Recursive Identification and Realization Algorithm of Input-output Bilinear Systems

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Abstract

In this paper a complete identification method for input-output bilinear systems have been proposed.

First, the general form of input-output equation has been given and a input-output difference equation corresponds to a class of canonical form has been obtained. Second, a recursive algorithm of the structural indices and the paramters identification has been proposed, and the realization algorithm has been given. Finally, the simulation example is given.

1. Introduction

Bilinear systems combine the versatility of a nonlinear structure with an intrinsic analytic simplicity which in many respects makes their theory similar to that of their linear counterparts. Many practical processes, in biology, socioeconomics and ecology for example, may be represented more adequately by bilinear models than by linear ones. Therefore many scholars are greatly interested.

Kotta and Nurges (1984) proposed a special class of bilinear systems called input-output bilinear system. This class is quite common among real life systems, and it is very simple for input-output model.

In this paper describes a complete procedure to obtain structure determination, paramters estimation and realization of an input-output bilinear multivariable systems from the input-output

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observed data. In this procedure the structural indices and the parameters of canonical representation are obtained simultaneously by recursive algorithm. Compared with the algorithm proposed by Kotta and Nurges, the present one is more convenient for computation. In addition we also give a general form of the input-output equation for a class of input-output bilinear systems and present a realization algorithm of canonical forms.

2. The General Form of Input-Output Relationship of an Input-output Bilinear Systems

Consider a bilinear discrete time stochastic system described by the equations

$$x(t+1)=Ax(t)+\sum_{i=1}^{m}N_i x(t)u_i(t)+Bu(t)+Mw(t) \tag{2.1}$$

$$y(t)=Cx(t)+v(t) \tag{2.2}$$

where $x(t)\in R^n$, $u(t)\in R^m$, $y(t)\in R^p$, The inputs $\{u(t)\}$ and outputs $\{y(t)\}$ are assumed to be stationary vector value stochastic sequences. The process and measurement disturbances $\{w(t)\}$ and $\{v(t)\}$ represent noise sequences with zero means, mutually independent and independent of the input $\{u(t)\}$, It is assumed that A, $N_i, i=1,2,\cdots,m,B,M,C$ are real constant matrices of appropriate dimensions and

$$\operatorname{rank}\begin{bmatrix}N\\C\end{bmatrix}=p, \qquad i=1,2,\cdots,m \tag{2.3}$$

The condition (2.3) implies

$$N_i=D_iC \qquad i=1,2,\cdots,m \tag{2.4}$$

and therfore the system (2.1), (2.2) can be rewritten as

$$x(t+1)=Ax(t)+\sum_{i=1}^{m}D_i y(t)u_i(t)+Bu(t)+Mw(t)-\sum_{i=1}^{m}D_i u_i(t)v(t) \tag{2.5}$$

$$y(t)=Cx(t)+v(t) \tag{2.6}$$

We call system (2.1), (2.2) which satisfies condition (2.3) the input-output bilinear system [1].

If the linear part (A, B, M, C) of the system (2.5), (2.6) is completely observable, then it is easy to show that there exists a transformation matrix T such that matrices A and C become canonical form through the transformation matrix T, and matrix T can be given by

$$T=E^*F \tag{2.7}$$

where $F=(C^T,A^TC^T,\cdots,A^{n-1T}C^T)^T$ is the observability matrix, E^* is a generalized selector matrix [2].

We can shown that by a similar way as [2] if the canonical form of a given system $(A,D_i,i=1,2,\cdots,m,B,M,C)$ is defined as $(A^*,D_i^*,i=1,2,\cdots,m,B^*,M^*,C^*)$ where $A^*=TAT^{-1}$, $B^*=TB,D_i^*=TD_i$, $M^*=TM,C^*=CT^{-1}$ and corresponding selector matrix is E^*, then the general input-output equation which equivalent to $(A^*,D_i^*,i=1,2,\cdots,m,B^*,M^*,C^*)$ can be expressed as

$$E^*\overline{y}(t+1)=A^*E^*\overline{y}(t)+Q\overline{u}(t)+\sum_{i=1}^{m}H^i\overline{z}_i(t)+E^*\overline{v}(t+1)$$

$$-A^*E^*\overline{v}(t)+\overline{K}w(t)-\sum_{i=1}^{m}H^i\overline{r}_i(t) \tag{2.8}$$

whree

$$Q=E^*\widetilde{R}-A^*E^*R,H^i=E^*\widetilde{G}_i-A^*E^*G_i,\quad i=1,2,\cdots,m,\quad K=E^*\widetilde{P}-A^*E^*P \tag{2.9}$$

$$R=\begin{pmatrix}0 & & & \\ CB & 0 & & \\ \vdots & \ddots & \ddots & \\ CA^{n-2}B & \cdots & CB & 0\end{pmatrix}\qquad \widetilde{R}=\begin{pmatrix}CB & & & \\ CAB & CB & & \\ \vdots & \ddots & \ddots & \\ CA^{n-1}B & \cdots & CAB & CB\end{pmatrix}$$

$$G_i=\begin{pmatrix}0 & & & \\ CD_i & 0 & & \\ \vdots & \ddots & \ddots & \\ CA^{n-2}D_i & \cdots & CD_i & 0\end{pmatrix}\qquad \widetilde{G}=\begin{pmatrix}CD & & & \\ CAD & CD & & \\ \vdots & \ddots & \ddots & \\ CA^{n-1}D & \cdots & CAD & CD\end{pmatrix}\quad i=1\cdots,m$$

$$i=1,\cdots,m$$

$$P=\begin{pmatrix}0 & & & \\ CM & 0 & & \\ \vdots & \ddots & \ddots & \\ CA^{n-2}M & \cdots & CM & 0\end{pmatrix}\qquad \widetilde{P}=\begin{pmatrix}CM & & & \\ CAM & CM & & \\ \vdots & \ddots & \ddots & \\ CA^{n-1}M & \cdots & CAM & CM\end{pmatrix}$$

$$\overline{y}(t)=(y^T(t),\cdots,y^T(t+n-1))^T\qquad \overline{u}(t)=(u^T(t),\cdots,u^T(t+n-1))^T$$

$$\overline{w}(t)=(w^T(t),\cdots,w^T(t+n-1))^T\qquad \overline{v}(t)=(v^T(t),\cdots,v^T(t+n-1))^T$$

$$\overline{z}_i(t)=(z_i^T(t),\cdots,z_i^T(t+n-1))^T\qquad \overline{r}_i(t)=(r_i^T(t),\cdots,r_i^T(t+n-1))^T$$

$$z_i(t)=y(t)u_i(t),\quad r_i(t)=v(t)u_i(t),\quad i=1,2,\cdots,n$$

For an arbitrary canonical form, corresponding selector matrix E^* given provided, then the input-output equations which correspond to the canonical form can be obtained from (2.8) by straightforward calculation.

3. Input-Output Equation

If the linear part of the system (2.5), (2.6) is completely observable, we take the generalized selector matrix

$$E^* = (E_1^T, E_2^T, \cdots, E_p^T)^T \tag{3.1}$$

where

$$E_j = (e_j^T, e_{j+p}^T, \cdots, e_{j+(n_j-1)p}^T)^T \qquad j = 1,2,\cdots,p$$

$$e_j = (\underbrace{0 \cdots 0\ 1}_{j}\ 0 \cdots 0) \tag{3.2}$$

$\{n_j\}$ are structure indices of system and $T = E^*F$, then it is easy to show that by the state transformation T, A^*, C^* have the following canonical form, respectively:

$$A^* = (A_{ij}) \tag{3.3}$$

$$A_{ii} = \begin{pmatrix} 0 & 1 & & \\ \vdots & \ddots & \ddots & \\ 0 & & 0 & 1 \\ \alpha_{ii,1} & \alpha_{ii,2} & \cdots & \alpha_{ii,n_i} \end{pmatrix} \qquad A_{\substack{ij \\ i \neq j}} = \begin{pmatrix} & & 0 & & \\ \alpha_{ij,1} & \cdots & \alpha_{ij,n_{ij}} & 0 \cdots 0 \end{pmatrix}$$

$$C^* = \begin{pmatrix} e_1 \\ e_{n_1+1} \\ \vdots \\ e_{n_1+n_2+\cdots+n_{p-1}+1} \end{pmatrix} \qquad n_{ij} = \begin{cases} \min(n_i+1, n) & \text{if } i<j \\ \min(n_i, n_j) & \text{if } i \geqslant j \end{cases} \tag{3.4}$$

We call system (2.5), (2.6) which matrices A and C with canonical form (3.3) and (3.4) respectively, POPOV's canonical form and referred to simply as PCF.

Replace A^*, E^* in (2.8) with (3.3), (3.1) respectively, the input-output difference equation which correspond to PCF can be obtained by straightforward calculation:

$$y_s(t+n_s) = \sum_{l=1}^{p} \sum_{j=1}^{n_{sl}} \alpha_{sl,j}\, y_l(t+j-1) + \sum_{l=1}^{m} \sum_{j=1}^{n_s} q_{sj,l}\, u_l(t+j-1)$$

$$+ \sum_{i=1}^{m} \sum_{l=1}^{p} \sum_{j=1}^{n_s} h_{sj,l}^{i}\, z_{li}(t+j-1) + v_s(t+n)$$

$$-\sum_{l=1}^{p}\sum_{j=1}^{n_{sl}}\alpha_{sl,j}v_l(t+j-1)+\sum_{l=1}^{m}\sum_{j=1}^{n_s}k_{sj,l}w_l(t+j-1)$$

$$+\sum_{i=1}^{m}\sum_{l=1}^{p}\sum_{j=1}^{n_s}h^i_{sj,l}\,r_{li}\,(t+j-1)\qquad s=1,\cdots,p \tag{3.5}$$

where y_i, u_i, v_i, w_i, r_{li}, z_{li}, $q_{sj,i}$, $h^l_{sj,i}$, $k_{sj,i}$ are the i-th elements of the vectors y, u, v, w, r_l, z_l, Q_{sj}, H^l_{sj}, K_{sj}, respectively, and which Q_{sj}, H^l_{sj}, K_{sj} satisfy the following relationships, respectively.

$$Q=(Q_{ij})\qquad i=1,\cdots,p,\ j=1,\cdots,n$$

$$Q_{ij}=\begin{cases} b_{s_i-j+1}-\displaystyle\sum_{l=1}^{p}\sum_{k=j+1}^{n_{il}}\alpha_{il,k}\,b_{s_{l-1}+k-j} & j=1,\cdots,n_i \\ 0 & j>n_i \end{cases} \tag{3.6}$$

$$H^i=(H^i_{kj})\qquad k=1,\cdots,p,\ j=1,\cdots,n,\ i=1,\cdots,m$$

$$H^i_{kj}=\begin{cases} d^i_{s_k-j+1}-\displaystyle\sum_{l=1}^{p}\sum_{t=j+1}^{n_{kl}}\alpha_{il,t}\,d^i_{s_{l-1}+t-j} & j=1,\cdots,n_k \\ 0 & j>n_k \end{cases} \tag{3.7}$$

$$K=(K_{ij})\qquad i=1,\cdots,p,\ j=1,\cdots,n$$

$$K_{ij}=\begin{cases} m_{s_i-j+1}-\displaystyle\sum_{l=1}^{p}\sum_{k=j+1}^{n_{il}}\alpha_{il,k}m_{s_{l-1}+k-j} & j=1,\cdots,n_i \\ 0 & j>n_i \end{cases} \tag{3.8}$$

where b_i, d^l_i, m_i are the i-th row vector of the matrices B^*, D^*_i, M^* in PCF, respectively, $s_i=\sum_{j=1}^{i}n_j$, $s_0=0$

4. The Recursive Algorithm of Structure Determination and Parameters Estimation

A recursive algorithm of the structural indices and the Parameters estimation is given based upon the estimates of auto-and crosscorreltion function of the input and output sequences.

The input and output signals are assumed to be stationary sequences. The autocorrelation function of the input u_k $(k=1,2,\cdots,m)$ then is

$$R_{u_k u_k}(j) = Eu_k(t)u_k(t-j)$$

the crosscorrelation function of the inputs u_k and u_l $(k,l=1,\cdots,m)$ is

$$R_{u_k u_l}(j) = Eu_k(t)u_l(t-j)$$

and the first and second order crosscorrelation function of input and the output are

$$R_{y_k u_l}(j) = Ey_k(t)u_l(t-j) \qquad R_{y_k u_l u_s}(i,j) = Ey_k(t)u_l(t-i)u_s(t-j)$$

Multiplying equation (3.5) by $u_l(t-k)$ and taking the mathematical expectations induces the difference equation for the correlation functions

$$R_{y_s u_l}(n_s+k) = S_{sl}^T(k)\Theta_s \tag{4.1}$$

where $S_{sl}^T(k) =$

$$\begin{aligned}
&[R_{y_1u_l}(k)\cdots R_{y_1u_l}(k+n_{s1}-1) \vdots \cdots \vdots R_{y_pu_l}(k)\cdots R_{y_pu_l}(k+u_{sp}-1)\\
&R_{u_1u_l}(k)\cdots R_{u_1u_l}(k+n_s-1) \vdots \cdots \vdots R_{u_mu_l}(k)\cdots R_{u_mu_l}(k+n_s-1)\\
&R_{y_1u_1u_l}(0,k)\cdots R_{y_1u_1u_l}(0,k+n_s-1) \vdots \cdots \vdots\\
&R_{y_pu_1u_l}(0,k)\cdots R_{y_pu_1u_l}(0,k+n_s-1) \vdots \cdots\cdots\\
&R_{y_pu_mu_l}(0,k)\cdots R_{y_pu_mu_l}(0,k+n_s-1)]
\end{aligned} \tag{4.2}$$

$$\begin{aligned}
\Theta_s^T = [&\alpha_{s1,1}\cdots\alpha_{s1,n_{s1}} \vdots \cdots \vdots \alpha_{sp,1}\cdots\alpha_{sp,n_{sp}} \vdots\\
&q_{s1,1}\cdots q_{sn_s,1} \vdots \cdots \vdots q_{s1,m}\cdots q_{sn_s,m} \vdots\\
&h_{s1,1}^1\cdots h_{sn_s,1}^1 \vdots \cdots \vdots h_{s1,p}^1\cdots h_{sn_s,p}^1 \vdots\\
&\cdots \qquad \cdots \qquad \cdots\\
&h_{s1,1}^m\cdots h_{sn_s,1}^m \vdots \cdots \vdots h_{s1,p}^m\cdots h_{sn_s p}^m]
\end{aligned} \tag{4.3}$$

where we have made use of the facts that $u(t)$, $v(t)$ and $w(t)$ are mutual independent and the disturbances have zero mean.

Using (4.1) for $k=0,1,\cdots,N$, one can obtain the following equations

$$\overline{R}_{y_s u_l}(n_s) = \overline{S}_{sl}(n_s-1)\Theta_s \qquad s=1,2,\cdots,p \tag{4.4}$$

where

$$\begin{aligned}
\overline{S}_{sl}(n_s-1) = [&\overline{R}_{y_1u_l}(0)\ \cdots\ \overline{R}_{y_1u_l}(n_{s1}-1)\ \cdots\ \overline{R}_{y_pu_l}(0)\ \cdots\ \overline{R}_{y_pu_l}(n_{sp}-1)\\
&\overline{R}_{u_1u_l}(0)\ \cdots\ \overline{R}_{u_1u_l}(n_s-1)\ \cdots\ \overline{R}_{u_mu_l}(0)\ \cdots\ \overline{R}_{u_mu_l}(n_s-1)
\end{aligned}$$

$$\begin{matrix} \overline{R}_{z_{11}u_l}(0) & \cdots & \overline{R}_{z_{11}u_l}(n_s-1) & \cdots & \overline{R}_{z_{p1}u_l}(0) & \cdots & \overline{R}_{z_{p1}u_l}(n_s-1) \\ & \cdots & & \cdots & & \cdots & \\ \overline{R}_{z_{1m}u_l}(0) & \cdots & \overline{R}_{z_{1m}u_l}(n_s-1) & \cdots & \overline{R}_{z_{pm}u_l}(0) & \cdots & \overline{R}_{z_{pm}u_l}(n_s-1)] \end{matrix} \tag{4.5}$$

$$\begin{aligned} &\overline{R}_{y_iu_l}(j)=(R_{y_iu_l}(j) \;\cdots\; R_{y_iu_l}(j+N))^T \quad i=1,\cdots,p \\ &\overline{R}_{u_iu_l}(j)=(R_{u_iu_l}(j) \;\cdots\; R_{u_iu_l}(j+N))^T \quad i=1,\cdots,m \\ &\overline{R}_{z_{ik}u_l}(j)=(R_{y_iu_ku_l}(j) \;\cdots\; R_{y_iu_ku_l}(j+N))^T \quad i=1,\cdots,p,\; j=1,\cdots,m \end{aligned} \tag{4.6}$$

The equation (4.4) means that if the structural index of s-th subsystem is n_s then vector $\overline{R}_{y_su_l}(n_s)$ is the linear combination of the column vectors in matrix $\overline{S}_{sl}(n_s-1)$.

We select the vector according to the order as following

$$\begin{aligned} &\overline{R}_{y_1u_l}(0) \cdots \overline{R}_{y_pu_l}(0)\ \overline{R}_{u_1u_l}(0) \cdots \overline{R}_{u_mu_l}(0)\ \overline{R}_{z_{11}u_l}(0) \cdots \overline{R}_{z_{p1}u_l}(0)\cdots \\ &\overline{R}_{z_{1m}u_l}(0) \cdots \overline{R}_{z_{pm}u_l}(0)\ \overline{R}_{y_1u_l}(1) \cdots \overline{R}_{y_pu_l}(1)\ \overline{R}_{u_1u_l}(1) \cdots \overline{R}_{u_mu_l}(1) \\ &\overline{R}_{z_{11}u_l}(1) \cdots \overline{R}_{z_{p1}u_l}(1) \cdots \overline{R}_{z_{1m}u_l}(1) \cdots \overline{R}_{z_{pm}u_l}(1)\cdots \end{aligned} \tag{4.7}$$

when some vector $\overline{R}_{y_su_l}(k)$ which is linear correlated with former selected vector has been found then $n_s=k$ and relevant n_{sj} $(j=1,\cdots,p,j\neq s)$ are obtained. Naturally, $\overline{R}_{y_su_l}(n_s)$ is not selected, and $\overline{R}_{y_su_l}(j)$ $(j>n_s)$ are not selected either, and so on, until all structural indices are determined.

Note: In order to the uncorrelative vector can be selected, large N is necessary. Generally, $N\geqslant n+(m+n-p)n_M-1$, $n_M=\max_i\{n_i\}$.

Now a recursive algorithm will be given. For the convenience, we rearrange the order of vectors as following matrix

$$\begin{aligned} \overline{S}_l=[&\overline{R}_{y_1u_l}(0)\overline{R}_{y_1u_l}(1)\cdots \vdots \cdots \vdots\ \overline{R}_{y_pu_l}(0)\overline{R}_{y_pu_l}(1)\cdots \vdots\ \overline{R}_{u_1u_l}(0)\overline{R}_{u_1u_l}(1)\cdots \vdots \\ &\cdots \vdots\ \overline{R}_{u_mu_l}(0)\overline{R}_{u_mu_l}(1)\cdots \vdots\ \overline{R}_{z_{11}u_i}(0)\overline{R}_{z_{11}u_i}(1)\cdots \vdots \cdots \overline{R}_{z_{pm}u_i}(0)\overline{R}_{z_{pm}u_l}(1)\cdots] \end{aligned} \tag{4.8}$$

and use $Q(\mu_1,\cdots,\mu_{p+m+m\cdot p})$ indicate the matrix formed by selected vector in according to priority, where μ_i is number of vectors selected from relevant submatrix of the matrix $\overline{S}_l$, $i=1,\cdots,p+m+mp$.

Let Q_i denote the matrix Q obtained at the end of i-th step of recursive scheme. If $Q_i=Q(\mu_1,\mu_2,\cdots,\mu_{p+m+mp})$

then
$$Q_{i+1}=[Q_i \ \vdots \ \overline{R}_{i+1}]$$

where $\overline{R}_{i+1}$ is the new vector selected at $i+1$-th step

$$\text{Let } S_i=Q_i^TQ_i, \qquad S_{i+1}=Q_{i+1}^T\,Q_{i+1}$$

provided S_i is nonsingular, it follows that

$$\det \quad S_{i+1}=\det S_i\,\det[\overline{R}_{i+1}^T(I-Q_iS_i^{-1}Q_i^T)\,\overline{R}_{i+1}] \tag{4.9}$$

If $\det S_{i+1}=0$ and $\overline{R}_{i+1}$ belong to j-th submatrix $(j\leqslant p)$ of the matrix S_l, then the structural index $n_j=\mu_j$ and the parameters of j-th subsystem

$$\hat{\theta}_j=S_i^{-1}\,Q_i^T\overline{R}_{i+1} \tag{4.10}$$

where $\hat{\theta}_j^T=(\alpha_{j1,1}\cdots\alpha_{jp,1} \ \vdots \ q_{j1,1}\cdots q_{j1,m} \ \vdots \ h_{j1,1}^1\cdots h_{j1,1}^m \ \vdots \cdots \vdots \ h_{j1,p}^1\cdots h_{j1,p}^m \ \vdots \cdots)$

$$\tag{4.11}$$

Note that the permutation order of the parameters in (4.11) is different from in (4.3).

If $\det S_{i+1}\neq 0$, then we can obtain

$$S_{i+1}^{-1}=\begin{pmatrix} S_i^{-1}+P2\overline{R}_{i+1}^TQ_iS_i^{-1} & -P2 \\ -P^T2 & P1 \end{pmatrix} \tag{4.12}$$

where

$$P^{-1}1=\overline{R}_{i+1}^T(I-Q_i\,S_i^{-1}\,Q_i^T)\,\overline{R}_{i+1} \tag{4.13}$$

$$P2=S_i^{-1}\,Q_i^T\overline{R}_{i+1}P1 \tag{4.14}$$

To combine (4.9), (4.12), (4.13), and (4.14), the recursive agorithm of the structural indices and the parameters of the input-output equation can be obtained. start from

$$S_1=Q^T(1\ 0\cdots 0)Q(1\ 0\cdots 0)$$

until some $\overline{R}_j$ which belongs to j-th submatrix of $\overline{S}_l$, is added to Q so that $\det S_i=0$, then n_j and $\hat{\Theta}_j$ are obtained, After that we drop the $\overline{R}_j$ and add the next vector in (4.7) to Q, until all indices are determined. The steps of algorithm see also [3].

5. The Realization Algorithm of B^*, D_i^* and N^*

After the structural indices $\{n_i\}$ and the parameters $\{\alpha_{ij,k}\}$, $\{q_{ij,k}\}\}$, $\{h^l_{ij,k}\}$ have been determined, the A^* and C^* of canonical form can be obtained immediatly. In this paragraph a realization algorithm of B^*, D_i^* and M^* is proposed.

From (3.6), by simple backward substitution B^* can be acquired. To be specific, using the equation (3.6), let

$i=1$, $j=n_1$, b_1 is obtained,

$i=2$, $j=n_2$, b_{s1+1} is obtained,

$\cdots$ $\cdots$

$i=p$, $j=n_p$, $b_{s_{p-1}+1}$ is obtained,

$i=1$, $j=n_1-1$, b_2 is obtained,

$\cdots$ $\cdots$

$i=p$, $j=n_p-1$, $b_{s_{p-1}+2}$ is obtained,

until to

$i=1$, $j=1$, b_{s1} is obtained,

$\cdots$ $\cdots$

$i=p$, $j=1$, b_{s_p} is ontained.

and so on, all rows of B^* are obtained.

Due to the structure of the equation (3.7) is as same as (3.6). so by similar procedure D_i^* can be acquired easily.

From (2.3) and (3.4), N_i^* can be acquired easily.

$$N_i^* = D_i^* C^* = (\underbrace{D_1^i 0}_{n_1} \quad \underbrace{D_2^i 0}_{n_2} \cdots \underbrace{D_p^i 0}_{n_p}) \qquad i=1,\cdots,m$$

where D_j^i is j-th column of the matrix D_i^*.

6. Simulation

Considering a two input and two output bilinear system

$$x(t+1) = Ax(t) + \sum_{i=1}^{2} D_i y(t) u_i(t) + Bu(t) + Mw(t)$$

$$y(t) = Cx(t) + Gv(t)$$

where

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -0.1 & 0.65 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0.67 & 1.67 & -0.25 & 1 \end{pmatrix} \quad B = \begin{bmatrix} 0 & 0.2 \\ 0.25 & 0.8 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \quad D = \begin{pmatrix} 1 & 0 \\ 0.1 & 0.2 \\ 0 & 1 \\ 1 & -1 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 1 \\ -2 & 0 \\ 0.1 & 0 \\ 0 & 1 \end{pmatrix} \quad M = \begin{pmatrix} 0.01 \\ 0.01 \\ 0.01 \\ 0.01 \end{pmatrix} \quad G = \begin{bmatrix} 0.08 \\ 0.09 \end{bmatrix}$$

The input are PRBS with amplitude 0.2 and 0.3, respectively. The process and measurement noise sequence $\{w(t)\}$ and $\{v(t)\}$ are the normal psedo-random sequences with zero mean and unit variance. The length of the data is 1000. Taking $N=100$, $\varepsilon=10^{-1}$ in the recursive algorithm, the procedure gives the following results

$$n_1 = 2, \quad n_2 = 2$$

$$A = \begin{pmatrix} .00000E+00 & .10000E+00 & .00000E+00 & .00000E+00 \\ -.10102E+00 & .63101E+00 & -.61370E-03 & -.12200E-03 \\ .00000E+00 & .00000E+00 & .00000E+00 & .10000E+01 \\ .67664E+00 & .16322E+01 & -.24788E+00 & .10038E+01 \end{pmatrix}$$

$$B = \begin{pmatrix} .93419E-01 & .20909E+00 \\ .29813E+00 & .77133E+00 \\ .19583E+00 & -.10886E+00 \\ .12289E+01 & .79422E+00 \end{pmatrix} \quad D1 = \begin{pmatrix} .10543E+01 & .56552E-02 \\ .94636E-01 & .20695E+00 \\ .18610E+00 & .10154E+01 \\ .11848E+01 & -.96782E+00 \end{pmatrix}$$

$$D2 = \begin{pmatrix} .19760E-01 & .10020E+01 \\ -.19638E+01 & -.91640E-02 \\ .16838E+00 & -.13950E-01 \\ .11873E+00 & .96577E+00 \end{pmatrix}$$

7. Conclusion

In this paper a recursive identification method for a class of bilinear systems have been proposed. It improved the method proposed by Kotta and Nurges (1984). In this recusive algorithm, the matrix inversion or the value of determinant is not needed, and the structural indices and the parameters are found out simultaneously.

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输入输出双线性系统的递推辨识与实现算法

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摘 要

本文对输入输出双线性系统给出了一套完整的辨识方法。首先给出了输入输出方程的一般形式，并对于一类规范型给出了相应地输入输出差分方程。然后给出了确定系统的结构指标和参数估计的递推算法，它改进了由Katta和Nurges（1984）给出的方法，大大缩短了计量时间。另外，还给出了状态方法的简单实现算法。最后，给出了仿真例子说明算法的有效性。