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Design Schemes for Robust Adaptive Control Systems*

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Abstract

Two design schemes for robust adaptive control systems are presented. In the first design scheme a model reference adaptive control system with a logic-decision device is proposed. The global stability of the whole system is guaranteed even when the structure of the controlled plant is not known accurately. The hybrid adaptive control system is used in the second design scheme. A new parameter estimator is used to determine the parameters of the main transfer function of the controlled plant. The parameters of the controller are adjusted at discrete instants by using the estimation results.

I. Introduction

Adaptive control theory has made much progress in recent years. For example, the global asymptotic stability of the adaptive control systems has been proved[1,2]. But up to now most of the design schemes for adaptive control systems are based on the assumptions that the structure of the controlled plant is accurately known, even though its parameters are unknown. These assumptions are rather too restrictive. Practically it is possible that some subsidiary characteristics of the controlled plant can't be considered in the design. Theoretical analysis and simulations[3] show that this subsidiary characteristic of the controlled plant may cause instability of the whole system. Therefore, the problem of robustness of the adaptive control systems has not yet been solved. It draws much attention

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during the recent years[4,5,6]. In this paper design schemes for robust adaptive control systems are presented.

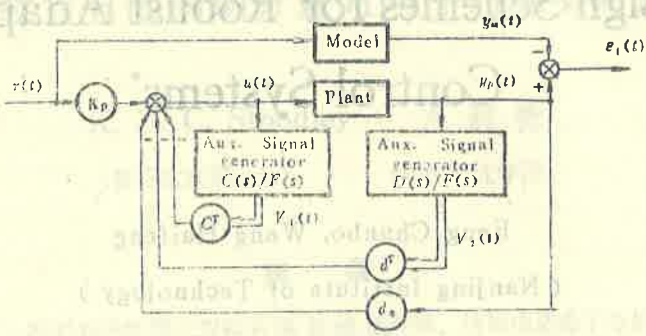

Fig. 1

II. Intelligent Model Reference Adaptive Control System (MRACS)

The general design scheme for MRACS is shown in Fig. 1. For the above design scheme the equivalent error model shown in Fig. 2 can be obtained[7].

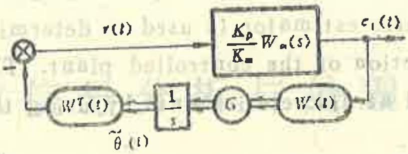

Fig. 2

The meanings of the nomenclature in Fig. 2 are as follows:

$W_m(S)$——transfer function of the reference model.

$$w^T(t) \triangleq [r(t), v_1^T(t), y_p(t), v_2^T(t)]$$

$$\tilde{\theta}^T(t) = \theta^T(t) - \theta^{*T}$$

$$\theta^T(t) \triangleq [k_p(t), c_1(t), \cdots, c_{n-1}(t), d_0(t), d_1(t), \cdots, d_{n-1}(t)]$$
$$= [k_p(t), C^T(t), d_0(t), d^T(t)]$$

θ^*——matched controller parameter vector.

G——constant positive-definite adaptation gain matrix.

The equivalent error model shown in Fig. 2 is obtained under the assumption that the structure of the controlled plant is exactly the same as that of the reference model. It is shown[8] by the Lyapunov function method or hyperstability theory that the whole system is globally asymptotically stable if $W_m(s)$ is strictly positive real. $W_m(s)$ can be chosen to be strictly positive real only when its pole excess is not greater than 1. If the pole excess of the plant is greater than 1 then some complicated design schemes[7,9] may be used. In all these design schemes the structure of the plant is assumed to be accurately known. If some subsidiary characteristics of plant are existent but unknown, then $W_m(s)$ in the error model will not be exactly equal to the transfer function of the reference model. Let us express the former by $W'_m(s)$ in this case. $W'_m(s)$ may not be strictly positive real. Therefore, the problem of global stability is not solved.

It is known that for causal systems the concepts for positivity, hyperstability and passivity are equivalent[10]. From the viewpoint of passivity analysis the global stability of the whole adaptive control system is guaranteed if the equivalent error model system as shown in Fig. 2 is strictly passive. The feedback loop in Fig. 2 is an integrating parameter adaptation loop. It is passive[10]. As shown in[11] that the necessary and sufficient condition of the strict passivity of the whole system shown in Fig. 2 is that the forward loop is strictly passive. But we have pointed out that $W'_m(s)$ can't be strictly passive if its pole excess is greater than 1, or it may be not strictly passive if there are some unconsidered subsidiary characteristics of the plant. If there is some scheme can make the forward loop strictly passive in any case, then the problem is solved. Such a scheme exists. It is shown in Fig. 3. SFS

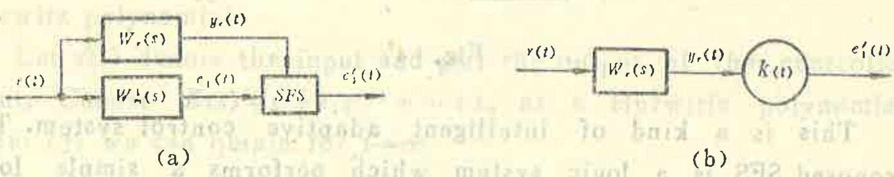

Fig. 3

in Fig. 3(a) is a sign-following system which is realized by a sign-comparison device connected to an ideal relay. SFS keeps the sign of the $e_1'(t)$ same as that of $y_r(t)$. Hence the system shown in Fig. 3(a) is equivalent to that shown in Fig. 3(b), where

$$k(t)=\left|\frac{\int_0^t h_m'(t-\tau)r(\tau)d\tau}{\int_0^t h_r(t-\tau)r(\tau)d\tau}\right| \tag{1}$$

and $h_m'(t-\tau)$ and $h_r(t-\tau)$ are impulse reponses of $W_m'(s)$ and $W_r(s)$ respectively. $k(t)$ is never negative. Now it is easy to see that the system shown in Fig. 3(a) is strictly passive if $Wr(s)$ is strictly passive no matter whether $W_m'(s)$ is strictly passive or not. The only requirement is that $W_m'(s)$ should be stable.

Now a design scheme for robust MRACS as shown in Fig. 4 can be suggested. In this scheme $W_m(s)$ has the same numbers of poles and zeros as those of the plant without consideration of the subsidiary characteristics of the plant. $W_m(s)$ may not be strictly passive. $W_r(s)$ has the same number of poles as that of $W_m(s)$, but it is chosen to be strictly passive. The $e_1'(t)$ instead of the $e_1(t)$ is now used for the adaptation algorithms.

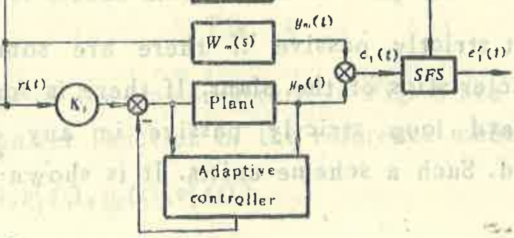

Fig. 4

This is a kind of intelligent adaptive control system. The proposed SFS is a logic system which performs a simple logic decision. If $e_1(t)$ is used in the algorithms for controller parameter adaptation it will cause the parameter adaptation process to be divergent in some intervals. SFS will change the sign of this model error in these intervals and thus make the parameter adaptation process always convergent. Thus the global stability of the whole system is guaranteed.

III. Hybrid Adaptive Control

The usual MRACS is a nonlinear time-varying system. The global stability of such a system is hard to guarantee. If the parameters of the adaptive controller are adjusted only at discrete instants, then the whole system will be linear and time-invariant during the intervals when the parameters of the controller are kept constant. The stability of such a system is much easier to solve. Such an adaptive control system is called a hybrid adaptive control system. A design scheme is shown in [12].

In this section a design scheme of the hybrid adaptive system is suggested. First of all a parameter estimator for the controlled plant is presented when it possesses subsidiary high frequency characteristics not considered in the model. The parameters of the adaptive controller are adjusted at individual instants by using the estimated parameters of the plant. The conditions of stability of the whole system will be given.

1. parameter estimation

Let the transfer function of the controlled plant be expressed by

$$P(s)=\frac{B_L(s)}{A_L(s)}\cdot\frac{B_H(s)}{A_H(s)} \tag{2}$$

where $B_L(s)/A_L(s)$ denotes the main characteristic of the controlled plant, and

$$\frac{B_L(s)}{A_L(s)}=\frac{b_1 s^{n-1}+b_2 s^{n-2}+\cdots+b_n}{s^n+a_1 s^{n-1}+\cdots+a_n}, \tag{3}$$

n is known. $H(s)=B_H(s)/A_H(s)$ denotes the subsidiary high frequency characteristic of the controlled plant, and $A_H(s)$ is a Hurwitz polynomial.

Let $u(t)$ denote the input and $y(t)$ the output of the controlled plant, Choose $K(s)=s^n+k_1 s^{n-1}+\cdots+k_n$ as a Hurwitz polynomial. From (2) we can obtain for $t\to\infty$

$$Y(s)=\frac{1}{K(s)}[K(s)-A_L(s)]Y(s)+\frac{B_L(s)}{K(s)}U(s)+[H(s)-1]\frac{B_L(s)}{K(s)}U(s).$$

Hence for sufficiently large t we have

$$y(t)=v^T(t)\theta^*+z(t) \tag{4}$$

where

$$\theta^*=[k_1-a_1,k_2-a_1,k_2-a_2,\cdots,k_n-a_n,b_1\cdots,b_n]^T \tag{5}$$

$$v(t) = L^{-1}\left\{ \left[\frac{1}{K(s)} [s^{n-1},\cdots,s,1]Y(s),\ \frac{1}{K(s)} [s^{n-1},\cdots,s,1]U(s) \right]^T \right\} \tag{6}$$

$$z(t) = L^{-1}\left\{ \frac{B_L(s)}{K(s)} [H(s)-1]U(s) \right\} \tag{7}$$

θ^* should be estimated and $v(t)$ is the signal vector which can be obtained from the outputs of the filter $1/K(s)$ excited by $u(t)$ and $y(t)$.

If we take the value of $y(t)$ and $v(t)$ at discrete instants t_k with the sampling period Δt in $[t_1, t_2]$, from (4) we have

$$y(t_k) = v^T(t_k)\theta^* + z(t_k) \tag{8}$$

Multiplying both sides of (8) by $v(t_k)$ and taking summation we obtain a matrix equation

$$Q\theta^* + \varphi = \eta \tag{9}$$

where

$$Q = \sum_{k=1}^{M} v(t_k)v(t_k)^T \in R^{2n\times 2n},\quad \Delta tM = t_2 - t_1,$$

$$\eta = \sum_{k=1}^{M} y(t_k)v(t_k) \in R^{2n\times 1},$$

$$\varphi = \sum_{k=1}^{M} z(t_k)v(t_k) \in R^{2n\times 1} \tag{10}$$

If the input signal $u(t)$ is sufficiently "rich" and Δt is chosen to be appropriately small, the matrix Q will be nonsingular (see Appendix I). Denote the estimated parameter vector of θ^* by $\hat{\theta}$. then from (4) we have

$$\hat{y}(t_k) = v(t_k)^T\,\hat{\theta} \tag{11}$$

From (9) we have

$$\hat{\eta} = Q\,\hat{\theta} \tag{12}$$

where $\hat{\eta} \triangleq \sum_{k=1}^{M} \hat{y}(t_k)v(t_k)$.

Denote the error vector of the estimated parameter vector by

$\widetilde{\theta}=\hat{\theta}-\theta^*$. Taking the following strict convex function

$$L(\widetilde{\theta})=1/2(\eta-\hat{\eta})^T(\eta-\hat{\eta})=1/2\,\widetilde{\theta}^TQ^TQ\widetilde{\theta}-\varphi^TQ\,\widetilde{\theta}+1/2\varphi^T\varphi \quad (13)$$

as performance index we can get

$$\hat{\theta}=Q^{-1}\eta \quad (14)$$

as a minimization solution of (13). Different numerical methods can be used to obtain $\hat{\theta}$ in a finite time[13].

From (9) and (14) we heve

$$\widetilde{\theta}=Q^{-1}\varphi \quad (15)$$

By using (14), (15) it is easy to show that the norm of the relative error of the parameter error vector satisfies the following inequaity

$$\frac{1}{\mathrm{Cond}(Q)}\frac{\|\varphi\|_2}{\|\eta\|_2}\leqslant\frac{\|\widetilde{\theta}\|_2}{\|\hat{\theta}\|_2}\leqslant\mathrm{Cond}(Q)\frac{\|\varphi\|_2}{\|\eta\|_2} \quad (16)$$

where $\mathrm{Cond}(Q)$ is the condition number of the matrix Q.

$\|\varphi\|_2$ can be estimated as follows (see Appendix II):

$$\|\varphi\|_2\leqslant C\,\sup_t|\dot{u}(t)|,\ C=\mathrm{Constant},\ (t\to\infty). \quad (17)$$

Eq. (17) shows that the parameter estimation error will be reduced if the high frequency component of the input is reduced. Another way to reduce the norm of the parameter error vector is to reduce $\mathrm{Cond}(Q)$. If M is increased this will be done[13].

2. Design of the hybrid adaptive control

The block diagram shown in Fig. 5 is used for the design of the hybrid MRACS. we assume that $B_L(s)$ is a Hurwitz polynomial.

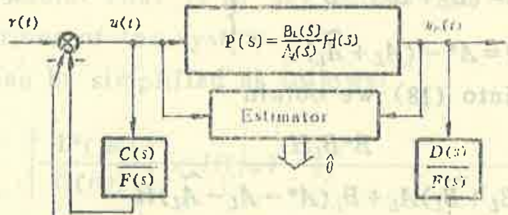

Fig. 5

Here $C(s)$ and $D(s)$ are two polynomials of order $n-1$. Their parameters can be adjusted at discrete instants. A reference model of order n is taken to be $W_m(s)=B^*(s)/A^*(s)$. Both $B^*(s)$ and $A^*(s)$ are Hurwitz polynomials. we take $B^*(s)=F(s)$. The transfer function of the controlled plant is expressed by

$$W(s)=\frac{B^*(s)B_L(s)H(s)}{[B^*(s)+C(s)]A_L(s)+B_L(s)D(s)H(s)} \qquad (18)$$

According to the block diagram shown in Fig. 5 the following design procedure is suggested:

(1) Estimate $A_L(s)$ and $B_L(s)$ by using the suggested estimator. If $r(t)$ is sufficiently "rich", then $u(t)$ is also sufficiently "rich" (see Appendix III), and the estimation of $A_L(s)$ and $B_L(s)$ can be performed. We denote them by $\hat{A}_L(s)$ and $\hat{B}_L(s)$.

(2) Adjust the parameters of the controller according to

$$C(s)=\hat{B}_L(s)-B^*(s), \qquad D(s)=A^*(s)-\hat{A}_L(s) \qquad (19)$$

If the whole system is stable the output error $e(t)$ between the reference model and the adaptive control system will be bounded. That is $|e(t)|<\sigma$, where σ is a certain constant.

(3) If the parameters of the plant have changed and the system become unstable, then $|e(t)|$ will increase and we may have $|e(t)|\geqslant\sigma$. For this moment we repeat steps (1) and (2) so as to stabilize the whole system once again.

3. Stability conditions

Let us analyze the stability of the proposed control system. Denote the estimation error of A_L and B_L by $\tilde{A}_L$ and $\tilde{B}_L$ respectively*. The estimated A_L and B_L are denoted by $\hat{A}_L$ and $\hat{B}_L$. From (19) we have

$$\left.\begin{aligned} C&=(B_L+\tilde{B}_L)-B^* \\ D&=A^*-(A_L+\tilde{A}_L) \end{aligned}\right\} \qquad (20)$$

Substituting (20) into (18) we obtain

$$W(s)=\frac{B^*B_LH}{(B_L+\tilde{B}_L)A_L+B_L(A^*-A_L-\tilde{A}_L)H}$$

$$=\frac{B^*B_LL}{B_L[A^*+D(H-1)]-B_L\tilde{A}_L+\tilde{B}_LA_L} \qquad (21)$$

*For simplicity the Laplace variable s is deleted hereafter

Since $H = B_H/A_H$, from (21) we have

$$W(s) = \frac{B^* B_L B_H}{B_L[A^* A_H + D(B_H - A_H)] - B_L \widetilde{A}_H A_L + \widetilde{B}_L A_L A_H} \tag{22}$$

The following lemma due to Rouchè is used to determine the stability conditions of the system:

Lemma (Rouche's Theorem[14]):

If on a closed loop in the complex plane we have

(1) $f(s) \neq 0$

(2) $|g(s)| < |f(s)|$

where $f(s)$ and $g(s)$ are polynomials of complex variable s, then $f(s)$ and $f(s)+g(s)$ have same number of zeros within the closed loop.

Applying this lemma to determine the stabilsty conditions, we get

$$|A^*(jw)A_H(jw)| > |D(jw)[B_H(jw) - A_H(jw)]|, \tag{23}$$

$$|B_L(jw)[A^*(jw)A_H(jw) + D(jw)B_H(jw) - A_H(jw))]| > |-B_L(jw)A_H(jw)\widetilde{A}_L(jw) + B_L(jw)A_L(jw)A_H(jw)| \tag{24}$$

Eq. (23) shows that $A^*A_H + D(B_H - A_H)$ and A^*A_H have the same number of zeros in the open left half complex plane. Eq. (24) shows that the denominator of $W(s)$ has the same number of zeros as that of $B_L[A^*A_H + D(B_H - A_H)]$ in the open left complex plane. Assume A_H is a Hurwitz polynomial of degree m, then $A^*A_H + D(B_H - A_H)$ has $n+m$ zeros in the open left half complex plane.Assume B_L is of degree 1, then the denominator of $W(s)$ has $(n+m+1)$ zeros in the open left half complex plane. Since the denominator of $W(s)$ is of degree $(n+m+1)$, therefore it is a Hurwitz polynomial, and the system is stable. That is to say Eqs. (23) and (24) are the stability conditions of the system.

Eq. (23) can be simplified as follows:

$$\left|\frac{A^*(jw)}{D(jw)}\right| > |H(jw) - 1| \tag{25}$$

Eq. (24) can be changed to:

$$\left|\frac{A^*(jw) + D(jw)[H(jw) - 1]}{A(jw)}\right| > 2\left|\frac{\widetilde{B}_L(jw)}{B_L(jw)}\right| \tag{26}$$

$$2 > \left| \frac{\tilde{A}_L(jw)}{A_L(jw)} \right| \qquad (27)$$

Eqs. (25) – (27) show that in order to enlarge the stability region we should:

(1) reduce the high frequency component of $r(t)$ so as to make $|H(jw)| \approx 1$;

(2) make $|A^*(jw)|$ as large as possible.

IV. Conclusion

Two design schemes for robust adaptive control system are presented in this paper. In the first scheme the parameters of the controller are adjusted continuously. A logic decison device introduced into this scheme guarantees the global stability of the whole system. This design scheme has a very strong degree of robustness. In the second design scheme the parameters of the controller are adjusted at discrete instants by using the results from the parameter estimation. The stability conditions of the system are given. These two design schemes possess respectively their own merits. It is evident that the combination of these two design schemes will give more satisfactory results. The first design scheme guarantees the global stability and the second scheme allows rapid adaptation.

Appendix I

If $u(t)$ is sufficiently "rich", then the elements of $v(t)$ will be independent each other in $[t_1, t_2]$[15]. Therefore matrix $\bar{Q} \triangleq \int_{t_2}^{t_1} v(t) v(t)^T dt = \{\bar{a}_{ij}\}$ $(i, j = 1, 2, \cdots, 2n)$ is nonsingular. Let $\underline{Q} \triangleq \sum_{k=1}^{M} v(t_k) v(t_k)^T \Delta t = \{\underline{a}_{ij}\}$. From the continuity principle we know that when Δt is appropriately small, there will exist a $\delta > 0$, when $|\underline{a}_{ij} - \bar{a}_{ij}| \leq \delta$, matrix $\underline{Q}$ will also be nonsingular. But we have $|\underline{Q}| = \Delta t^{2n} |Q|$, hence Q is also nonsingular.

Appendix II

From (10) we have

$$\|\varphi\|_2 \leq \max_k |z(t_k)| \sum_{k=1}^{M} \|v(t_k)\|_2 \tag{II.1}$$

Let

$$[H(s)-1]U(s) = \frac{B'_H(s)}{A_H(s)} \cdot sU(s) \tag{II.2}$$

where $B'_H(s)/A_H(s)$ is strictly proper.

Since $K(s)$ and $A_H(s)$ are Hurwitz polyomials, therefore from (II.1), (II.2) and (7) we can obtain inequality (17).

Appendix III

Since the system shown in Fig. 5 is linear during $[t_1, t_2]$, we can write

$$\frac{U(s)}{R(s)} = \frac{B^* A_L A_L}{(B^* + \widetilde{B}_L) A_L A_H + \widetilde{A}_L B_L B_H} \tag{III.1}$$

If $r(t)$ is sufficiently "rich", there will exist no polynomials $p_1(s)$ and $q_1(s)$ of order $(4n+2m-2)$ which satisfy[15]

$$p_1(s)R(s) = q_1(s) \tag{III.2}$$

Therefore, if $u(t)$ is not sufficiently "rich", there would exist two polynomials $p_2(s)$ and $q_2(s)$ of order $(2n+m-1)$ which satisfy

$$p_2(s)U(s) = q_2(s) \tag{III.3}$$

Subsitituting (III.1) into (III.3) we can get a contradiction to (III.2). Hence, if $r(t)$ is sufficiently "rich", $u(t)$ will also be sufficiently "rich".

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鲁棒自适应控制系统的设计

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摘 要

本文提出了鲁棒自适应控制系统的两种设计方案．第一方案为具有逻辑决策环节的模型参考自适应控制系统，即使在被控对象结构不确知的情况下此方案仍能保证全局稳定．在第二种方案中采用混合式自适应控制，采用一种新的参数估计器，利用参数估计结果在离散时刻调节控制器参数．

网络与系统数学理论会议简况

网络与系统的数学理论（MTNS）国际讨论会，主要探讨控制论的有关问题．这种会会议每两年举行一次，它旨在把工程师和数学家集中在一起，讨论应用系统理论中的一些数学问题，诸如最优控制、滤波、网络与系统建模、随机与自适应控制、反馈系统的稳定性、时间序列分析、人工智能、机器人和大规模集成电路设计等．与会者包括对系统理论有兴趣的应用数学专家和从事系统理论研究的人员．会议讨论的问题虽然都有实用背景，但大多是探讨系统科学中的理论或数学方法，直接涉及工程应用则很少．

第七届网络与系统的数学理论(MTNS—85)国际讨论会于1985年6月10至14日在瑞典首都斯德哥尔摩皇家工学院召开．参加会议的约有三百余人，约有近三百篇学术报告．大会特邀了一些知名学者作大会报告．会上的报告有：一族对象模型的同时稳定（西德J.Ackermann），控制及信号处理的专家系统（美国G.Blankenship），机械手的控制（美国R.W.Brockett），自适应控制的必要条件（美国C.I.Byrnes），随机实现理论在统计物理的一些问题中的应用（意大利G.Picci），黎卡提方程的相图（美国M.Shayman），线性系统的建模、复杂性及近似（荷兰J.C.Willems），离散事件系统的控制（加拿大W.M.Wonham），H^{∞}空间中反馈系统的组成及其复杂性（加拿大G.Zames）．

除大会报告外，共分54组进行了专题学术报告，这些报告涉及的主要问题，包括线性系统、一般系统理论、非线性系统一些表示方法与分析、系统设计的“鲁棒性”、非线性系统的结构、反馈控制的新动向、自适应控制、系统辨识与过滤、随机控制、大规模系统、时滞系统、分布参数系统等等．从总的情况看，我认为人们对以下一些问题较为重视．

①如何使反馈控制系统有较高的鲁棒性；

②一些特定形式的非线性系统的表示方法、分析和控制系统的设计；

③系统的建模、估计和自适应控制；

④线性系统的新问题如时滞系统、分布参数、几何方法等． （冯纯伯）