第3卷 第3期 控制理论与应用 Vol. 3, No.3
1986年9月 CONTROL THEORY AND APPLICATIONS Sept., 1986

Decision Making in Some Large Scale Systems

Guo Zhongwei B. McInnis
(Tsinghua Usiversity, Beijing) (University of Houston, Texas, U. S. A.)

Abstract

Using general system theory, we present large scale system models of decentralized, centralized, and coordinated decision making systems. An equation set for a simple case is obtained and optimization techniques are applied to determine optimal strategies. The case of stochastic multi-step systems is also discussed. A numerical example relevant to current conditions in China is given.

I. Introduction

By using the theory of large scale systems[1,2,3], general systems[4], shadowp rices[5,6] and the decision methods of Y. C. Ho and G. J. Olsder[7], we developed a technique for studying large scale systems. The main idea of this work is to develop a model structure that enables us to integrate market-regulation and centralized planning in order to achieve the global maximal benefit for a large scale system. Although this paper is based upon existing theoretical concepts, we believe the application to economic problems given here is new and provides a theoretic basis for studying some economic planning problems, e. g., the recent economic reform in China.

II. Analysis of decentralized decision making system

The information structure of a decentralized decision making system, which corresponds to the typical free-market economy, may be shown as in Fig. 1. The set of economic subsystems is the set of

Manuscript recevied Nov. 2, 1984, revised April 23, 1985.

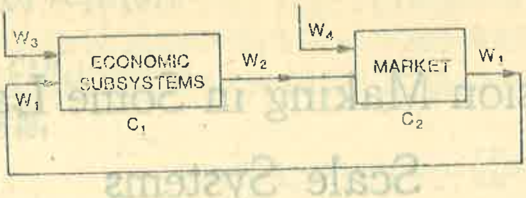

Fig. 1 Block Diagram of Decentralized Decision Making System

related producers and consumers (including different groups, collectives and persons). The arrow indicates the direction of information flow (not flow of goods). The set C_1 represents the state of the set of economic subsystems, set W_1 represents the market prices and other related market information, set W_3 shows non-market information that goes to economic subsystems and set W_2 shows the supply and demand information that goes into the market.

The response relationship of the economic subsystems is

$$R_1: \ C_1 \times (W_1 \times W_3) \rightarrow W_2 \tag{1}$$

where "$\times$" denotes the Cartesian product.

The decision method of the economic subsystems can be expressed by the relationship

$$R_2: \ C_1 \times (W_1 \times W_3) \times W_2 \rightarrow P \tag{2}$$

where P is a set of performance measures. The i-th economic subsystem will evaluate its performance P_i according to the input subsets W_{1i}, W_{3i}, the output W_{2i} and the state subset C_{1i}. Then the optimal value of the output $W^*_{2i} \in W_{2i}$ can be found by maximizing the set P_i, such that

$$p^*_i = \max_{W_{2i}} \{p : p \in P_i\} \tag{3}$$

Similarly, for the market system we have

$$R_3: \ C_2 \times (W_2 \times W_4) \rightarrow W_1 \tag{4}$$

where R_3 is a function that models the market and W_4 represents exogeneous influences. Then the free-market system as a whole can be expressed as a relation S

$$S \subset (W_3 \times W_4) \times (W_1 \times W_2) \tag{5}$$

The characteristics of the system S may be interpreted as following:

1. Because of (2) and (3) the initiative and enthusiasm of the

economic units (subsystems) are brought into full play and high productivity is achieved.

2. Because the subsystems are subjected to many constraints and interconnection in the market, and for the sake of system dynamics, the regulation ability of the whole system is limited. As shown by experience, sometimes it not only cannot achieve optimality, but even leads to serious economic errors and global performance of the system will not be satisfactory. Hence it is quite desirable to introduce some degree of planning.

Ⅲ. Analysis of ceniralized decision making system

Generally, centralized decision systems have a hierarchical structure. The information structure of a two-level system is shown in Fig. 2.

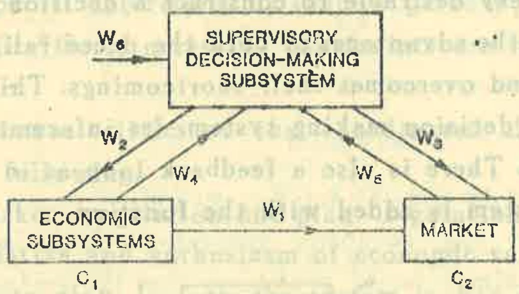

Fig. 2 Block Diagram of Centralized Decision Making System

According to the input information set W_6 and the information sets W_4 and W_5 the supervisory decision-making subsystem determines decision sets W_2 and W_3. Then the expression representing the supervisory decision-making subsystem is

$$S_1 \subset (W_6 \times W_4 \times W_5) \times (W_2 \times W_3) \tag{6}$$

The economic subsystems determine the information set W_1 according to W_2 and its own state set C_1, so the expression of the economic subsystems is

$$S_2 \subset (C_1 \times W_2) \times W_1 \times W_4 \tag{7}$$

Similarly for the market

$$S_3 \subset C_2 \times (W_1 \times W_3) \times W_5 \tag{8}$$

The main characteristics of this decision making system are:

1. This system is a typical large scale system and its optimal

(or satisfactory) control can be achieved in some ideal cases, but generally the uncertainty and number of factors are so large that the optimal solution will not easy to be found.

2. Because the W_1 (see Fig. 2) is determined to a large extent by W_2 and equations (2) and (3) do not hold, the initiative and enthusiasm of the economic sybsystems are seriously limited, and productivity will not be as high as possible. Furthermore, because there is no market regulation, if the supervisory decision subsystem makes any mistake, the system performance will be far from satisfactory or may even cause serious damage to the economy. Therefore, it is very desirable and reasonable to use market considerations in the centralized decision making system.

Ⅳ. A coordinated decision making system

It will be very desirable to construct a decision making system which possesses the advantages of both the decentralized and centralized systems and overcomes their shortcomings. This is the idea of the coordinated decision making system. Its information structure is shown in Fig. 3. There is also a feedback loop as in Fig. 1, but an algorithm sybsystem is added with the function

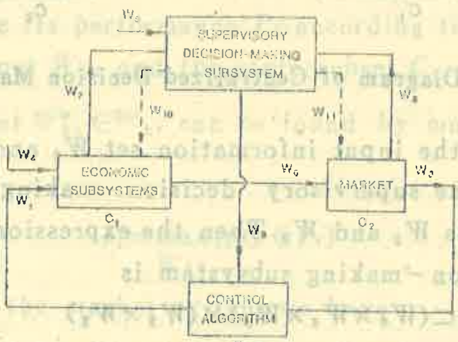

Fig. 3 Block Diagram of Coordinated Decision Making System

$$R_1: W_3 \times W_6 \to W_1. \tag{9}$$

As compared with Fig. 2, the supervisory decision subsystem in Fig. 3 does not provide mandatory decisions directly to the economic and market snbsystems, but only provides the decision set W_9 to the control algorithm, i. e.

$$R_2: \quad W_6 \times W_7 \times W_8 \to W_9. \tag{10}$$

The economic subsystems do not work directly with the information set W_3 from the market, but with the modified information set W_1, which can be considered to be like a set of shadow prices. The function of the economic subsystems is

$$R_3: \quad C_1 \times (W_1 \times W_4) \to W_2 \times W_7 \tag{11}$$

The decision method is similar to Eq. (2) and (3) and there is also a set of optimal (satisfactory) subsystems

$$R_4: \ C_1 \times (W_1 \times W_4) \times W_2 \to P \tag{12}$$

and
$$p_i^* = \max \ \{p: \ p \in P_i\}$$
$$w_{2i} = \overset{*}{w}_{i2} \tag{13}$$

By this method the economic subsystems will pursue not the market profit but the profit determined by shadow price W_1 and it is called the "social profit." Some necessary constraints are imposed by W_6, W_{10} and W_{11}.

The main characteristics of the coordinated decision making system are the following:

1. The market-regulation and plan-regulation are integrated, so that the initiative and enthusiasm of economic subsystems may be brought fully into play. In fact. the system in Fig. 3 is a feedback system with an optimal control algorithm and adjustable parameters, so it has stronger corrective ability and can achieve better system performance than the system in Fig. 1 and 2.

2. The smaller quantity of elements in W_9 than in the centralized method (W_2 and W_9 in Fig. 2) leads to much easier optimization of the whole large scale system.

V. Equation set for a simple case

We will consider a simple case of a coordinated decision system. Let a certain supervisory decision making subsystem manage l factories and we will limit the discussion to one period ΔT (e. g. one week, one month or one year).

The states of each factory can be described by consumption coefficients matrix H_i. Let the vector e_n represents the consumption per unit output and e_m the unit consumption vector (e. g. energy, labor, and materials). The system equation for ith economic subsystem is

$$e_n = H_i e_m \qquad i = 1, \cdots, l \tag{14}$$

where

$$H_i = \begin{bmatrix} h_{i11} & h_{i12} & h_{i1r} \\ \vdots & \vdots & \vdots \\ h_{is1} & h_{is2} & h_{isr} \end{bmatrix}$$

$$e_n^T = [e_{n1}, \cdots\cdots, e_{ns}]$$

and $$e_m^T = [e_{m1}, \cdots\cdots, e_{mr}]$$

The elements of H_i represents the consumption for unit product of i-th factory.

Let $n_i^T = [n_{i1}, n_{i2}, \cdots\cdots, n_{is}]$ be the output vector (i. e. it represents the quantity of the products) and let $m_i^T = [m_{i1}, m_{i2}, \cdots, m_{ir}]$ be the input (consumption) vector.

From (14) we can get

$$m_i = H_i^T n_i \tag{15}$$

where (according to Fig. 3) $n_i \in W_2$, $m_i \in W_2$.

Assume that the economic subsystems have an interest in pursuing the maximal social profit, so that the performance measure of each factory is the social profit L_i:

$$L_i = \int_0^{n_i(\Delta T)} b^T V dn_i(t) - \int_0^{m_i(\Delta T)} a^T U dm_i(t) \tag{16}$$

$$i = 1, \cdots, l$$

with constraints: $f[n_i(t), m_i(t)] \geq 0$ (17)

where:

$a \in W_0$ is the decision vector ($a \in R^r$) for consumption

$b \in W_0$ is the decision vector for the output ($b \in R^s$)

$U \in W_3$ is the unit market price diagonal matrix for comsumption and $V \in W_3$ is that for output:

$$U = \begin{pmatrix} u_1 & & & 0 \\ & u_2 & & \\ & & \ddots & \\ 0 & & & u_r \end{pmatrix} \qquad V = \begin{pmatrix} v_1 & & & 0 \\ & v_2 & & \\ & & \ddots & \\ 0 & & & v_s \end{pmatrix}$$

The optimal output n_i^* for maximal social profit L_i^* can be obtained from Eq. (15) and (16) and by maximization:

$$L_i^* = \max_{n_i = n_i^*} L_i\ (z, H_i, n_i, U, V, W_4) \tag{18}$$

where z is the set of optimal decisions:

$$z^T = [a^T, b^T, \cdots]$$

performance measure for the supervisory decision making subsystem is:

$$R = \sum_{i=1}^{l} \left[\int_0^{n_i(\Delta T)} b^T V dn_i - \int_0^{m_i(\Delta T)} a^T U dm_i \right] \tag{19}$$

For consistency in the calculations let

$$\sum_{i=1}^{l} \int_0^{n_i(\Delta T)} b^T V dn_i = \sum_{i=1}^{p} \int_0^{n_i(\Delta T)} v^T dn_i \tag{20}$$

and
$$\sum_{i=1}^{l} \int_0^{m_i(\Delta T)} a^T u dm = \sum_{i=1}^{l} \int_0^{m_i(\Delta T)} u^T dm_i \tag{21}$$

where $v^T = [v_1, v_2, \cdots, v_s]$, $u^T = [u_1, u_2, \cdots, u_r]$

s.t. constraints:

$$g\left(\sum_1^l n_i(t),\quad \sum_1^l m_i(t) \right) = 0 \tag{22}$$

or
$$y\left(\sum_1^l n_i(t),\quad \sum_1^l m_i(t) \right) \geq 0$$

and
$$R^* = \max_{z = z^*} R(z, H_i, n_i^*, u, v, W_6). \tag{23}$$

The set of Eq. (16) to (23) is referred to as the equation set for the coordinated decision making system.

Ⅵ. Multi-stage stochastic control

The information structure of the multi-stage stochastic control is shown in Fig. 4. where "ˇ" and "^" denote the related estimated

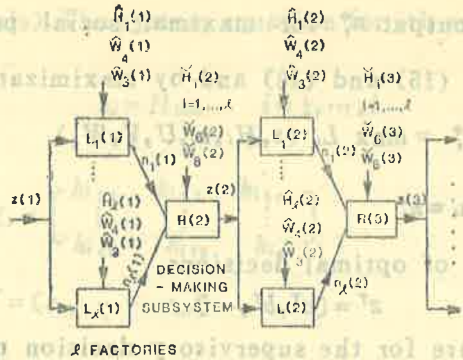

Fig. 4 Block Diagram of Multi-stage Stochastic Control System

values of the supervisory decision making subsystem and economic sybsystems, respectively, and the numbers in brackets show the index of the period considered.

It is reasonable to assume perfect measurement and perfect memory. Then the maximum J_R^* for the supervisory decision making subsystem can be obtained by the method of dynamic programming

$$J_R^*(j)=\max_{z(j)} E\left\{R[j,z(j)]+J_R^*[j+1,z^*(\overline{j+1\to k})]/Y_1(i-1)\right\} \tag{24}$$

where Y_1 $(j-1)$ is the memory set and $z^*(\overline{j+1\to k})$ is the optimal decision set from stage $j+1$ forward

$$z(\overline{j+1\to k})=[z^*(j+1),z^*(j+2),\cdots,z^*(k)]$$

where k is the number of calculated stages. Similarly for each factory we have

$$J_{Li}^*(j)=\max_{n} E\{L_i[j,n(j),z(j)]$$

$$+J_{Li}^*[j+1,n^*(\overline{j+1\to k}),z^*(\overline{j+1\to k})]/Y_{2i}(j)\} \tag{25}$$

Ⅶ. Example — Economic readjustment

Coordinated decision making systems can be applied successfully to the problems of economic readjustment, which are major problems in China today.

Let a certain company manage 10 factories with similar product and the production capacity of each factory be the same:

$$n_{i\max} = 1000 \text{ pieces/month} \qquad i = 1, \cdots, 10$$
$$m_i = h_i n_i \qquad \text{[monetary unit]}$$

where h_i is the consumption coefficient. Values of h_i are shown in Tab. 1

Table 1

i	1	2	3	4	5	6	7	8	9	10
h_i[m.u./p]	0.75	0.81	0.815	0.82	0.825	0.83	0.835	0.84	0.845	0.85

The gross production is 10000p/month. If the market demand is only one-half this amount, i. e., 5000p/month, how should the production be redistributed?

Because the first 5 factories have less consumption, the supervisory decision making subsystem will have the other 5 factories stop their production. The same result will be obtained by decentralized decision making, because the last 5 factories will go bankrupt in free competition. Thus the result is

$$n_i = \begin{cases} 1000 & \text{if } i = 1, \cdots, 5 \\ 0 & \text{if } i = 6, \cdots, 10 \end{cases} \tag{26}$$

But if we take the gross profit of the whole large scale system into account, the decision as shown in Eq. (26) will not be optimal. For example, if we consider the different ability of transforming one kind of product into the other of each factory, the decision in Eq. (26) might be the worst one.

Assume that the better factories have greater ability to transform, we define the function q_i as the transformation-ability of the i-th factory to adjust its output with the same consumption because of changing the kind of products. Assume the q_i is inversely proportional to h_i, i. e.,

$$q_i = q_1 \frac{h_1}{h_i} \qquad (i = 1, \cdots, 10) \tag{27}$$

For the i-th factory the profit L_i [monetary unit] is

$$L_i = vn_i - h_i n_i + vq_i(1000 - n_i) - h_i(1000 - n_i) \tag{28}$$

where

v is the price of product [monetary unit/piece]

n_i is the quantum of products according to a certain decision method, q_i $(1000-n_i)$ is the quantum of new products converted to the quantum of original products.

In Eq. (28) the first two terms on the right side indicate the profit from producing original products, and the last two the profit from producing new products.

Assume

$$q_1=\frac{1}{1.04} \text{ and } v=1 \text{ monetary unit/piece}$$

It means the first factory should decrease 4% of its production because of transform to new products. According to the decision of Eq. (26) the profit of each factory is shown in Tab. 2 and is accounted in monetary unit

Table 2

i	1	2	3	4	5	6	7	8	9	10	R
L_i	250	190	185	180	175	38.9	28.6	18.5	8.4	0	1074.4

where R is the gross profit.

By using mathematical programming, we can obtain the decision for maximized R,

$$n_{im}=\begin{cases}0, & \text{if } i=1,\cdots,5\\ 1000, & \text{if } i=6,\cdots,10\end{cases} \tag{29}$$

The profit of each factory and gross profit are shown in Tab. 3

Table 3

i	1	2	3	4	5	6	7	8	9	10	R
L_i	211.5	80	69.9	59.5	49.1	170	165	160	155	150	1270

It shows that the decision of Eq.(26) is the worst one for the global system. Although the decision of Eq. (29) is superior for the global system, it is quite unreasonable and unacceptable because the better factories 2~5 (with smaller h_i) obtain less profit than worse ones 6~10 (with larger h_i).

Coordinated decision making method can be successfully applied to the problem. According to the equation set of the coordinated decision making system, we have:

$$L_i = b_1 n_i - h_i n_i + b_2 q_i (1000 - n_i) - h_i (1000 - n_i) \quad (30)$$

$$L_i^* = \max(b_1, b_2, n_i, h_i) \quad (31)$$
$$n_i = n_i^*$$

$$R = \sum_1^{10} L_i^* \quad (32)$$

$$R^* = \max_{b_1, b_2} R(b_1, b_2, h_i) \quad (33)$$

s. t.

$$\sum_1^{10} n_i^* = 5000 \quad (34)$$

$$\sum_1^{10} [b_1 n_i^* + b_2 q_1 (1000 - n_i^*)] = \sum_1^{10} [n_i^* + q_i (1000 - n_i^*)]. \quad (35)$$

Using the nonlinear programming the solution is

$$b_1 = 0.934$$
$$b_2 = 1.0735$$

and the social profit L_i^* is shown in Tab. 4.

Table 4

i	1	2	3	4	5	6	7	8	9	10	R*
L_i^*	282.2	145.7	134.8	124	113.3	104	99	94	89	84	1270

As compared to Tab. 2, the gross profit has been increased 18. 2% and as compared to Tab. 3, the profit of each factory is reasonably arranged; furthermore, by using coordinated decision making, the initiative and activeness of each factory are not constrained.

Ⅷ. Conclusion

The advantages of coordinated decision making method become clear when analyzed as shown in this paper. We have analyzed also another example of optimal decision making for economic uniti with a limited power supply[10]. It is shown that the global optimum is achieved by that the economic sybsystems are pursuing the social benefits accounted for by shadow prices.

References

[1] Sage, A. P., Methodology for Large Scale Systems, New York, McGraw-Hill, (1977).

[2] Jamshidi, M., Large-scale Systems: Modeling and Control, New York, North-Holland, (1983).

[3] Mesarovic, M. D. and Mihajlo, D., Theory of Hierarchical, Multilevel Systems, New York, Academic Press, (1970).

[4] Mesarovic, M. D. and Takahara, Y., General Systems Theory: Mathematical Foundations, New York, Academic Press, (1975).

[5] Dantzig, G. B. and Wolf, P., The Decomposition Algorithm for Linear Programming, Econometrica, 29, 4. (1961).

[6] Little, I. M. D. and Scott, M. FG. ed., Using Shadow Prices, London Heinemann, (1976).

[7] Ho, Y. C., Introduction to Decision Analysis, Information and Control, China, 3, 4, (1983).

[8] Chow, G. C., Analysis and Control of Dynamic Economic Systems, New York, John Wiley, (1975).

[9] Yuan-li Wu, Planning, Management and Economic Development in Communist China, Comparative Economic Systems, Models and Cases, New York, Richard D. Irwin, Inc, (1969).

[10] Guo Zhongwei, Systems Analysis of Some Typical Economic Systems and softplan Decision Method, Proceedings of Second International Conference on Technology for Development, Cairo, Egypt, (1982).

一类大系统的决策方法

郭仲伟 B. McInnis

（清华大学） （Univ. of Houston, Texas, U. S. A.）

摘 要

本文中利用一般系统理论讨论了分散、集中和协调系统的大系统模型。对于一种简化了的案例给出了为确定最优策略的方程组。对多级多段随机控制系统也进行了讨论。结合中国当前的具体情况给出了一个数字例子。