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The Real-Time Monitoring of Time Series With Discontinuities and Quasi-Periodic Components

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Abstract

A new criterion is described for choosing the Parameters of the forecasting algorithm corresponding to the linear growth model, where quasi-periodic components exist in the series. The forecasts based on the resulting algorithm allow prediction of the mean level and slow trend of a process in spite of large random variations, stochastic trend changes and possible quasi-periodic components in the series. A new slope change detection method is also discussed based on a dual model approach. This method in conjunction with previously developed methods for detection of step changes and transients allows monitoring of a time series with quasi-periodic components and subject to discontinuities.

1. Introduction

The linear growth model is widely used in the medical and commercial fields. The general linear growth model (GLGM) forecasting algorithm is well-known Holt algorithm (1957) which contains two parameters. The restricted linear growth model (RLGM) forecasting algorithm was developed by Stoodley (1979) and contains one parameter. The values of the parameters in these algorithms are usually determined by a least squares criterion which minimises the sum of squares of the one-step ahead forecasting errors. In practice some time series contain periodic components at more than one frequency, and in addition the periods of such components may not

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be an integral multiple of the sample interval of the series. In these cases, the least squares criterion will not give satisfactory values for these parameters. Seasonal models also appear to be inadequate for modelling such quasi-periodic series. This paper describes a new criterion for modelling such series. The forecasts based on the criterion allow prediction of the mean level of and slow trends in the series in spite of large random variations, discontinuities and possible multi-periodic components in the series. The approach of Stoodley and Mirnia is developed to detect changes in this type of series. In addition a new slope detection method is developed which has many advantages over the existing methods. These developments have been successfully applied in a patient monitoring system (Lu 1983).

2. Frequency response functions of the forecasting algorithm

It is well known that random variations are represented by a power spectrum with almost equal power over a wide frequency band, and slow trends are represented by a low frequency component in the frequency domain. For a quasi-periodic series the periodic components usually occur around or above some particular frequency. It is most necessary to study the response of the forecasting algorithm to these different frequency components in the series. In this situation the forecasting algorithm may be regarded as a filter in a broad sense. Holt algorithm is defined as follows:

$$\begin{aligned} \hat{z}_t(k) &= \hat{m}_t + k\hat{b}_t \\ \hat{m}_t &= \hat{m}_{t-1} + \hat{b}_{t-1} + A_1(z_t - \hat{z}_{t-1}(1)) \\ \hat{b}_t &= \hat{b}_{t-1} + A_2(z_t - \hat{z}_{t-1}(1)) \end{aligned} \tag{1}$$

Here $\hat{z}_t(k)$ is the k-sthp ahead forecast at time t. $\hat{m}_t$ and $\hat{b}_t$ may be interpreted as the estimated mean level and slope of the series at time t. A_1 and A_2 are two parameters to be determined. If A_2 is zero, the algorithm corresponds to the RLGM. By some algebraic manipulation, it is easy to show the relationship between the one-step ahead forecast and actual readings in term of the parameters A_1 and A_2. Furthermore, the backward shift operator B is simply $e^{-j\omega T}$ in the frequency domain, where T is the sampling interval. Therefore the frequency response function for Holt algorithm can be obtained as:

$$F_1(jwT)=\frac{\hat{z}_t(1)}{z_t}=\frac{A_1+A_2-A_1e^{-jwT}}{1-(2-A_1-A_2)e^{-jwT}+(1-A_1)e^{-2jwT}} \quad (2)$$

The frequency response function for the RLGM can be directly obtained from formula (2) by letting $A_2=0$. Thus:

$$F_2(jwT)=\frac{A_1}{1-(1-A_1)e^{-jwT}} \quad (3)$$

3. Frequency response criterion for choosing the parameter in the RLGM forecasting algorithm

The frequency domain analysis suggests an alternative criterion for the choice of parameters in the forecasting algorithms of the GLGM and RLGM. After plotting the amplitude spectrum($|F_2(jwT)|$) with different values of A_1 in the case of the RLGM, the following facts become clear:

a) If A_1 is less than one, the forecasting algorithm behaves like a low pass filter. The amplitude decreases constantly with increasing w. There are no oscillations (Gibb's phenomenon).

b) If A_1 is larger than one, the amplitude increases with ω.

It can be shown that the RLGM may be equivalently represented as an ARIMA(0, 1, 1) model, if and only if A_1 is less than one and positive. It can also be shown (Lu, stoodley 1985) that A_1 may be derived from the limiting form of the Bayesian updating algorithm for the RLGM and is equal to:

$$A_1=(2+R-\sqrt{R^2+4R})/2$$

$$\text{where:}\quad R=\sigma_r^2/\sigma_\varepsilon^2$$

Again A_1 is positive and less than one. Hence theoretically A_1 should only take positive values of less than one. In practice the least squares criterion is sometimes found to give a value of A_1 greater than unity which is unacceptable. The new criterion of choosing A_1 is as follows.

We define $|F_2(jw_cT)|=c$, where c is a constant value. Then (1-c) may be regarded as the degree of decay of the amplitude of the forecasts at cut-off frequency w_c. For a given c, w_c is given in terms of A_1 as:

$$w_cT = \cos^{-1}\left(A_1^2 \frac{(1-c^2)-2A_1c^2-2c^2}{-2c^2(1-A_1)} \right) \tag{4}$$

In order to avoid aliasing, w_cT must be less than π. A_1 and c must be positive and less than one. Under these conditions, the value of A_1 and w_c have a one-to-one relationship. As a criteion, formula (4) is plotted in Fig-1 with $c=0.37$. The choice of A_1 is straightforward. For example, suppose that a series is sampled at one second intervals and frequency analysis of the series shows that the quasi-periodic components are above 0.1 Hz. The value of A_1 required is 0.22 from Fig-1.

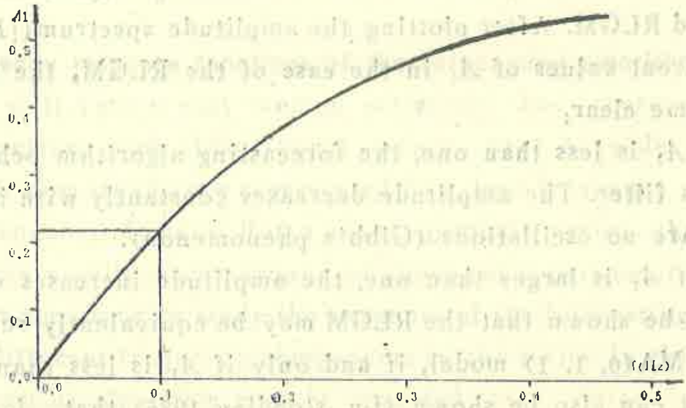

Fig.1 Criterion for A_1(63%decay of amplitude)

4. Frequency response criterion for choosing the parameters in Holt algorithm

The case of the GLGM is more complicated, because the frequency response function contains two parameters, A_1 and A_2. However the same approach can be used as in the RLGM case. In practice the value of A_1 is usually much larger than the value of A_2. For example, the modelling of economic time series often leads to a value of A_1 less than 0.3 and A_2 less than 0.03.

On plotting the amplitude spectrum ($|F_1(jwT)|$), similar behaviour is observed to that discussed above for the RLGM. Further more it is very interesting to notice from Fig-2 that incorporation of A_2 in the Holt algorithm enchances the amplitude of the forecasts at very low frequencies. Under the condition that the A_1 is less than one and A_2 is much less than A_1, the frequency at which the amplitude response function has a maximum value is defined as w_m. It is well known that stochastic trends are represented by very low frequencies in the frequency domain. This enchancement reflects the fact in the frequency domain that the GLGM allows the modelling of stochastic trends in time series.

Study of the frequency response function (2) shows that both w_m and w_c are determined by the value of A_1 and A_2, as well as c. It is cumbersome to represent w_m and w_c analytically in term of A_1 and A_2 for a given value of c. Instead a computer program is used to search for these frequency values at different values of A_1 and A_2 for a given c.

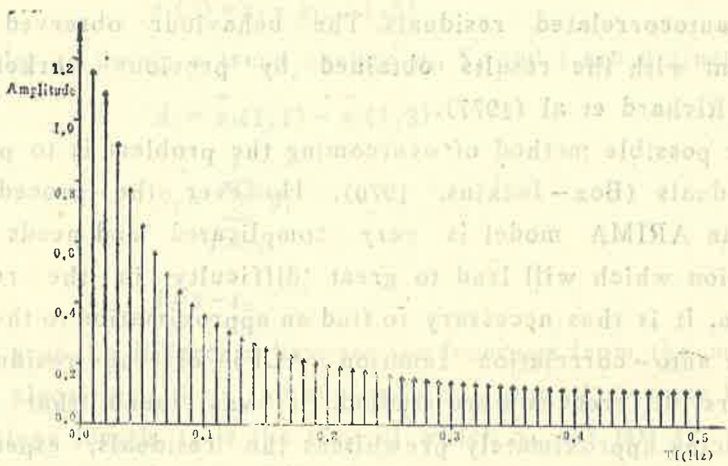

Fig. 2 Frequency response of Holt algorithm with $A_1 = 0.2$ and $A_2 = 0.01$

The new criterion should be used for modelling the linear growth model in the following situations:

1) Whenever the forecasts are required to predict the true level and slow trends, rather than to follow quasi-periodic variations of the process.

2) Whenever a statistical study of the series indicates that the series is not fitted by the correct subset of the ARIMA (0, 0, 1), ARIMA (0, 1, 1) and ARIMA (0, 2, 2) models.

3) Whenever the series has one or more periodic components at high frequencies.

However, the criterion has the following limitations:

1) In formula (2) and (3), only one-step ahead forecasts are considered. If forecasting needs to be carried out at more than one step ahead, an extension of the method, following the same approach as above, is necessary.

2) The method does not guarantee a minimum value for the sum of squares of the residuals.

5. Detection of step change and transients in quasi-periodic series

Stoodley and Mirnia (1979) first suggested a method for detection of changes in a discontinuous time series based applying the backward CUSUM scheme to the one step ahead forecasting errors. However, when the method is directly used in the type of series being discussed in this paper, it was found that the scheme is not robust. A further study has shown that it is because of the highly autocorrelated residuals. The behaviour observed here is consistent with the results obtained by previous workers, Page (1955), Richard et al (1977).

One possible method of overcoming the problem is to prewhiten the residuals (Box-Jenkins, 1970). However the procedure for fitting an ARIMA model is very complicated and needs operator interaction which will lead to great difficulty in the real-time situation. It is thus necessary to find an approximation to the method.

The auto-correlation function (ACF) of the residuals and their first differences were studied. It was found that the first differencing approximately prewhitens the residuals, especially if there exists a periodic component in the residuals. Therefore two-sided CUSUM scheme is implemented on the first difference of the residuals. In practice it is found that the modified scheme is extremely sensitive to step changes and transients.

6. Detection of Slope Changes in Qu Asi-Periodic Series

A drawback of the above scheme is that the detection of a slope change is not guaranteed because of the difference of the residuals used. However a new method has been developed to detect slow trend changes which not only compensates the drawback of the modified scheme, but also has certain advantages over existing methods. The performance of the new method is independent of the residuals and it appears to be very robust.

With the new method two forecasting algorithms used are defined as the main and auxiliary algorithms and are applied to each series as follows:

main forecasting algorithm:

$$\begin{aligned} \hat{z}_t(1,1) &= \hat{m}_t(1) + \hat{b}_t(1) \\ \hat{m}_t(1) &= \hat{m}_{t-1}(1) + \hat{b}_{t-1}(1) + A_1 e_t(1) \\ \hat{b}_t(1) &= \hat{b}_{t-1}(1) + A_2 e_t(1) \\ e_t(1) &= z_t - \hat{z}_{t-1}(1,1) \end{aligned} \tag{5}$$

auxiliary forecasting algorithm:

$$\begin{aligned} \hat{z}_t(1,2) &= \hat{m}_t(2) + \hat{b}(2) \\ \hat{m}_t(2) &= \hat{m}_{t-1}(2) + \hat{b}(2) + A_3 e_t(2) \\ e_t(2) &= z_t - \hat{z}_{t-1}(1,2) \end{aligned} \tag{6}$$

In order to detect a trend change, d_t, S_t and I are defined by:

$$\begin{aligned} d_t &= \hat{z}_t(1,1) - \hat{z}_t(1,2) \\ S_t &= \sum_{i=t_0}^{t} d_i \\ I &= t - t_0 \end{aligned} \tag{7}$$

Here d_t is the difference between the forecasts from the main and auxiliary algorithms. S_t is the accumulated value of d_t since time t_0. I is the slope length. t_0 is the time at which S_t was last set to zero. S_t is set to zero under the following circumstances; 1) a step change is detected, 2) any type of transient is detected, 3) the sign of d_t changes, 4) a trend change is signalled.

If there is no slope change in the series, d_t has a zero mean and S_t fluctuates around this mean. Whenever a trend change occurs, the value of S_t increases or decreases quadratically. A slope change is signalled when the absolute value of S_t exceeds a preset limit ($LIM = k\sigma_e$), where k is a constant and σ_e is the standard deviation of the first difference of the residuals.

7. Theoretical analysis of the dual model slope detection method

In this section, the new slope detection method described above is analysed theoretically. An obvious alternative expression of d_t in formula (7) is:

$$d_t = \hat{m}_t(1) + \hat{b}_t(1) - \hat{m}_t - \hat{b}(2) \qquad (8)$$

In order to assess the performance of the method, it is necessary to find a mathematical model for d_t. It can show that d_t may be expressed in terms of the parameters A_1, A_2, A_3 and observation z_t. Then after some manipulation, we may have

$$R_1R_2d_t = [(A_1 + A_2 - A_3) + (A_3 - A_1)B](1 - B)\,z_t - A_2\,\hat{b}(2)$$

where: $R_1 = 1 - (2 - A_1 - A_2)B + (1 - A_1)B^2 \qquad (9)$

$$R_2 = 1 - (1 - A_3)B$$

It is assumed that:

$$A_2 \ll (A_1 - A_3) \qquad (10)$$

and certainly $A_2 < A_1$. Since all significant slope changes are detected within short time, it is reasonable to assum that the pattern of the trend does not change within this time. Therefore it is assumed that z_t is generated from a RLGM originally with trend b_0. With condition (10) taken into account, it can be shown that equation (9) becomes:

$$[1 - (1 - A_1)B][1 - (1 - A_3)B]d_t = (1 - B)^{-1}A_2(b_0 - \hat{b}(2)) + (A_1 - A_3)(1 - \theta B)d_t$$

where: $\theta = [\,2 + R - \sqrt{R^2 + 4R}\,]/2$

$$R = \sigma_\gamma^2/\sigma_\varepsilon^2, \quad d_t \sim N(0, \sigma_\varepsilon^2/\theta) \qquad (11)$$

Equation (11) gives a theoretical ARMA(2,1) model for d_t. It is worth pointing out that b_0 is the gradient of the series and $b(2)$ is the gradient of the auxiliary model. If there is no trend change, $\hat{b}(2)$ is equal to b_0. Then the first term (trend term) on the right hand side of equation (11) is zero. If there is a trend change, then the series d_t certainly has a deterministic trend. Hence:

1) When a trend in a monitored series changes, d_t will have a deterministic trend, and S_t will then decrease or increase quadratically. Thus S_t is a sensitive indicator of a trend change.

2) The deterministic trend in d_t is proportional to the gradient change in z_t. Therefore with a given preset limit, the larger the trend changes, the quicker the detection will be.

These results are confirmed by extensive computer simulations and practical experience.

8. Conclusions

This paper has been concerned with modelling quasi-periodic time series and detecting possible changes in such series. A new criterion has been developed for choosing the parameters in the forecasting algorithms. The criterion overcomes many restrictions associated with commom seasonal modelling techniques. The two-sided backward CUSUM scheme has been modified to allow the detection of transients and step changes. A new method based on a dual model approach has been developed to detect short term trend changes. The methods developed here have been successfully tested on simulations and used in a patient monitoring system.

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非连续含有准周期分量时间序列的实时监测

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摘 要

本文提出了一种新的线性增长型数字模型选值标准。该标准适合于分析含有准周期分量的时间序列。根据这种新标准选值，尽管时间序列含有随机的倾向变化和准周期分量，线性增长型模型仍可预报该随机过程的期望和倾向。基于双模型法，本文还提出了一种新的披露微小倾向变化的方法。应用该方法及对阶跃变化和瞬间变化的探测，使我们有可能对含有伪周期分量的非连续时间序列进行实时的监测和预报。

书刊评介

《信号分析与处理》
——《现代控制系统理论小丛书》之一

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由关肇直、许文源、贾沛璋编著的《现代控制系统理论小丛书》之一——《信号分析与处理）一书，将由科学出版社出版。

该书叙述信号分析的基本理论与处理方法，包括信号分类、Z变换与离散系统、Fourier变换与Hilbert变换、数字滤波器设计原理、同态滤波与最小平方滤波、谱分析、频谱有限信号、反褶积等内容。所讨论的信号包括确定性和随机性两大类。全书除包括信号分析与处理的经典内容外，同时吸收了近年来该领域的新进展，也包括了作者在这方面的部分研究成果。

该书理论较严谨，观点较新颖，在叙述各种处理方法时又注意便于读者直接应用。该书适于高等院校有关专业的教师、研究生及有关工程技术人员阅读参考。