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Robustness of Reduced-order Observer-based Control Systems*

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Abstract

The robustness of reduced-order observer-based linear control systems is discussed in this paper. Three important frequency domain properties and a theorem for these systems are derived. And a pole-assignment procedure for reduced-order observers is given.

For the full-order observer-based control systems, Doyle and Stein pointed out in [1] that in order to achieve the same robustness as the full-state feedback implementation, one should make some observer poles towards stable plant zeros and the rest towards infinity.

The robustness of reduced-order observer-based control systems is discussed in this paper. The robustness results of [1] are extended to the reduced-order observer case successfully.

Consider controllable and observable linear time-invariant multivariable plant

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} u(t) \tag{1}$$

$$y(t) = [O \quad I] \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = x_2(t)$$

Where $x_2(t) = y(t) \in R^p$ is the output vector, $x_1(t) \in R^{n-p}$ is the unavailable state vector, $u(t) \in R^m, m = p$, rank$(B) = m$

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In analogy to the transmission zeros of linear multivariable systems[2], we have

Definition The set of generalized transmission zeros of the subsystem of (1) (A_{11}, B_1, A_{21}, B_2) is

$$\left\{ \lambda_i: \ \mathrm{rank}\begin{pmatrix} \lambda_i I - A_{11} & B_1 \\ -A_{21} & B_2 \end{pmatrix} < n \right\} \tag{2}$$

Through calaulation, the generalized transmission zeros of the subsystem (A_{11}, B_1, A_{21}, B_2) are the zeros of polynomial

$$f_i(s) = \det(sI - A_{11})\det(B_2 + A_{21}(sI - A_{11})^{-1}B_1) \tag{3}$$

Suppose all the generalized transmission zeros of the subsystem (A_{11}, B_1, A_{21}, B_2) have negative real parts.

The Laplace transformations of equation(1) are

$$x_1(s) = (sI - A_{11})^{-1}A_{12}x_2(s) + (sI - A_{11})^{-1}B_1 u(s)$$
$$x_2(s) = (sI - A_{22})^{-1}A_{21}x_1(s) + (sI - A_{22})^{-1}B_2 u(s) \tag{4}$$
$$y(s) = x_2(s) \tag{5}$$

and those of the reduced-order observer[3] are

$$\Psi(s) = (sI - A_{11} + MA_{21})^{-1}\{(B_1 - MB_2)u(s) + [A_{12} - MA_{22} + (A_{11} - MA_{21})M]y(s)\} \tag{6}$$

$$\hat{x}_1(s) = \Psi(s) + My(s) \tag{7}$$

Where M is the reduced-order observer correction matrix.

Fig. 1 and Fig. 2. show the full-state feedback implementation and the reduced-order observer-based implementation respectively.

When the variations of the plant parameters occur, the separation property of the observer-based system can not be held. If we want to study the stability of the overall system under influence of variations of plant parameters, we must discuss the open loop transfer function matrix with broken loop at "X" rather than "XX" in Fig. 2. Now we have three frequency domain properties.

property 1 For the linear time-invariant systems, the closed-loop transfer function matrices from command input $y_{re}(s)$ to state vector x(s) are identical in both implementations Σ_1 and Σ_2.

property 2 For the linear time-invariant systems, the open-loop transfer function matrices from control signal u′(s) to control signal u(s) (loops broken at point "XX") are identical in both implementations Σ_1 and Σ_2.

property 3 For the linear time-invariant systems, the open

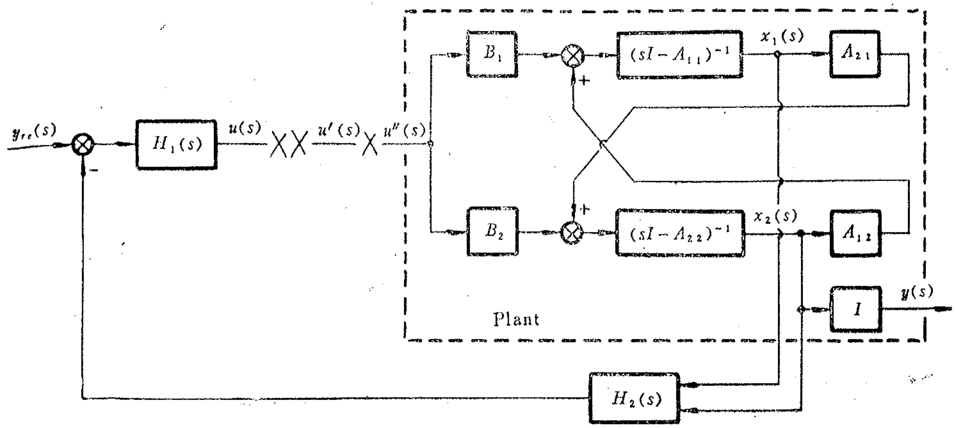

Fig. 1 Full-state Feedback ImplementationΣ_1

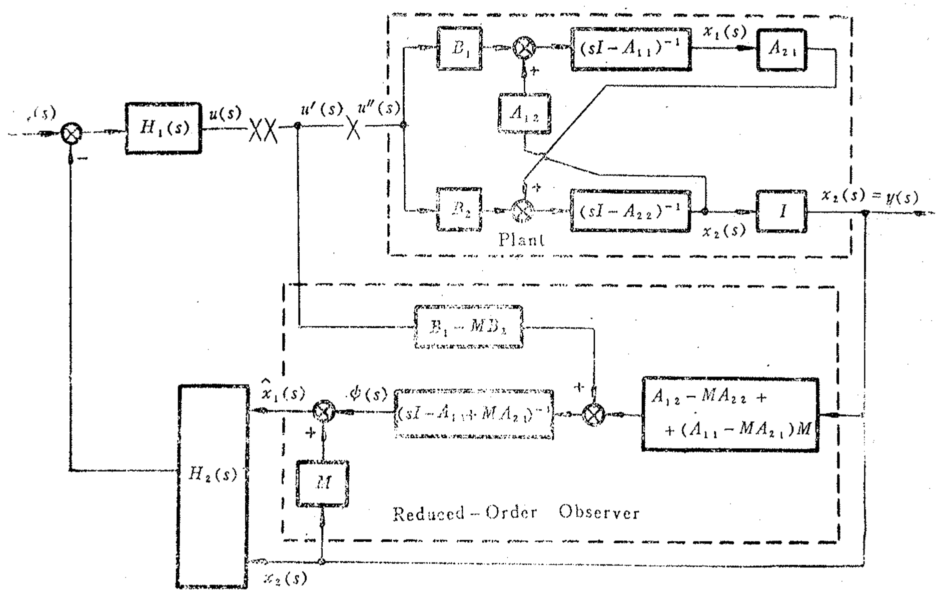

Fig. 2 Reduced-order Observer-Based ImplementationΣ_2

-loop transfer function matrices from control signal $u''(s)$ to control signal $u'(s)$ are generally different. They are identical if we choose the observer correction matrix M to satisfy:

$$B_1[B_2+A_{21}(sI-A_{11})^{-1}B_1]^{-1}=M[I+A_{21}(sI-A_{11})^{-1}M]^{-1} \tag{8}$$

for all s, where A_{11}, B_1, A_{21}, B_2 are the matrices in(1). when $B_2\equiv 0$ or $MB_2\equiv 0$, the condition (8) becomes (8′)

$$B_1[A_{21}(sI-A_{11})^{-1}B_1]^{-1}=M[I+A_{21}(sI-A_{11})^{-1}M]^{-1} \tag{8′}$$

property 1 and property 2 are very well known. The proof of property 3 is given in Appendix I.

An adjustment procedure is also given in (8). If (8) holds for all s, the open-loop transfer function matrices are identical in both implementations Σ_2(see Fig. 2) and Σ_1 (see Fig. 1). Therefore, implementation Σ_2 has the same robustness as implementation Σ_1.

Unfortunately, (8) is a nonlinear matrix equation of M. we suppose M is parametered as a function M(q) of a scalar variable q as mention ed in [1]. M(q) should be so selected as to satisfy

$$\frac{M(q)}{q} \xrightarrow{q\to\infty} (B_1 - MB_2)W$$

for a nonsingular matrix w and the stability of the observer. Then (8) will be satisfied asymptotically as $q\to\infty$. The finite poles of the reduced-order observer will be zeros of polynomial(see [4]).

$$f(s) = \det(sI - A_{11})\det[B_2 + A_{21}(sI - A_{11})^{-1}B_1] \qquad (9)$$

which is identical to (3). Then we have

Theorem In implementation Σ_2, the finite poles of the reduced-order observer should be coincided with all zeros of the generalized transmission zero polynomial f_1 (s) of the subsystem (A_{11}, B_1, A_{21}, B_2), and the rest should be selected as fast as possible to achieve the same robustness as implementation Σ_1

So, the poles of the reduced-order observer should be assigned as that some finite observer poles should be driven towards the stable generalized transmission zeros of the subsystem (A_{11}, B_1, A_{21}, B_2) and the rest should be selected as fast as possible under the noise restriction.

Example The controllable and observable plant is

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{pmatrix} = \begin{pmatrix} -1 & 2 & 0 \\ -6 & -2 & 0 \\ 4 & 1 & -10 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u(t)$$

$$y(t) = [0 \ 0 \ 1][x_1(t)x_2(t)x_3(t)]^T = x_3(t)$$

The subsystem (A_{11}, b_1, A_{21}) is

$$A_{11} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{11} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ -6 & -2 \end{pmatrix}$$

$$b_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad A_{21} = [4 \quad 1]$$

The generalized transmission zero of the subsystem is $s=-0.5$. We use LQSF design method and obtain an optimal feedback gain matrix $k^* = [9, 1.2, 2]$. In order to elucidate the influence of the chang of the observer poles on the robustness of the closed-loop system, we discuss 8 cases here. The computer simulation results are showed in Table 1. We see that case 2, in which a pole of the reduced-order observer $s=-0.5$ is identical with the generalized transmission zero of the subsystem, has the largest parameter stable region. While case 3 to case 8, which do not assign the observr poles as the procedure proposed above, the faster the observer poles are assigned, the smaller the parameter stable region.

Table 1

No	poles of the observer	m_1	m_2	$\frac{\delta a_{21}}{a_{21}} 100\%$	valuations of robustness
1	full-state feedback	/	/	156.7%	ideal case
2	$-0.5, -100$	24.75	-1.5	156.7%	best
3	$-5, -100$	-198	894	86.7%	↓
4	$-20, -100$	-940.5	3879	73.3%	
5	$-50, -100$	-2425.5	9849	50%	poor
6	$-10 \pm j5$	-53	229	96.6%	
7	$-20 \pm j5$	-193	809	76.6%	
8	$-20 \pm j15$	-293	1209	73.3%	

So, we have the following conclusion:

where $M = [m_1 \quad m_2]^T$ is the observer correction matrix.

δa_{21} is the perturbation of a_{21}.

When we design the reduced-order observer, in order to achieve the same robustness as the full-state feedback system, the poles of the reduced-order observer should be assigned as some of which towards the stable generalized transmission zeros of the subsystem and speed up the rest as fast as possible under the noise restrictions.

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Appendix I Proof of Property 3

From Fig.1 and Fig.2, we know that the open - loop transfer function matrices from control signal $u''(s)$ to $x_2(s)$ (with loop broken at point "X")in both implementations Σ_1 and Σ_2 are identical. Hence, it is clear that all we want to prove is that the transfer function matrix from $u''(s)$ to $x_1(s)$ in Σ_1 is identical with that from $u''(s)$ to $\hat{x}_1(s)$ in Σ_2 if(8) is satisfied for all s.

$$\text{Let } \phi(s) \triangleq (sI - A_{11})^{-1}$$

For implementation Σ_1(see Fig.1), from (4)

$$x_1(s) = \phi(s)A_{12}x_2(s) + \phi(s)B_1u''(s) \tag{I.1}$$

and for implementation(see Fig.2), from (6) and (7)

$$\hat{x}_1(s) = (\phi^{-1}(s) + MA_{21})^{-1}(B_1 - MB_2)u'(s) + (\phi^{-1}(s) + MA_{21})^{-1}(A_{12} - MA_{22} + (A_{11} - MA_{21})M)x_2(s) + Mx_2(s)$$

After some simplifications, we have

$$\hat{x}_1(s) = (\phi^{-1}(s) + MA_{21})^{-1}(B_1 - MB_2)u'(s) + (\phi^{-1}(s) + MA_{21})^{-1}(A_{12} + M(sI - A_{22}))x_2(s) \tag{I.2}$$

From implementation

$$(sI - A_{22})x_2(s) = B_2u''(s) + A_{21}\phi(s)A_{12}x_2(s) + A_{21}\phi(s)B_1u''(s)$$

Substituting the above into (I.2), we have

$$\begin{aligned}\hat{x}_1(s) &= (\phi^{-1}(s) + MA_{21})^{-1}(B_1 - MB_2)u'(s) \\ &\quad + (\phi^{-1}(s) + MA_{21})^{-1}(A_{12}x_2(s) + MB_2u''(s) \\ &\quad + MA_{21}\phi(s)A_{12}x_2(s) + MA_{21}\phi(s)B_1u''(s)) \\ &= (\phi^{-1}(s) + MA_{21})^{-1}B_1u'(s) + (\phi^{-1}(s) + MA_{21})^{-1}MB_2(u''(s) - u'(s))\end{aligned}$$

$$+\phi(s)A_{12}x_2(s)+(\phi^{-1}(s)+MA_{21})^{-1}MA_{21}\phi(s)B_1u''(s) \tag{I.3}$$

From (I.1) and (I.3), we have

$$\begin{aligned}
&\hat{x}_1(s)-x_1(s)\\
&=(\phi^{-1}(s)+MA_{21})^{-1}B_1u'(s)+(\phi^{-1}(s)+MA_{21})^{-1}MB_2(u''(s)-u'(s))\\
&\quad+\{(\phi^{-1}(s)+MA_{21})^{-1}MA_{21}\phi(s)B_1-\phi(s)B_1\}u''(s)\\
&=(\phi^{-1}(s)+MA_{21})^{-1}B_1u'(s)+(\phi^{-1}(s)+MA_{21})^{-1}MB_2(u''(s)-u'(s))\\
&\quad+(\phi^{-1}(s)+MA_{21})^{-1}(MA_{21}-\phi^{-1}(s)-MA_{21})\phi(s)B_1u''(s)\\
&=(\phi^{-1}(s)+MA_{21})^{-1}(B_1-MB_2)(u'(s)-u''(s))
\end{aligned} \tag{I.4}$$

Considering the matrix which appears the right side of (I.4) and noting that

$$(\phi^{-1}(s)+MA_{21})^{-1}=\phi(s)-\phi(s)M(I+A_{21}\phi(s)M)^{-1}A_{21}\phi(s)$$

we have

$$\begin{aligned}
&(\phi^{-1}(s)+MA_{21})^{-1}(B_1-MB_2)\\
&\qquad=\phi(s)[(B_1-MB_2]-M(I+A_{21}\phi(s)M)^{-1}A_{21}\phi(s)(B_1-MB_2)]
\end{aligned}$$

If we want to obtain $\hat{x}_1(s)=x_1(s)$, the above is identically equal to zero. Then we have

$$B_1-MB_2=M(I+A_{21}\phi(s)M)^{-1}A_{21}\phi(s)(B_1-MB_2)$$

After some simplifications, we get

$$\begin{aligned}
B_1&=M(I+A_{21}\phi(s)M)^{-1}A_{21}\phi(s)(B_1-MB_2)+MB_2\\
&=M(I+A_{21}\phi(s)M)^{-1}A_{21}\phi(s)B_1\\
&\quad+M(I+A_{21}\phi(s)M)^{-1}(-A_{21}\phi(s)M+I+A_{21}\phi(s)M)B_2\\
&=M(I+A_{21}\phi(s)M)^{-1}A_{21}\phi(s)B_1+M(I+A_{21}\phi(s)M)^{-1}B_2\\
&=M(I+A_{21}\phi(s)M)^{-1}(B_2+A_{21}\phi(s)B_1)
\end{aligned}$$

Because the inversion of matrix $(B_2+A_{21}\phi(s)B_1)$ exists, we have

$$B_1(B_2+A_{21}\phi(s)B_1)^{-1}=M(I+A_{21}\phi(s)M)^{-1} \tag{I.5}$$

That is (8). If we select the observer correction matrix M to make (8) hold for all s, the open-loop transfer function matrices in both implementations Σ_1 and Σ_2 (with loop broken at point "X") are identical. The property 3 is proved. Q.E.D.

带降维状态观测器控制系统的鲁棒性

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摘　要

本文讨论了带降维状态观测器控制系统的鲁棒性。给出了带降维状态观测器控制系统的三个重要的频域性质和一个定理，同时给出了降维状态观测器极点配置方法。