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Bang-Bang Control of a Robot Arm with Three-degrees of Freedom

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Abstract

In this paper, a three-degrees of freedom robot arm is used to model an experimental manipulator for controller design purpose. The mathematical models of the robot arm, the bang-bang controller and the electrical drives are then formulated. Calculation of the switching times for the bang-bang controller and the simulation of the arm motion are both performed on the VAX 11/750 computer. Implementation of the bang-bang controller was made possible using the existing M6800-based single board micro-computer. Experimental tests revealed that the actual results agreed reasonably well with the simulated ones.

Notation

e = maximum voltage applied to the armature circuit

I = moment of inertia of link about its center of gravity

l = length of link

l_g = length between the center of gravity of link and its respective joint

m = mass of link

m_l = mass of load at the end of link 3 simulating the total mass of two stepping motors, wrist, end-effector, payload etc.

M_m = mass of load at joint 3 simulating the total mass of servomotor, encoder etc.

r = reduction ratio of gear box

T = load torque applied to output shaft

θ = joint displacement

Introduction

With a minimum-time controller, the manipulator can move from

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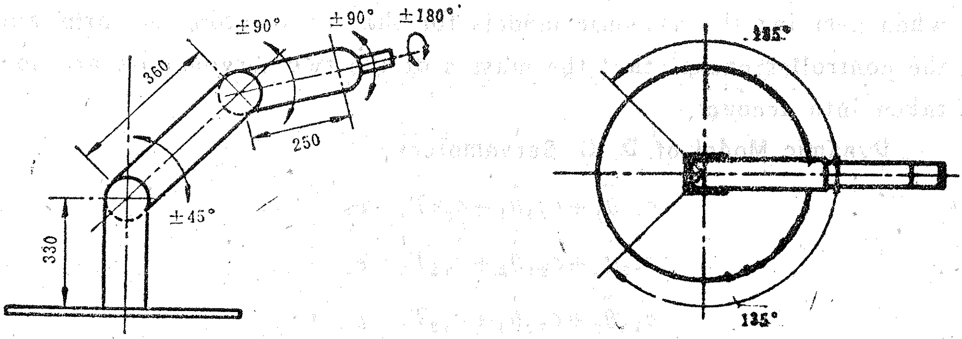
one point to another, such as pick-and-place operation, in shortest time. The design of th eminimum-time controller using maximum principle (1) involved complicated calculations even for a two-link device. Simplifications such as no path constraint and two saturated positions for the driving source lead to bang-bang control. Design of the bang-bang controller for a planar manipulator with two-degrees of freedom was studied initially by Sato et. al.(2) and its subsequent simulation on a desk-top microcomputer was realized by Fung and Leung (3). The present investigation is indeed an extension of the previous work (3). It deals with the design of a bang-bang controller for the three-degrees of freedom robot arm which is used to model the experimental five-axis servo-controlled manipulator (4). The arm is commanded to move in a way such that all the three joints would reach their final desired positions simultaneously. The dynamics of the robot arm are now simulated on the VAX 11/750 computer using FORTRAN 77 compiler and GIGI graphics package.

Mechanical Construction of Manipulator

The pilot articulated robot manipulator shown in Figure 1 is located in the Control & Automation Laboratory of Hong Kong Polytechnic. It consists of three main structural elements: the arm, with three-degrees of freedom for positioning; the wrist, with two-degrees of freedom for orientation; and the gripper hand or other end- effector. The five degrees of freedom achieved are;

- Slew rotation of ± 135 degrees from the central position.
- Inner-arm movement of ± 45 degrees from the vertical.
- Outer-arm movement of ± 90 degrees from the axis of the inner arm.
- End effector pitching angles of ± 90 degrees from the axis of the outer arm.
- End effector rolling angles of ± 180 degrees from any arbitrary position

The robot arm is the part of the robot which performs placing movements. The arm consists of three rigid members connected by two rotary joints and mounted on a rotary base, giving the three degrees of freedom. The three movements are actuated by dc servo-motors through reduction gears. The driving torques required at the base and the two joints were determined so that the arm can support a 1 kg load any-



(a) side-view (b) top-view

Fig.1 Robot Manipulator

where within the robot work volume. Servo-motors of suitable sizes were then chosen. The end-effector is connected to the main frame of the robot through the wrist which adds two more degrees of freedom to the three-link device. The principal mechanism of the wrist is a differential-gear assemly, driven by a pair of stepping motors through chains and sprockets.

Modelling of Manipulator

Physical Model

The physical model of the arm is shown in Figure 2. It is a two-link device mounted on an upright column. Each link is modelled by a light rod with concentrated mass located at a certain distance from the respective joint. Additionally, there are two extra masses M_m and M attached to the links as depicted in Figure 2. Mass M_m at joint 3 simulates the total mass of the servomotor, encoder etc. while M at the end of link 3 simulates the total mass of the two stepping motors, wrist, end-effector and payload etc..

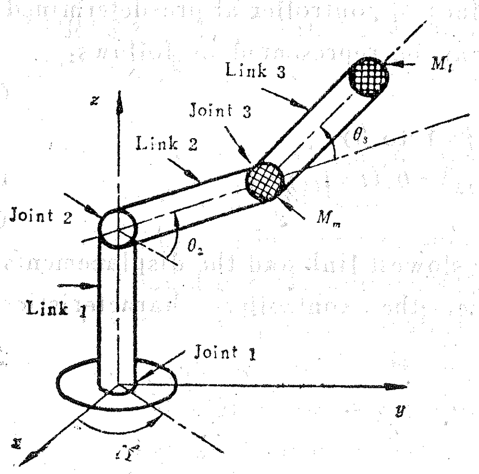


Fig.2 Physical Model of Robot Arm

Mathematical Model

The assumptions stated in the previous work (3) are still valid when deriving the dynamic models for the servomotor, the arm and the controller except that the masses of the two servomotors are now taken into account.

Dynamic Model of D. C. Servomotors:

$$c_{11}\ddot{\theta}_1 + c_{21}\dot{\theta}_1 + c_{31}T_1 = e_1 \tag{1}$$

$$c_{12}\ddot{\theta}_2 + c_{22}\dot{\theta}_2 + c_{32}T_2 = e_2 \tag{2}$$

$$c_{13}\ddot{\theta}_3 + c_{23}\dot{\theta}_3 + c_{33}T_3 = e_3 \tag{3}$$

Derivation of the above equations can be found in (3). The suffix of 'e', 'T' or 'θ' denotes the particular link the motor drives; '1' for link 1, '2' for link 2 and '3' for link 3.

Dynamic Model of Robot Arm:

$$T_1 = A_{11}\ddot{\theta}_1 + B_{112}\dot{\theta}_1\dot{\theta}_2 + B_{113}\dot{\theta}_1\dot{\theta}_3 \tag{4}$$

$$T_2 = G_2 + A_{22}\ddot{\theta}_2 + A_{23}\ddot{\theta}_3 + B_{211}\dot{\theta}_1^2 + B_{223}\dot{\theta}_2\dot{\theta}_3 + B_{233}\dot{\theta}_3^2 \tag{5}$$

$$T_3 = G_3 + A_{32}\ddot{\theta}_2 + A_{33}\ddot{\theta}_3 + B_{311}\dot{\theta}_1^2 + B_{322}\dot{\theta}_2^2 \tag{6}$$

The formulation of the above dynamic equations using Lagrange-Euler method is also outlined in (3)

Bang-Bang Controller Model: The bang-bang controller is designed so that the three joint positions of the robot arm can reach their final values at the same time with zero terminal velocities by switching between the boundary values of controller at pre-determined switching times. The requirements can be represented as follows:

$$\text{(a)} \quad \theta_j(t=0) = \theta_{ij} \text{ and } \quad \theta_j(t=t_{fj}) = \theta_{fj} \qquad (j=1 \text{ to } 3) \tag{7}$$

$$\text{(b)} \quad \dot{\theta}_1(t=t_{f1}) = \dot{\theta}_2(t=t_{f2}) = \dot{\theta}_3(t=t_{f3}) = 0 \tag{8}$$

$$\text{(c)} \quad t_{f1} = t_{f2} = t_{f3} \tag{9}$$

Assuming that the link n is the slowest link and the displacements ($\theta_{fj} - \theta_{ij}$, $j=1,2,3$) are all positive, the controller characteristics become:

$$e_n = \begin{cases} +e_{n\max} & 0 < t \leq t_{gn} \\ -e_{n\max} & t_{gn} < t \leq t_{fn} \\ 0 & t_{fn} < t \end{cases} \tag{10}$$

$$e_j = \begin{cases} -e_{j\max} & 0 < t \leq t_{xj} \\ +e_{j\max} & t_{xj} < t \leq t_{gj} \\ -e_{j\max} & t_{gj} < t \leq t_{fj} \\ 0 & t_{fj} < t \end{cases} \qquad (j=1 \text{ to } 3 \text{ except } n) \tag{11}$$

Combining equations (1) to (6), we obtain

$$e_1 = (c_{11} + c_{31}A_{11})\ddot{\theta}_1 + c_{21}\dot{\theta}_1 + c_{31}B_{112}\dot{\theta}_1\dot{\theta}_2 + c_{31}B_{113}\dot{\theta}_1\dot{\theta}_3 \qquad (12)$$

$$e_2 = (c_{12} + c_{32}A_{22})\ddot{\theta}_2 + c_{22}\dot{\theta}_2 + c_{32}A_{23}\ddot{\theta}_3 + c_{32}B_{211}\dot{\theta}_1^2$$

$$+ c_{32}B_{223}\dot{\theta}_2\dot{\theta}_3 + c_{32}B_{233}\dot{\theta}_3^2 + c_{32}G_2 \qquad (13)$$

$$e_3 = (c_{13} + c_{33}A_{33})\ddot{\theta}_3 + c_{23}\dot{\theta}_3 + c_{33}A_{32}\ddot{\theta}_2 + c_{33}B_{311}\dot{\theta}_1^2$$

$$+ c_{33}B_{322}\dot{\theta}_2^2 + c_{33}G_3 \qquad (14)$$

The mathematical model of the robot arm together with the bang-bang controller is described by equations (7) to (14).

Simulation of Robot Arm

The computer simulation of the motion of robot arm under bang-bang control was performed on the VAX 11/750 computer in the Department of Mechanical and Marine Engineering of the Hong Kong Polytechnic. The main program was written to calculate the switching times and the joint displacements, velocities and accelerations for any path of the robot arm. The other two subprograms were used for graph plotting and three dimensional representation of robot arm trajactory. All programs were written in FORTRAN language using FORTRAN 77 compiler and the graphic utilities were developed using GIGI graphic package. The execution procedure takes the following steps:

(a) It accepts manually input data such as initial and final arm position, and load.

(b) The routine first computes the coefficients of the differential equations (Equations (12)—(14)).

(c) The routine next evaluates the approximate switching times (t_{on}, t_{xi}, t_{gi}) of the three motor voltages. The method of calculation is very similar to the one employed in the case of two-link device (3).

(d) Exact switching times are found from the mathematical model (Equations (7)—(14)) by iteration using Fourth-order Runge-Kutta Method.

(e) The three-dimensional arm configuration together with the graphs of joint position, velocity and acceleration versus time are displayed on the graphic terminal. Tabulated results are output to the printer.

Implementation of Bang-Bang Controller

The existing microcomputer system for the controller implementation is composed of six major components: single-board microcomputer (M68MM01), Microbug and RS-232C board, Analogue-Digital and Digital-Analogue (A/D and D/A) cards, Peripheral Interface Adaptor (PIA), display console, power supply and card cage. The mono-board microcomputer, which is MC6800-based, has 1 K and 8 K of Random Access Memory (RAM) after expansion. The monitor Microbug occupies 2 K of EPROM and the self-developed software occupies 529 bytes of a 2K EPROM. The software program written in assembly language performs initialization task and sends out three timing pulses to the three amplifiers of the servomotors via the D/A convertors.

The Motorola Exorciser II development system is used as the tool in the software design and development stage. After the program has been thoroughly tested, it is then programmed onto the EPROM which will later be inserted into the socket of the M68MM01. With the other supporting microsystems such as microbug and RS232C card, A/D and D/A cards, PIA card, power supply and display console, the dedicated microcomputer control system is expected to act individually as the controller for the experimental manipulator.

Results

The parameters of the physical model representing the mechanical manipulator are given in Table 1 of the Appendix. An illustrative set of results is presented in this paper for the trajectory with initial joint coordinates (−10°, 90°, 0°) and final joint coordinates (20°,120°,20°), and $M_m=3.575$ kg and $M_l=5.5$kg. Figures 3 to 5 show the simulation and experimental results of the three joint displacements respectively against time. It can be noted the three joints reached their final positions at the same time which was about 1.6s. The joint error at their final positions ranged from 2° to 3° which is considered reasonable in the case of open-loop bang-bang control. The simulation results predicted with reasonable accuracy

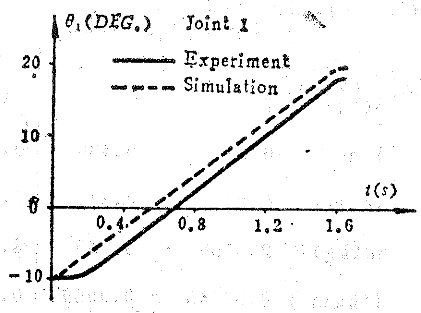


Fig.3 Displacement of Joint *1 from -10° to 20°

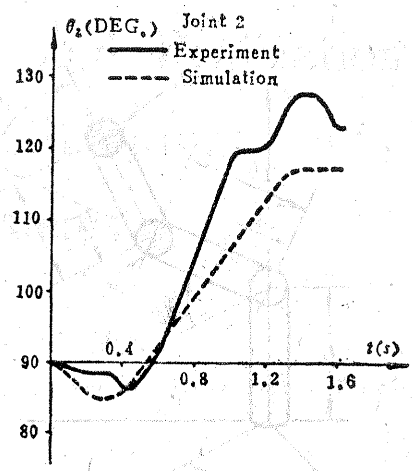


Fig.4 Displacement of Joint *2 from 90° to 120°

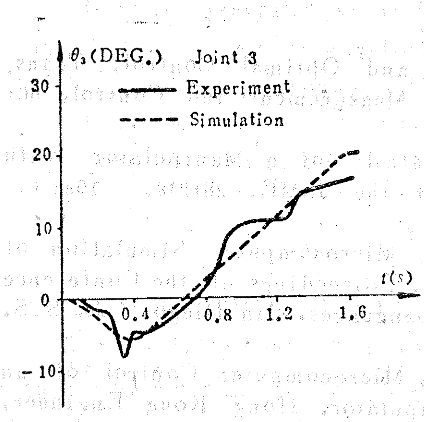


Fig.5 Displacement of Joint *3 from 0° to 20°

the actual robot motions and the discrepancies are attributed to the fact that the effects of gear friction, backlash and flexible link structure on the robot dynamics were ignored in the modelling, and the uncertainties in the parameter identification.

Conclusions

In this paper, it has been demonstrated that the design method for the bang-bang control of a two-link device can be applied to a three-degrees of freedom robot arm. Practical implementation of the controller using single-board microcomputer to the experimental mechanical manipulator was found to be feasible. Reasonably good agreement between the computer simulation results and the experimental ones was obtained.

Appendix

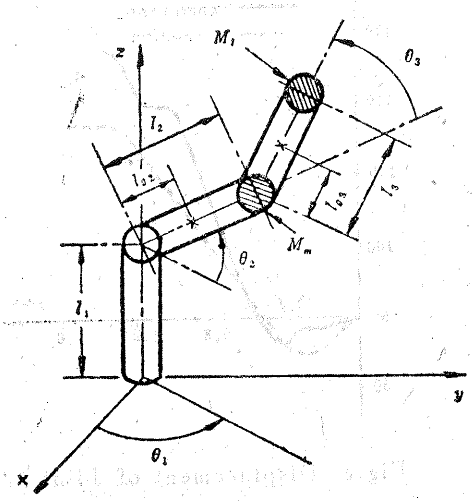


Fig.A.1 Robot Arm

Table 1 Parameters of Robot Arm

	Link1	Link2	Link3
e(v)	2	2	1
l(m)	0.38	0.436	0.35
lg(m)	0.21	0.24	0.175
m(kg)	23.155	8.745	8.2
I(kgm²)	0.07745	0.09599	0.04115
r(m)	0.01667	0.012	0.01061

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三自由度机械臂的继电式控制

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摘 要

本文介绍了一个具有三个自由度机器人手臂的继电式控制器的设计方法。通过公式，表达了该机器人手臂的数学模型、继电式控制器和其电气拖动装置的特征。利用VAX 11/750计算机进行继电式控制器开关时间的计算，以及手臂移动的模拟。而利用现有的M6800单板机作继电式控制器，对一台试验性机器人手臂进行了实际的控制试验。这台试验性机器人手臂同数学模型机是相似的。这个试验，证实了实际结果同模型机器人的计算结果是相当一致的。