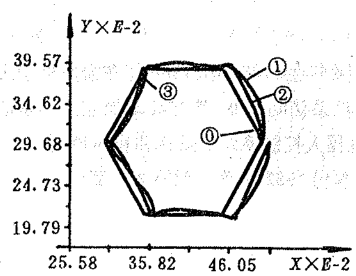

Fig. 3 Responses following the desired trajectory

From Table 1, a better tracking precision has been obtained by the design approach for discrete-time systems with less control torque compared with that for continuous-time systems in [1]. However, as above-metioned in section 2.3, in terms of the design approach for continuous-time systems, the matching condition, with the parameter $r_1=0.2$ which is more than the permission value $r_1=0.18$ used in the design approach for discrete-time systems, is not satisfactory as the sampling interval is considered. By using disturbance compensation scheme, a desired tracking is obtained with remarkable performance improvement (see Table 1).

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机械手的离散滑动模控制器设计

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摘要：本文针对机器人系统的计算机控制问题,将带有边界层的滑动模控制器设计方法[1]推广到了离散系统.本文研究了采样周期对系统性能的影响并给出其选择方法,为进一步提高跟随精度,本文提出了一种新的补偿方法.最后,本文提出方法的有效性和优越性在PUMA560六自由度机械手的数字仿真中获得验证.

关键词：采样系统;机器人;变结构控制

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