第4卷 第2期
1987年 6月

控制理论与应用
CONTROL THEORY AND APPLICATIONS

Vol. 4, No.
June, 198

A Note on the Solutions of Popov Integral Inequality

Wu Junjie
(University of Science and Technology of China, Hefei)

Abstract

This paper completes the proof of Popov inequality in hyperstable adaptive systems, and reveals the relationship between Lyapunov and hyperstability approachs to adaptive control systems.

To design the model reference adaptive systems (MRAS) using the hyperstability approach, we must solve the following integral inequality

$$\int_{t_0}^{t_1} u^T(t)\int_{t_0}^{t} [S'(u(t'),t',t)+S''(t_0)]y(t)dt'dt \geq -\alpha^2 \tag{1}$$

where $y,u\in R^m$, $S'(u(t'),t',t)$, $S''(t)\in R^{m\times m}(t_0\leq t'\leq t)$, and α^2 is a finite positive constant. Landau has indicated the solution of ineq. (1), that is

$$S'(u(t'),t',t)=H(t-t')u(t')\cdot y^T(t')\cdot G_0 \tag{2}$$

where $H(t-t')$ is a positive definite square matrix kernel whose Laplace transformation is a positive real transfer matrix with pole at $s=0$, and G_0 is a positive definite constant matrix with appropriate dimension. This result which have shown up in current books and papers is not very strict and has some mistakes, so this paper will give the correct result.

Replacing $S'(u(t'),t',t)$ in ineq.(1)by its expression given by eq. (2), one must solve the following integral inequality instead of (1)

$$\int_{t_0}^{t_1} f^T(t)\left[\int_{t_0}^{t} H(t-t')f(t')dt'+V_0\right]dt\geq -\alpha_0^2 \tag{1'}$$

where $f(t),V_0\in R^m$ and V_0 is a constant vector, α_0^2 is a positive constant.

Manuscript received June 9, 1984, revised Aug. 18, 1986.

If $V_0=0$, ineq. (1′) is holding. For $V_0\neq 0$, the author used the following relation

$$V_0=\int_{t_0-h}^{t_0} H(t-t')f^* dt' \tag{3}$$

here f^* is constant vector, $0<h<\infty$. Eq. (3) holds only if $L(H(\cdot))=H(s)=\frac{1}{s}G$ $(G>0)$. A common positive definite square matrix kernel $H(t-t')$ whose Laplace transformation has a pole at $s=0$ depends on time t, so eq. (3) is invalid. For example,

$$H(s)=\begin{bmatrix}\frac{1}{s} & 0\\ 0 & \frac{1}{s+1}\end{bmatrix},\quad H(t-t')=\begin{bmatrix}1 & 0\\ 0 & e^{-(t-t')}\end{bmatrix}1(t-t')$$

where $1(t-t')$ is the unite step function. We have

$$\int_{t_0-h}^{t_0}\begin{bmatrix}1 & 0\\ 0 & e^{-(t-t')}\end{bmatrix}\cdot f^* dt'=\begin{bmatrix}h & 0\\ 0 & e^{-t}(e^{t_0}-e^{t_0-h})\end{bmatrix}\cdot f^*$$

for $V=[3,2]^T$ and any $f^*\in R^m$, eq. (3) is not holding.

To attain the goal, we separate pole $s=0$ from others of $H(s)$, thus

$$H(s)=H_1(s)+\frac{1}{s}G$$

where $G=\lim\limits_{s\to 0} sH(s)$ $(G\geq 0)$ and hence $H_1(s)$ is a positive real transfer function matrix. We rewrite ineq. (1′) as

$$\int_{t_0}^{t_1} f^T(t)\left[\int_{t_0}^{t} H_1(t-t')f(t')dt'\right]dt$$

$$+\int_{t_0}^{t_1} f^T(t)\left[\int_{t_0}^{t} G\cdot 1(t-t')f(t')dt'+V_0\right]dt\geq -\alpha_0^2 \tag{4}$$

Because $H_1(s)$ is positive real, the Popov inequality holds for the first term on the left-hand side of ineq. (4). Assuming $V_0\in$ Range G, there exists a constant vecter $f_0^*\in R^m$ such that

$$V_0=G\cdot f_0^*$$

and for any $h>0$, we have

$$V_0=\int_{t_0-h}^{t_0} G\cdot\frac{1}{h}\cdot f_0^* dt'$$

$$=\int_{t_0-h}^{t_0} G\cdot 1(t-t')\cdot\frac{1}{h}\cdot f_0^* dt', \qquad t\geq t_0$$

Let

$$f'(t)=\begin{cases} f(t) & t\geq t_0 \\ \frac{1}{h}f_0^* & t_0-h\leq t<t_0 \\ 0 & \text{otherwise}\end{cases}$$

We get

$$\int_{t_0}^{t_1} f^T(t)\left[\int_{t_0}^{t} G\cdot 1(t-t')\cdot f(t')dt'+V_0\right]dt$$

$$=\int_{t_0-h}^{t_1} f'^T(t)\left[\int_{t_0-h}^{t} G\cdot f'(t')dt'\right]dt-\frac{1}{h^2}\int_{t_0-h}^{t_0} f_0^{*T}\left[\int_{t_0-h}^{t} G\cdot f_0^* dt'\right]dt$$

$$=\int_{t_0-h}^{t_1} f'^T(t)\left[\int_{t_0-h}^{t} G\cdot f'(t')dt'\right]dt-\frac{1}{2}f_0^{*T}G\cdot f_0^*$$

Note that G is a positive square matrix kernel, so

$$\int_{t_0-h}^{t_1} f'^T(t)\left[\int_{t_0-h}^{t} G\cdot f'(t')dt'\right]dt-\frac{1}{2}f_0^* Gf_0^*\geq-\tilde{\alpha}_0^2$$

where $\tilde{\alpha}_0^2$ is a positive constant. Since V_0 as well as $S''(t_0)$ corresponds with unknown parameters and ineq. (1′) should be hold for arbitrary V_0, it is necessary that

$$\mathrm{Range}G=R^m$$

That is to say $G>0$. So the sufficient conditions under which ineqs. (1) and (1′) hold good are that the Laplace transformation of matrix kernel $H(t-t')$ is positive real whose residual matrix at pole $s=0$ must be a strictly positive definite matrix instead of with a pole at $s=0$.

Now we attempt to discuss the differences in the adaptive law of MRAS between the two schemes. In the asymptotically stable model reference adaptive control systems, the adaptive law deduced from the Lyapunov approach is an integrator whose transfer matrix is $\frac{1}{s}G(G>0)$, while using the hyperstability approach it is composed of an integrator which is equivalent to that one, and a linear dissipation subsystem in parallel. Those are shown in Figs. 1 and 2. Since the time constants of the additional part in Fig. 2 are smaller than that of the integrator, the system shown in Fig. 2 may have higher adaptive speed. Even through both scheme are sensitive to disturb owing to the existence of the integrator, the latter has the potential to design a robust system.

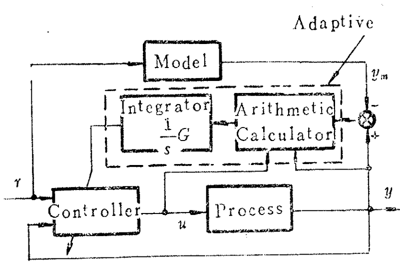

Fig.1 Block diagram of MRAS by Lyapunov method

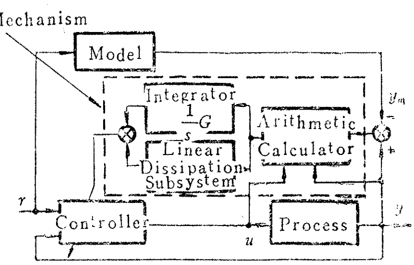

Fig.2 Block diagram of MRAS by hyperstability method

References

〔1〕 Landau, I. D., Adaptive Control——The Model Reference Approach, Dekker, (1979).

〔2〕 Landau, I. D., Algorithmes d′ Adaptation Hyperstables Peur une Classe de Systems Adaptatifs Echantillonnes, C. R. Acad. Sci. Paris, Series A, 275, (1972), 1391—1394.

〔3〕 Narendra, K. S. and Valavani, L. S., A Comparison of Lyapunov and Hyperstability Approaches to Adaptive Control of Continuoues Systems, IEEE Trans. Autom. Contr., AC—**25**, 2,(1980), 243—246.

〔4〕 Anderson, B. D. O., A Simplified Viewpoint on Hyperstability, IEEE Trans. Autom. Contr., AC—**13**, 1,(1968), 292—294.

关于 Popov 积分不等式解的注记

武 俊 杰

（中国科学技术大学，合肥）

本文给出了用渐近超稳定定理综合自适应系统所出现的 Popov 积分不等式的严格解，揭示了用李亚普诺夫和超稳定两种方法所得自适应律之间的差别。