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Observer Design for Bilinear Multivariable Systems with Unknown Disturbance

Sun Youxian, Wang Qingguo
(Department of Chemical Engineering, Zhejiang University, Hangzhou)

Abstract

The problem of designing an observer for bilinear multivariable systems in the presence of unknown disturbances is considered. A composite observer is constructed such that it asymptotically estimates both system states and disturbance states at the same time. A unified control law consisting of disturbance-feedforward and state-feedback is designed and its application to the control of a three-component distillation column system is illustrated.

1. Introduction

In the recent years, there has been an increasing interest in the study of bilinear systems control. In particular, the stabilization of a bilinear system by quadratic state feedback has been considered by Gutman (1981) and Jacobson (1977). As the linear system, if the state variables of a bilinear system are not all available, the use of an observer is indispensable in order to implement a quadratic state feedback. For this purpose, minimal order state observers for bilinear systems, whose estimated error is independent of inputs, have been found by Hara and Furuta (1976). However, in order to construct such an observer, it is necessary that all system inputs should be known or measurable. In many practical cases some system inputs, including system disturbances, are completely unknown or unmeasurable. Under this situation, Hara and Furuta's observer is not applicable. It is well known that a disturbance feedforward is quite effective to the control of a system. It would be desirable that some observer can be used to estimate both states and disturbances so that state feedback and disturbance feedforward

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can be realized simultaneously.

In this paper, the problem of designing an observer for bilinear systems in the presence of unknown disturbances is considered. A composite observer is constructed such that it asympototically estimates both system states and disturbance states at the same time and then an unified controller consisting of disturbance-feedforward and state-feedback is designed (Section 2). The application of the observer and controller to the control of a three-component distillation system is illustrated (Section 3).

2. Observer Design

We now consider an n-dimensional bilinear system Σ_p with unmeasurable disturbances

$$\dot{X} = A^0 X + \sum_{i=1}^{m} A^i u_i X + BU + FV \qquad (1)$$

$$Y = CX + DV \qquad (2)$$

where $X \in R^n$, $U \in R^m$, $Y \in R^p$ and $V \in R^t$ are state, input, output and disturbance vectors, respectively. It is assumed that V is generated by

$$\dot{X}_W = A_W X_W \qquad (3)$$

$$V = C_W X_W \qquad (4)$$

where X_W is an n_W-dimensional disturbance state vector, and that V consists of l_1 measurable disturbances and l_2 unmeasurable disturbances, where $l_1 + l_2 = t$. Without loss of generality, let $V^T = [V^T_1 \quad V^T_2]$ where V_1 is l_1-dimensional measurable disturbance subvector and V_2 the all unmeasurables. Then (4) is rewriten as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{pmatrix} C_{W_1} \\ C_{W_2} \end{pmatrix} X_W \qquad (5)$$

where C_{W_1} is the first l_1 rows of C_w.

The problem then becomes to construct an $(n+n_w-p-l_1)$-dimensional system Σ_0 which will be represented by

$$\dot{Z} = \hat{A}^0 Z + \sum_{i=1}^{m} \hat{A}^i u_i Z + \sum_{i=1}^{m} \hat{B}^i u_i [Y^T \quad V_1^T]^T + \hat{J} U \qquad (6)$$

$$W = \hat{C} Z + \hat{D} [Y^T \quad V_1^T]^T \qquad (7)$$

such that it is a composite observer for Σ_p, i.e.,

$$\lim_{t\to\infty}\frac{d^j}{dt^j}(W(t)-[X^T \quad X_W^T]^T)=0, \quad j=0,1,2,\cdots$$

and independent of $U(\cdot)$, $X(0)$, $X_W(0)$ and $Z(0)$. To this end, Σ_p is rewriten as

$$\dot{\overline{X}}=\overline{A}^0\overline{X}+\sum_{i=1}^{m}\overline{A}^i\overline{u}_i\overline{X}+\overline{BU} \tag{8}$$

$$\overline{Y}=\overline{CX} \tag{9}$$

where

$$\overline{X}=\begin{bmatrix} X \\ X_W \end{bmatrix}, \quad \overline{U}=U, \quad \overline{Y}=\begin{bmatrix} Y \\ V_1 \end{bmatrix}, \quad \overline{A}^0=\begin{bmatrix} A & FC_W \\ 0 & A_W \end{bmatrix}$$

$$\overline{A}^i=\begin{bmatrix} A^i & 0 \\ 0 & 0 \end{bmatrix}, \quad \overline{B}=\begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \overline{C}=\begin{bmatrix} C & DC_W \\ 0 & C_{W1} \end{bmatrix}$$

Let $\overline{n}=n+n_W$, $\overline{m}=m$, $\overline{p}=p+l_1$, and Rank $(\overline{C})=\overline{p}$. Then there is a nonsingular state transformation matrix $T(\widetilde{X}=T\overline{X})$ which transforms Σ_p into Σ_p^* represented by

$$\dot{\widetilde{X}}=\begin{pmatrix} \widetilde{A}^0_{11} & \widetilde{A}^0_{12} \\ \widetilde{A}^0_{21} & \widetilde{A}^0_{22} \end{pmatrix}\widetilde{X}+\sum_{i=1}^{m}\begin{pmatrix} \widetilde{A}^i_{11} & \widetilde{A}^i_{12} \\ \widetilde{A}^i_{21} & \widetilde{A}^i_{22} \end{pmatrix}\overline{u}_i\widetilde{X}+\begin{bmatrix} \widetilde{B}_1 \\ \widetilde{B}_2 \end{bmatrix}\overline{U} \tag{10}$$

$$\overline{Y}=[I_p \quad 0]\widetilde{X} \tag{11}$$

where $\widetilde{A}^i_{11}$, $\widetilde{A}^i_{12}$, $\widetilde{A}^i_{21}$, $\widetilde{A}^i_{22}$, $\widetilde{B}_1$ and $\widetilde{B}_2$ are $\overline{p}\times\overline{p}$, $\overline{p}\times(\overline{n}-\overline{p})$, $(\overline{n}-\overline{p})\times\overline{p}$, $(\overline{n}-\overline{p})\times(\overline{n}-\overline{p})$, $\overline{p}\times\overline{m}$ and $(\overline{n}-\overline{p})\times\overline{m}$ matrices. Then the system Σ_p^*, will have no unknown input. By applying the result of *Hara* and Furuta (1976) to Σ_p^*, we then obtain the following.

Theorem 1 A minimal-order composite observer for Σ_p exists if and only if there is an $(\overline{n}-\overline{p})\times\overline{p}$ matrix H satisfying the following two conditions

(*a*) $\widetilde{A}^i_{22}+H\widetilde{A}^i_{12}=0$, $\quad i=1,2,\cdots,\overline{m}$

(*b*) $\sigma(\widetilde{A}^0_{22}+H\widetilde{A}^0_{12})\in C^-$

where $\sigma(T)$ denotes the set of all eigenvalues of T and C^- the left half of the complex plane. Furthermore if (a) and (b) hold true, the parameter matrices of a minimal-order observer Σ_0 is given by

$$\hat{A}^i = 0, \qquad i = 1, 2, \cdots, m$$

$$\hat{B}^i = H\tilde{A}^i_{22} + \tilde{A}^i_{21}, \qquad i = 1, 2, \cdots, m$$

$$\hat{A}^0 = \tilde{A}^0_{22} + H\tilde{A}^0_{12}$$

$$\hat{B}^0 = H\tilde{A}^0_{11} + \tilde{A}^0_{21} - (\tilde{A}^0_{22} + H\tilde{A}^0_{12})H$$

$$\hat{J} = H\tilde{B}_1 + \tilde{B}_2$$

$$\hat{C} = \begin{bmatrix} 0 \\ I_{n-p} \end{bmatrix}, \quad \hat{D} = \begin{bmatrix} I_p \\ -H \end{bmatrix}$$

To further simplify the conditions (a) and (b), first (a) can be rewriten as

$$H[\tilde{A}^1_{12} \quad \tilde{A}^2_{12} \cdots \tilde{A}^m_{12}] = [-\tilde{A}^1_{22} \quad -\tilde{A}^2_{22} \cdots -\tilde{A}^m_{22}] \tag{12}$$

Let T_1 be a nonsingular matrix such that

$$[\tilde{A}^1_{12} \quad \tilde{A}^2_{12} \cdots \tilde{A}^m_{12}]T_1 = [G_1 \quad 0]$$

where G_1 has full column rank with Rank $(G_1) = \text{Rank}[\tilde{A}^1_{12} \quad \tilde{A}^2_{12} \cdots \tilde{A}^m_{12}] = r$. postmultiplying (12) by T_1 yields

$$H[G_1 \quad 0] = [Q_1 \quad Q_2] \tag{13}$$

where Q_1 has the same number of columns as that of G_1. Therefore, condition (a) is satisfied if and only if

$$Q_2 = 0 \tag{14}$$

If it is the case, all solutions can be given in a parametric form. Since G_1 has full column rank, its first r rows can be assumed nonsingular, i. e.

$$G_1 = \begin{pmatrix} G_{11} \\ G_{21} \end{pmatrix}$$

where G_{11} is nonsingular. Also H can be partitioned as

$$H = [H_1 \quad H_2]$$

where H_1 is the first r columns of H. With (14) holding, all solutions H of (12) can be given by

$$H = [(Q_1 G_{11}^{-1} - H_2 G_{21} G_{11}^{-1}) \quad H_2] \tag{15}$$

where H_2 is an arbitrary matrix. Using the condition (b) of Theorem 1. It follows from (15) that

$$\tilde{A}_{22}^0 + H\tilde{A}_{12}^0 = (\tilde{A}_{22}^0 + Q_1 G_{11}^{-1}\tilde{A}_{121}^0) + H_2(\tilde{A}_{122}^0 - G_{21}G_{11}^{-1}\tilde{A}_{121}^0)$$

where

$$\tilde{A}_{12}^0 = \begin{pmatrix} \tilde{A}_{121}^0 \\ \tilde{A}_{122}^0 \end{pmatrix}$$

and $\tilde{A}_{121}^0$ has r rows. Let

$$Q = \tilde{A}_{22}^0 + Q_1 G_{11}^{-1}\tilde{A}_{121}^0$$

$$G = \tilde{A}_{122}^0 - G_{21}G_{11}^{-1}\tilde{A}_{121}^0$$

Then it follows from well-know pole assignment theory that the following holds.

Theorem 2 The conditions (a) and (b) of Theorem 1 hold if and only if (14) is satisfied and (Q,G) is a detectable pair. Furthermore, all eigenvalues of the observer can be arbitrarily assigned if and only if (Q,G) is an observable pair.

For a bilinear system Σ_p given by (1) to (4), if X and X_W are all available, then a controller, which uses all available plant informations as its inputs, will have the form

$$U = F_X(X) + F_W(X_W) \tag{16}$$

where $F_X(X)$ is feedback control part and F_W (X_W) is feedforword control part. For the feedback part F_X (X), it has been shown by Jacobson (1977) and Gutman (1981) that if A^0 has no eigenvalue in the right-half plane, the following quadratic state feedback control law will stabilize the system Σ_p.

$$u_i = -(A^i X + b_i)^T P X, \qquad i = 1,2,\cdots,m \tag{17}$$

where b_i is the ith column of B and P is a symmetric and positive definite solution of the following equation

$$PA^0 + (A^0)^T P = -Q \tag{18}$$

with Q being arbitrary positive definite matrix.

For the feedforward contol part F_W (X_W), if $D=0$ for Σ_p, we will determine the following control law

$$U = K_W X_X \tag{19}$$

such that disturbance state X_W has no effect on the system state X. This means that

$$[A^1X + b_1 \quad A^2X + b_2 \cdots A^m X b_m] K_W + FC_W = 0$$

which has a solution for K_W if and only if

$$\text{rank}[A^1X + b_1 \quad A^2X + b_2 \cdots A^m X + b_m]$$
$$= \text{rank}[A^1X + b_1 \quad A^2X + b_2 \cdots A^m X + b_m \quad FC_W] \tag{20}$$

for all X.

If X and X_W are not all available, they can be replaced by their estimates which are obtained by a composite observer discussed in the last section.

3. Application

In this section, the application of the observer and controller design to a three-component distillation system is presented.

Process Description The plant to be controlled is a three-component distillation column system, shown in Fig. 1. It consists

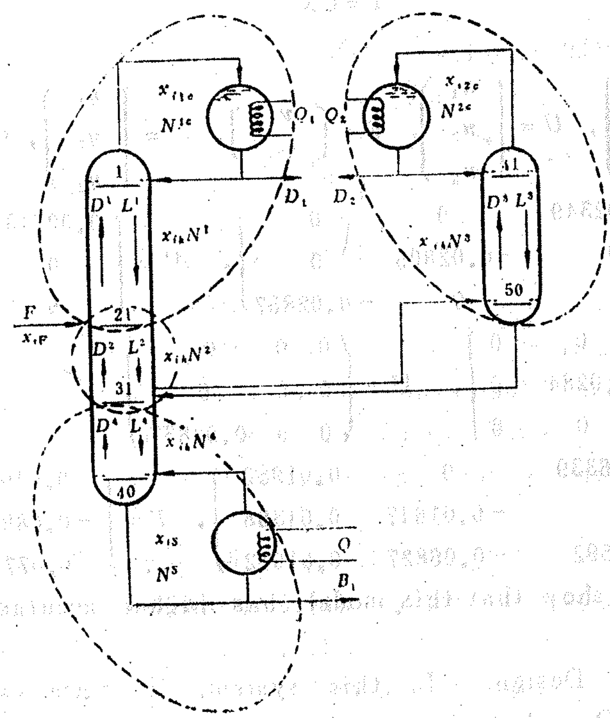

Fig.1

of two cascade distillation columns. The feed is a mixture of methanol, ethanol and proponol. production goal is to make the three product concentrations as high as possible. Control variables are reflux ratioes u_1 and u_2 of first and second columns, and the steam flow rate u_3 at the bottom of the first column. Main disturbances are the feed flow rate v_1 and feed component v_2. The physical analysis of distillation process shows that the largest concentration variations occurs in the middle section of the column. If the column operating pressure is constant, the plate temperature and concentration are one-to-one correspondence. Since the former is more sensitive to the production condition variation than the concentration. Three measured temperatures y_1, y_2 and y_3 in the middle section of the column are chosen as controlled variables.

Process Model This distillation column system has a nonlinear property. Thus a bilinear model is established by physical analysis as follows

$$X = A^0 X + \sum_{i=1}^{3} A^i u_i X + BU + FV \tag{21}$$

$$Y = CX \tag{22}$$

where

$$A = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad U = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \quad V = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}, \quad C = I_3$$

$$A^0 = \begin{bmatrix} -0.02349 & 0 & 0 \\ 0 & -0.02805 & 0 \\ 0 & 0 & -0.02867 \end{bmatrix}, \quad A^1 = \begin{bmatrix} 0.02513 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.0284 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.02837 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.06339 & 0 & 0.01936 \\ 0 & -0.01647 & 0.01388 \\ -0.2592 & -0.06827 & 0.01012 \end{bmatrix}, \quad F = \begin{bmatrix} -0.01096 & -0.01196 \\ -0.08987 & 0 \\ -0.07766 & -0.04349 \end{bmatrix}$$

Experiments show that this model has higher accuracy than the linear model.

Observer Design In this system, all state variables are measurable. One disturbance, the feed flow rate v_1, can easily measured, but the other disturbance, the feed component v_2, is difficult and expensive to measure. Therefore, an observer is used to estimate it. Assuming constant disterbance vector, the disturbance equations are

$$X_W = A_W X_W \tag{23}$$

$$V = C_W X_W \tag{24}$$

where $X_W = V$, $A_W = O$, and $C_W = I_2$. By theorem 1, one can show that the eigenvalue of the observer can be arbitrarily assigned. It is specified as -0.1 for the practical reason. After some simple calculations, the observer is

$$\dot{Z} = \hat{A}^0 Z + \hat{B}^0 \overline{Y} + \sum_{i=1}^{3} \hat{B}^i u_i \overline{Y} + \hat{J} U \tag{25}$$

$$W = \hat{C} Z + \hat{D} \overline{Y} \tag{26}$$

where

$$\overline{Y}^T = [y_1 \quad y_2 \quad y_3 \quad v_1]$$

$$\hat{A}^0 = -0.1, \quad \hat{B}^0 = [0 \quad 0 \quad 0.1641 \quad -0.1786]$$

$$\hat{B}^1 = [0 \quad 0 \quad 0 \quad 0], \quad \hat{B}^2 = [0 \quad 0 \quad 0 \quad 0]$$

$$\hat{B}^3 = [0 \quad 0 \quad 0.6525 \quad 0], \quad \hat{J} = [-0.5962 \quad -0.157 \quad 0.2328 \quad 0]$$

$$\hat{C} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \hat{D} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -2.3 & 0 \end{pmatrix}$$

It is clear that w_5 is an estimate of v_2.

Feedforward Control Since this system operates in the neighbourhood of the steady state, X is thus small. When $\max\{|x_1|, |x_2|, |x_3|\} \leqslant 0.2$, the matrix $[A^1X + b_1, \ A^2X + b_2, \ A^3X + b_3]$ is of full rank. For this reason, we can obtain, after some simplifications,

$$K_W = [A^1 X + b_1, \ A^2 X + b_2, \ A^3 X + b_3]^{-1} F$$

$$= \begin{pmatrix} (0.5081 + 0.0341x_2 + 0.1383x_3) & (-0.141 - 0.7502x_2 + 0.1509x_3) \\ (2.609 - 5.447x_1 + 4.366x_3) & (0.1291 + 0.4099x_1) \\ (9.577 - 3.311x_1 + 0.2485x_2) & (0.153 + 0.513x_1 + 0.697x_2) \end{pmatrix} \tag{27}$$

The feedforward control thus becomes

$$F_W(X_W) = K_W V \tag{28}$$

where $V^T = [v_1 \quad w_5]$.

Feedback Control We choose $Q = \mathrm{diag}\{0.5, \ 0.5, \ 0.5\}$, P can be solved from (26). The resulting feedback control is

$$U = F_x(X) \tag{29}$$
$$= -\begin{pmatrix} (0.2673x_1 - 0.6744) & 0 & (-2.259) \\ 0 & (0.2531x_2 - 0.1467) & (-0.5952) \\ (0.2056) & (0.1245) & (0.247x_3 + 0.8823) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Simulation Results The simulated system consists of the exact 106th-order plant model, observer (25) and (26), feedforward control (27) and feedback control (29). By system simulations and experiments, this designed system is compaired with the system without feedforward control (27). The result is shown in Fig.2.

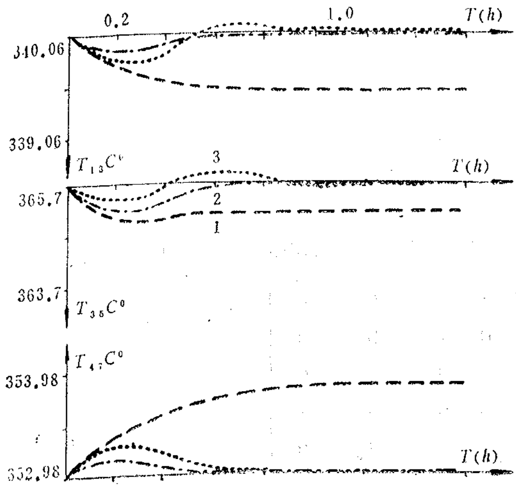

Fig.2

1. Simulated without observer
2. Simulated with observer
3. Experimented with observer

4. Conclusion

In this paper, an observer design for bilinear multivariable systems in the presence of unknown disturbances is presented. The resulting composite observer asymptotically estimates both system states and disturbance states, while the estimate error is independent of inputs and initial states, with this observer, an unified controller consisting of disturbance-feedforward and state-feedback can be designed and implementaed. From the application to a distillation system, the designed observer-controller system exhibits a great improvement over only state-feedback control system.

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具有未知扰动的双线性多变量系统的观测器设计

孙优贤　王庆国

（浙江大学化学工程系，杭州）

摘　要

本文讨论了存在未知扰动情况下如何设计双线性多变量系统的观测器问题．文中开发了一个联合观测器，它能同时渐近估计系统的状态和扰动的状态，实现扰动前馈和状态反馈控制．文中还给出了上述观测器和控制器设计在一个大型三组份精馏塔群中的应用结果。