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Speed-tracker with Digital Control System and Its Simulation*

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Abstract

Several practical control programs for a speed-tracker with digital control computer are studied in this paper. The stability and adaptability of the closed-loop system are discussed. A real-time hybrid computer simulation is presented.

1. Problem Formulation

A general structure of the tracking system is shown in Fig. 1.

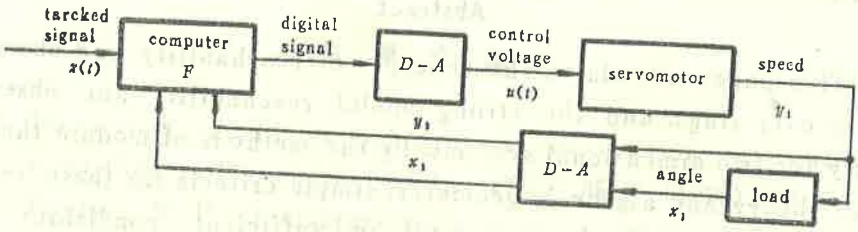

Fig. 1

Its simplified mathematical model is

$$\dot{x}_1 = k_1 y_1 , \tag{1.1}$$

$$s\dot{y}_1 + y_1 = k_2 u , \tag{1.2}$$

$$u = F(x_1, y_1, z). \tag{1.3}$$

where $z(t)$——azimuth of the tracked target; $x(t)$——angle of the tracking servomechanism; $y_1(t)$——rotational speed of the servomo-

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tor; $u(t)$——control voltage; k_1, k_2 and s are constants depending on these servomotor and load; F——unknown operator to be determined.

The control voltage u is bounded, as

$$|u| \leq U \tag{1.4}$$

If we ignore this constraint some absurd results will be obtained. The admissible $u(t)$ can be any piecewise continuous function.
We assume the velocity of motion $z(t)$ is constant in the period of time required for tracking, i. e.

$$z(t) = a + bt, \quad \text{b is constant,} \tag{1.5}$$

where a is the initial deviation. Note that the maximum velocity of x_1 is kU, $k = k_1 k_2$, so in order to assure tracking we must assume

$$|b| \leq kU. \tag{1.6}$$

Without losing generality we always assume

$$x_1(0) = 0, \qquad \dot{x}_1(0) = 0. \tag{1.7}$$

The problem is to design such an operator F that

$$x_1(T) = z(T), \quad \dot{x}_1(T) = \dot{z}(T), \text{ and } x_1(t) = z(t) \text{ for all } t > T, \tag{1.8}$$

and T is minimal. We call T the Speed-Catch Time.

2. Mathematical Analysis

Let

$$x = x_1 - z, \quad y = k_1 y_1 - b, \quad v = ku - b, \quad k = k_1 k_2. \tag{2.1}$$

then the system (1.1) - (1.4) becomes

$$\left.\begin{aligned} \dot{x} &= y, \ x(0) = -a, \\ s\dot{y} + y &= v, \ y(0) = -b, \end{aligned}\right\} \tag{2.2}$$

$$-kU - b \leq v \leq kU - b, \tag{2.3}$$

and we must find the optimal F and the minimum T that

$$x(T) = 0, \ y(T) = 0, \text{ and } x(t) = y(t) = 0 \text{ for all } t > T. \tag{2.4}$$

By means of the speed control theory (Ref. 1), the optimal switchcurve L consists of the lower half of the trajectory satisfying

$$\left.\begin{aligned} \dot{x} &= y, \ x(0) = 0, \\ s\dot{y} + y &= kU - b, \ y(0) = 0, \end{aligned}\right\} \tag{2.5}$$

and the upper half one satisfying

$$\left.\begin{aligned} \dot{x} &= y, \ x(0) = 0, \\ s\dot{y} + y &= -kU - b, \ y(0) = 0, \end{aligned}\right\} \tag{2.6}$$

The curve L is formulated as

$$\left.\begin{aligned} L_+:\ & x+sy=s(kU+b)\,Ln\left(1+\frac{y}{kU+b}\right), \quad as\ y>0, \\ L_-:\ & x+sy=s(b-kU)\,Ln\left(1-\frac{y}{kU-b}\right), \quad as\ y<0. \end{aligned}\right\} \tag{2.7}$$

Expending Eq. (2.7) we obtain an approximate formula as

$$\left.\begin{aligned} L_+:\quad & x=-\frac{sy^2}{2(kU+b)}, \quad as\ y>0, \\ L_-:\quad & x=\frac{sy^2}{2(kU-b)}, \quad as\ y<0, \end{aligned}\right\} \tag{2.8}$$

Note that L depends on b and is not symmetric as $b\neq 0$. A family of curves L_i, $i=-1, 0, 1$, corresponding to $b_{-1}<b_0=0<b_1$ respectively is shown in Fig. 2. The output of the control computer should be

$$u=F(x, y, z)=\begin{cases} U, & \text{as } (x, y) \text{ below } L \text{ or in its lower half,} \\ -U, & \text{as } (x, y) \text{ above } L \text{ or in its upper half,} \\ \dfrac{b}{k}, & \text{as } (x, y)=(0, 0). \end{cases} \tag{2.9}$$

For the optimal phase trajectory we may refer to Ref. 1 and the associated optimal controlled transient process is shown in Fig. 3.

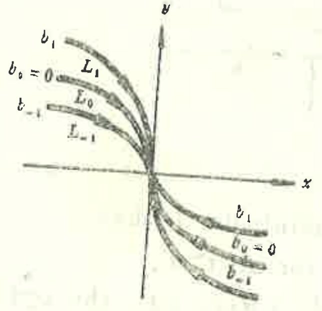


Fig.2

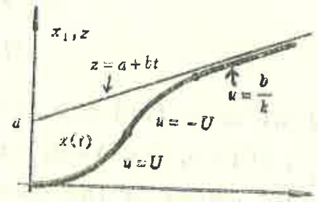


Fig.3

3. Speed Control Problem

Let the sample period be T_0 and then we have approximately

$$b=\dot{z}(t)\doteq\frac{z(t_i+T_0)-z(t_i)}{T_0}=\frac{z_{i+1}-z_i}{T_0} \tag{3.1}$$

where $z_i=z(t_i)$ —— the i-th sample value of z. By (2,7) and (2,9),

$$u=\begin{cases} U, \text{ as } y\geq 0 \text{ and } s(kU+b)\,\mathrm{Ln}\left(1+\dfrac{y}{kU+b}\right)-sy-x<0, \\ -U, \text{ as } y\geq 0 \text{ and } s(kU+b)\,\mathrm{Ln}\left(1+\dfrac{y}{kU+b}\right)-sy-x\geq 0, \\ \dfrac{b}{k}, \text{ as } x=0 \text{ and } y=0, \\ U, \text{ as } y<0 \text{ and } s(kU-b)\,\mathrm{Ln}\left(1-\dfrac{y}{kU-b}\right)+sy+x\leq 0, \\ -U, \text{ as } y<0 \text{ and } s(kU-b)\,\mathrm{Ln}\left(1-\dfrac{y}{kU-b}\right)+sy+x>0, \end{cases} \tag{3.2}$$

or by (2.8) and (2.9),

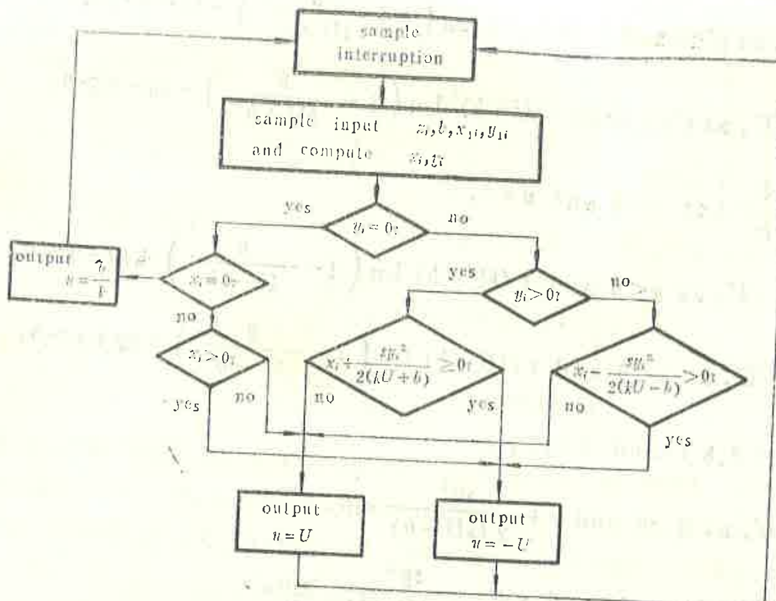
$$u=\begin{cases} U, \text{ as } y\geq 0 \text{ and } x+\dfrac{sy^2}{2(kU+b)}<0, \\ -U, \text{ as } y\geq 0 \text{ and } x+\dfrac{sy^2}{2(kU+b)}\geq 0, \\ \dfrac{b}{k}, \text{ as } x=0 \text{ and } y=0, \\ U, \text{ as } y<0 \text{ and } x-\dfrac{sy^2}{2(kU-b)}\leq 0 \\ -U, \text{ as } y<0 \text{ and } x-\dfrac{sy^2}{2(kU-b)}>0. \end{cases} \tag{3.3}$$

The real-time speed control scheme is shown in Flow Diag. 1.

4. Measured Optimal Switch-curve

Although the above control system is optimal in theory, we can not conclude that it is better than the used one designed by means of classical regulating theory. Because in a practical tracker, 1) these system parameters can not be measured exactly; 2) the model (1.1)-(1.2) is only a simplified one; 3) there exist friction, backlash, nonlinearity, etc. According to the Maximum Principle, the optimal switchcurve itself is a state trajectory passing through the origin, so we provide a method for measuring the real optimal switch-curve as follows.

1). Use an integrator generating an output $a+bt$ to replace $z(t)$ and let the initial state be $x_1(0)=y_1(0)=0$, then in the deviation plane we have $x(0)=-a$ and $y(0)=-b$. The structure is shown in Fig. 4



Flow diag. 1

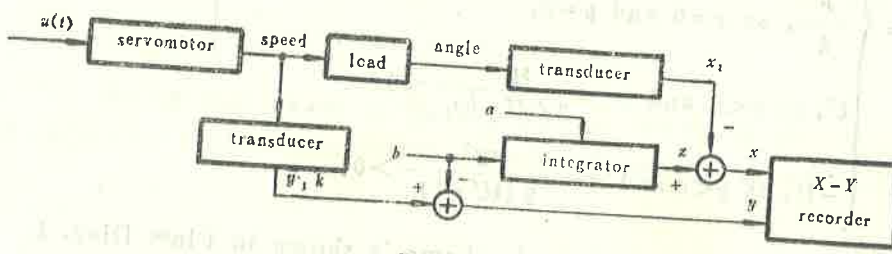


Fig. 4

2) The control $u(t)$ (open loop) starts with $u=U$ and is changed to $u=-U$ from some instant t_0 until the trajectory intersects the x-axis for the second time. There are three possibilities as shown in Fig. 5.

3) Varying t_0 such that the corresponding trajectory reaches(0,0). Then the segment A0 of this trajectory (see Fig. 5) is the upper half of the optimal switch-curve.

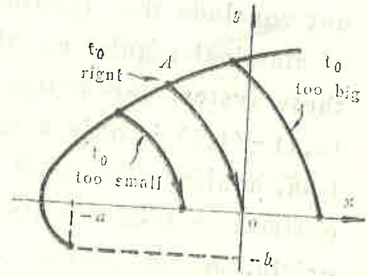


Fig.5

4). The lower half one can be obtained in a similar way.

Note that the optimal switch-curve depends on b. If the memory in the computer is big enough, we can generate a family of switchcurves with different values of b. Otherwise the following correcting formula can be employed.

Let $y=y^*_{b'}(x)$ be the measured optimal switch-curve, then for any value of b we have approximately (see eq. (2.8))

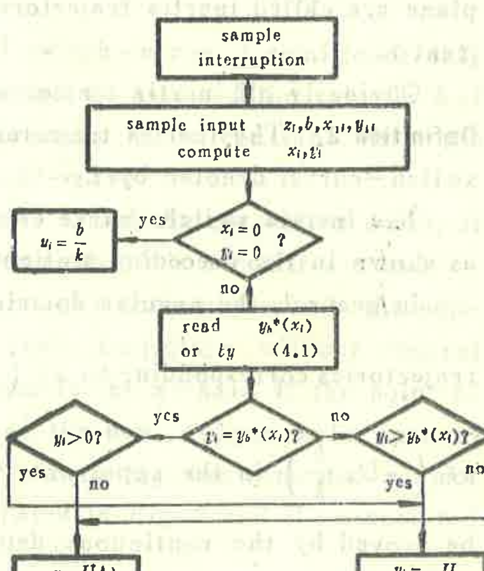
$$y^*_b(x)=y^*_{b'}(x)\left(\frac{kU+b'}{kU+b}\right)^{\frac{1}{2}}, \quad \text{as } x<0;$$

$$y^*_b(x)=y^*_{b'}(x)\left(\frac{kU-b'}{kU-b}\right)^{\frac{1}{2}}, \quad \text{as } x>0, \tag{4.1}$$

where $y^*_b(x)$ is the optimal switch-curve corresponding to b.

No doubt, the measured switch-curve is more reliable than the one computed by (2.7) or (2.8), because the nonlinear influence in the case of large disturbances is implied and any linear model as (1.1)-(1.2) is only approximately correct under small deviations.

The real-time speed control program is shown in Flow Diag.2. The simulation experiment shows that, to a certain extent, some nonlinearities can be treated by employing the measured switch-curve.



Flom diag. 2

5. Program for Ensuring Stability

The control systems given above are stable in theory but they can not guarntee stability in practice. Two reasons for this are: 1), the sample period exists so the state points sampled can not precisely fall on the switch-curve; 2), in the region near the origin there are many complicated factors influencing the stability.

To deal with case 1), we shorten the sample period when the state point is close to the switch-curve. By the practical implications of x and y (y means the velocity of x), we denote

$$T_i = \frac{|\triangle x_i|}{|y_i|}, \tag{5.1}$$

where $|\triangle x_i|$ is the level distance between the measured state point and the switch-curve. Then, if $T_i < T_0$, we change the sample period T_0 into some $T_0^i < T_i$ (for example, let $T_0^i = \frac{1}{2}T_i$).

To deal with case 2), we note that the followed signal z has been almost caught by this time, then rapidity is already a secondary question compared to the stability. Denote

$$O_c = \{(x,y);\ |x| < c,\ |y| < c\} \tag{5.2}$$

Definition 1. If $v = 0$ $\left(i.\ e.\ u = \frac{b}{k}\right)$, all the trajectories on the $x-y$ plane are called inertia trajectories (corresponding to free movement).

Obviously all inertia trajectories approach the x-axis.

Definition 2. The inertia trajectory reaching (0, 0) is called inertia switch-curve, denoted by L_0.

The inertia switch-curve can be measured by the similar way as shown in the preceding section.

In general, the angular domain between L_0 and L is full of these trajectories corresponding to $u \in \left(\frac{b}{k}, U\right)$ in the lower plane and $u \in \left(-U, \frac{b}{k}\right)$ in the upper one (see Fig. 6). This conclusion can be proved by the continuous dependence of phase trajectories on parameters.

We divide O_c into four parts denoted by A, B, C, and D as shown in Fig. 6. The control u in O_c is given as

$$u=\begin{cases}-U, & \text{as } (x,\ y)\in A,\\ \dfrac{b}{k}-\dfrac{y-y^0(x)}{y^*(x)-y^0(x)}\left(U+\dfrac{b}{k}\right), & \text{as } (x,\ y)\in B,\\ U, & \text{as } (x,\ y)\in C,\\ \dfrac{b}{k}+\dfrac{y^0(x)-y}{y^0(x)-y^*(x)}\left(U-\dfrac{b}{k}\right), & \text{as } (x,\ y)\in D,\end{cases} \tag{5.3}$$

6. Adaptive Control Problem

Based on the method for measuring optimal switch-curve shown in Sec. 4 we give a program for doing them by the computer in the control process. Then the speed control system is adaptive.

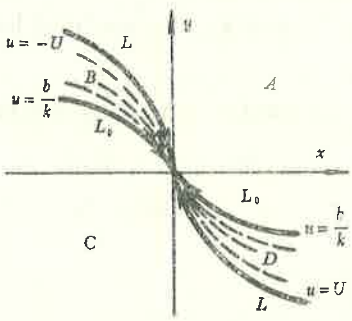


Fig. 6

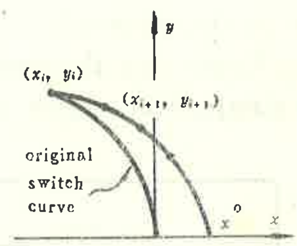


Fig. 7

The correction of the optimal switch-curve is significant only when the deviation is large. Although we assumed the velocity b of $z(t)$ to be constant, it is only necessary to be constant while the state point is moving on the switch-curve.

Definition 3. A state point (x_i, y_i) is called the correction point, if it satisfies, 1) $|y_i|>h$; 2) $y_iu_i\leq 0$; 3) $x_iy_i<0$. (h is a positive cons.)

After the state point passes through a correction point, the computer needs only to record the state trajectory without control output until this trajectory intersects the x-axis. If the point of intersection is $(x^0,\ 0)$, $x^0\neq 0$, then the new approximate optimal switch-curve can be obtained by translating the recorded trajectory into $-x^0$. This method is illustrated in Fig. 7 and the associated flow diagram is not very hard to make.

After adding this adaptive program to the preceding control system we can use any straight line $x=cy$, $c\leq 0$, instead of the optimal switch-curve at the beginning, and this system will be self-optimizing and self-stabilizing.

7. Calculation of the Speed-Catch Time T

Let (x_i, y_i) be a given initial state, the problem is to estimate the speed-catch time $T=T(x_i, y_i)$. It arises in the following situations: 1) The track transient time T is useful to the command computer which is usually separate from the control computer, 2) A group of trackers are controlled by one control computer.

By (1.1-2) and (2.7) the speed-catch time T can be estimated as

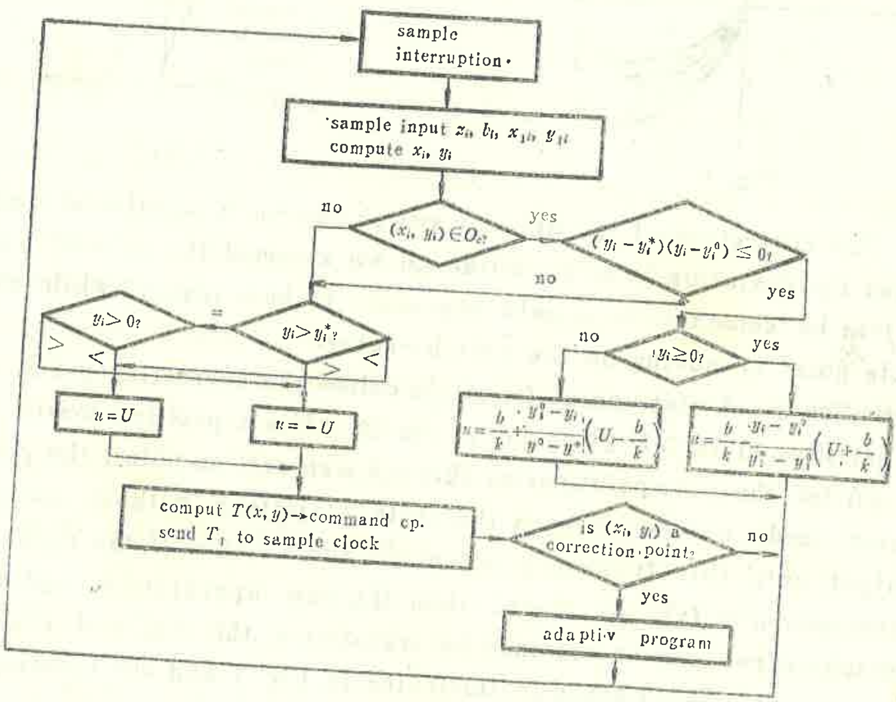
$$T=T_1+T_2 \tag{7.1}$$

where T_1——the time to get to the switch-curve from (x_i, y_i);

T_2——the time to get to the origin along L.

The detailed formulas for computing T_1 and T_2 are omitted here for saving space.

Combining the preceding discussions an entire control program is obtained as shown in Flow Diag. 3



Flow diag. 3

8. A Real-time Simulation of the Track Control System

A hybrid computer simulation scheme of the entire speed control system is given in Fig. 8. The purpose of this simulation is three fold: 1) Comparision of this control program with the used one; 2) Performance test (rapidity, stability, adaptability); 3) Choice of parameters.

The main results are briefly summarized as follows.

1) The bigger the time constant s or the greater the initial deviation $(x_0, \dot{x}_0)$, is the more superior the speed-tracking system to the classical one willbe. When $s=1$ and initial values $x_0=2, \dot{x}_0=0$, the speed-catch time is shortened by $\frac{1}{5}-\frac{1}{3}$.

2) As the target velocity $b \neq 0$, the nonsymmetry of the optimal control schemes (see Fig. 2) is ignored in the classical designs, so that the superiorty of this speed-tracker is apparent.

3) In small deviation region the scheme given by (5.3) is feasible, the advantage of this speed-tracker is merely its simplicity.

4) Applying the measured optimal switch-curve method to gain adaptability is much simpler than applying the ordinary identification techniques.

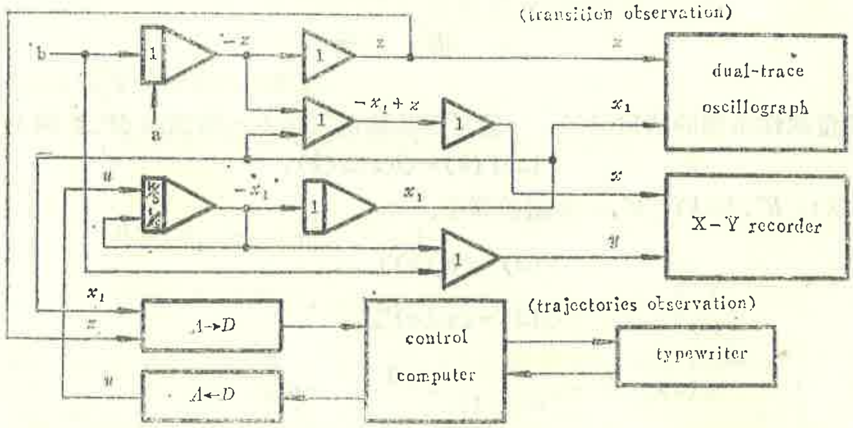


Fig. 8

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