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Detection-Oriented Kalman Filtering for Bernoulli-Gaussian Sequence

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Abstract

A detection-oriented Kalman filter is employed for on-line estimating the arrival times and heights of a Bernoulli-Gaussian impulse sequence which is observed in the presence of noise. The results are illustrated by a simulation study.

1. Introduction

A well-known problem in statistical communication theory, control theory, reflection seismology and some other fields is the problem of detecting the arrival times and estimating the heights of a randomly arriving impulse sequence which is observed in the presence of noise [1, 2]. Kwakernaak presented an estimation algorithm for this problem using Rissanen's modification of the maximum likelihood principle [3]. Mendel, Kormylo and Chi have discussed this problem through their work on seismic deconvolution [2, 4, 5].

All their methods are off-line in nature. In this paper, we develop a modified Kalman filter, named the "detection-oriented Kalman filter", for on-line estimation. Our method is motivated by some schemes which detect deterministic sudden changes in dynamic systems [6, 7]. The performance of our algorithm is demonstrated by computer simulation.

2. Problem Formulation

Our starting point is the following discrete-time convolution

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model:

$$z(k)=\mu(k)*v(k)+n(k)=\sum_{i=1}^{k}v(k-i)\mu(i)+n(k), \qquad (1)$$

where $k=1,2,\cdots,N$. In Eq. (1), $\mu(k)$ is an input random spike sequence which is to be estimated; it is assumed to be Bernoulli-Gaussian, and can be expressed in the following product form:

$$\mu(k)=q(k)r(k), \qquad (2)$$

in which $q(k)$ is a Bernoulli sequence with parameter λ,

$$P_r[q(k)]=\begin{cases}\lambda & : \ q(k)=1\\ 1-\lambda & : \ q(k)=0\end{cases}, \qquad (3)$$

and $r(k)$ is a zero-mean, white and Gaussian sequence with variance σ_r^2. Sequences $r(k)$ and $q(k)$ are statistically independent.

In Eq.(1), $v(k)$ can be viewed as the impulse response of a linear, time-invariant, discrete-time system. Additionally, measurement noise $n(k)$ is assumed to be zero-mean, white and Gaussian sequence with variance σ_n^2, and is statistically independent of sequence $\mu(k)$.

The problem of estimating the arrival times and heights of Bernoulli-Gaussian sequence $\mu(k)=q(k)r(k)$ is: given the impulse response data $v(k)$ $(k=0,1,\cdots)$ and the statistical parameters $(\lambda, \sigma_r^2, \sigma_n^2)$, detect the Bernoulli sequence $\{q(1),\cdots,q(N)\}$ and estimate the amplitudes $\{r(1),\cdots,r(N)\}$ by processing all of the observed data $\{z(1),\cdots,z(N)\}$. In the case of real-time applications, however, in order to estimate $q(k)$ and $r(k)$ $(k=1,2,\cdots,N)$, we can only process all of the observed data up to t_k: $\{z(1),\cdots,z(k)\}$.

In this paper, we apply a modified Kalman filter for on-line detection and estimation. So the convolution model (1) must be realized by a suitable state-variable model. when $v(0)\neq 0$, the state-variable model is:

$$x(k)=\Phi x(k-1)+\gamma q(k)r(k), \qquad (4)$$

$$z(k)=h'x(k)+n(k), \qquad (5)$$

where $x(k)$ is the $n\times 1$ state vector and $x(0)=0$, Φ is the $n\times n$ transition matrix, γ is the $n\times 1$ input dist-ibution vector and h' is the $1\times n$ observation vector. The triples $\{\Phi, \gamma, h\}$ as well as the order n can be determined via the approximate realization technique by processing the sequence $v(k)$ [8], so

$$h'\Phi^k\gamma \doteq v(k), \quad (k=0,1,\cdots,N) \tag{6}$$

in which $v(0)=h'\gamma\neq 0$.

Sometimes, however, $v(0)$ is zero but $v(1)\neq 0$. At that case, the convolution model (1) can be approximately realized by the following state-variable model [8]:

$$x(k)=\Phi x(k-1)+\gamma q(k-1)r(k-1), \tag{7}$$

$$z(k)=h'x(k)+n(k), \tag{8}$$

in which

$$h'\Phi^{k-1}\gamma \doteq v(k), \quad (k=1,2,\cdots,N) \tag{9}$$

and $v(1)=h'\gamma\neq 0$. In the case of real-time applications, in order to estimate $q(k-1)$ and $r(k-1)(k=1,2,\cdots,N)$, we can only process all of the observed data up to t_k: $\{z(1),\cdots,z(k)\}$.

In the next section, we first discuss the case of $v(0)\neq 0$, then the case of $v(0)=0$ and $v(1)\neq 0$.

3. Detection-Oriented Kalman Filter

The on-line detection procedure presented in this paper is to employ a modified Kalman filter, named "detection-oriented Kalman filter", for state-variable model (4) and (5). The scheme is, in order to detect whether $q(k)=0$ or 1 at each value of t_k, to run first the Kalman filter to obtain the innovation $\tilde{z}(k|k-1)$ and then to compare $\tilde{z}^2(k|k-1)$ to a certain threshold. If $\tilde{z}^2(k|k-1)$ exceeds this threshold we set $\hat{q}(k|k)=1$ and set the input variance, which appears in the covariance equations, equal to $E\{\mu^2(k)|q(k)=1\}=E\{r^2(k)\}=\sigma_r^2$; otherwise we set $\hat{q}(k|k)=0$ and set the input variance equal to $E\{\mu^2(k)|q(k)=0\}=0$. The equations of the detection-oriented Kalman filter are as follows:

- Innovation:

$$\tilde{z}(k|k-1)=z(k)-h'\hat{x}(k|k-1), \tag{10}$$

$$\hat{x}(k|k-1)=\Phi\hat{x}(k-1|k-1). \tag{11}$$

- Detector:

$$\tilde{z}^2(k|k-1) \underset{\hat{q}(k|k)=0}{\overset{\hat{q}(k|k)=1}{\gtrless}} T(k). \tag{12}$$

• Filter:

$$\hat{x}(k|k)=\hat{x}(k|K-1)+K_q(k)\tilde{z}(k|k-1), \tag{13}$$

$$k_q(k)=P_q(k|k-1)h\eta_q^{-1}(k). \tag{14}$$

• Covariances:

$$\eta_q(k)\triangleq V_{ar}\{\tilde{z}(k|k-1)\}=h'P_q(k|k-1)h+\sigma_n^2, \tag{15}$$

$$P_q(k|k-1)=\begin{cases}P_0(k|k-1)=\Phi P(k-1|k-1)\Phi', & :\hat{q}(k|k)=0\\ P_1(k|k-1)=\Phi P(k-1|k-1)\Phi'+\sigma_r^2\gamma\gamma' & :\hat{q}(k|k)=1,\end{cases} \tag{16}$$

$$P(k|k)=[I-K_q(k)h']P_q(k|k-1), \tag{17}$$

where $k=1,2,\cdots,N$, and $T(k)$ is the threshold.

We now determine the appropriate threshold $T(k)$ via the maximum a posteriori (MAP) detection rule [9]. The likelihood function for detecting $q(k)$ given $\tilde{z}(k|k-1)$ is

$$L\{q(k)|\tilde{z}(k|k-1)\}\propto p[\tilde{z}(k|k-1)|q(k)]P_r[q(k)]. \tag{18}$$

Under the Gaussian assumptions described in section 2 and the known estimation sequence $\{\hat{q}(1|1),\hat{q}(2|2),\cdots,\hat{q}(k-1|k-1)\}$, we can assume that $p[\tilde{z}(k|k-1)|q(k)]$ is an approximate Gaussian probability density function; i. e.,

$$p[\tilde{z}(k|k-1)|q(k)]\doteq[2\pi\eta_q(k)]^{-\frac{1}{2}}\exp[-\tilde{z}^2(k|k-1)/2\eta_q(k)] \tag{19}$$

where we have used the fact that innovation $\tilde{z}(k|k-1)$ is zero-mean. The MAP detection rule is [9]:

$$\ln\Lambda(k)=\ln\frac{p[\tilde{z}(k|k-1)|q(k)=1]P_r[q(k)=1]}{p[\tilde{z}(k|k-1)|q(k)=0]P_r[q(k)=0)}\underset{\hat{q}(k|k)=0}{\overset{\hat{q}(k|k)=1}{\gtrless}}0. \tag{20}$$

We have:

Theorem 1 A real-time maximum a posteriori threshold detector for state-variable model (4) and (5) is Eq. (12), in which the threshold $T(k)$ is

$$T(k)=\nu^2(0)\sigma_r^2\ \xi(k)[1+\xi(k)]\left\{\ln\left[1+\frac{1}{\xi(k)}\right]-2\ln\left(\frac{\lambda}{1-\lambda}\right)\right\} \tag{21}$$

where

$$\xi(k)\triangleq\frac{\eta_0(k)}{\nu^2(0)\sigma_r^2}=\frac{h'\Phi P(k-1|k-1)\Phi'h+\sigma_n^2}{\nu^2(0)\sigma_r^2}. \tag{22}$$

Proof: From Eq. (19), we see that

$$\ln\frac{p[\tilde{z}(k|k-1)|q(k)=1]}{p[\tilde{z}(k|k-1)|q(k)=0]}=\frac{1}{2}\ln\left[\frac{\eta_0(k)}{\eta_1(k)}\right]$$

$$+\frac{1}{2}\tilde{z}^2(k|k-1)\left[\frac{1}{\eta_0(k)}-\frac{1}{\eta_1(k)}\right]. \tag{23}$$

Substituting Eqs. (3) and (23) into Eq. (20), we find that the MAP detection rule is

$$\ln\Lambda(k)=\frac{1}{2}\tilde{z}^2(k|k-1)\frac{\eta_1(k)-\eta_0(k)}{\eta_0(k)\eta_1(k)}+\frac{1}{2}\ln\left[\frac{\eta_0(k)}{\eta_1(k)}\right]$$

$$+\ln\left(\frac{\lambda}{1-\lambda}\right)\ \underset{\hat{q}(k|k)=0}{\overset{\hat{q}(k|k)=1}{\gtrless}}\ 0. \tag{24}$$

From Eqs. (15) and (16), we have

$$\eta_0(k)=h'P_0(k|k-1)h+\sigma_n^2=h'\Phi P(k-1|k-1)\Phi' h+\sigma_n^2,$$

and

$$\eta_1(k)=h'P_1(k|k-1)h+\sigma_n^2=h'\Phi P(k-1|k-1)\Phi' h+v^2(0)\sigma_r^2+\sigma_n^2,$$

where we have used the fact that $v(0)=h'\gamma=\gamma' h$. Since $\eta_1(k)-\eta_0(k)=v^2(0)\sigma_r^2>0$, from Eq. (24), the MAP detection rule can be expressed as

$$\tilde{z}^2(k|k-1)\ \underset{\hat{q}(k|k)=0}{\overset{\hat{q}(k|k)=1}{\gtrless}}\ \frac{\eta_0(k)\eta_1(k)}{\eta_1(k)-\eta_0(k)}\left\{\ln\left[\frac{\eta_1(k)}{\eta_0(k)}\right]\right.$$

$$\left.-2\ln\left(\frac{\lambda}{1-\lambda}\right)\right\},$$

or

$$\tilde{z}^2(k|k-1)\ \underset{\hat{q}(k|k)=0}{\overset{\hat{q}(k|k)=1}{\gtrless}}\ v^2(0)\sigma_r^2\ \xi(k)[1+\xi(k)]\left\{\ln\left[1+\frac{1}{\xi(k)}\right]\right.$$

$$\left.-2\ln\left(\frac{\lambda}{1-\lambda}\right)\right\}$$

where we have used the facts that $\eta_1(k)=\eta_0(k)+v^2(0)\sigma_r^2$ and $\xi(k)\triangleq\eta_0(k)/v^2(0)\sigma_r^2$.

After we have detected whether $q(k)=0$ or 1 at each value of t_k, i. e., after we have obtained $\hat{q}(k|k)$, we immediately have:

Theorem 2 The real-time maximum a posteriori estimate of $\mu(k)$, $\hat{\mu}(k|k)$, for state-variable model (4) and (5) is

$$\hat{\mu}(k|k)=A_q(k)\,\tilde{z}(k|k-1), \tag{25}$$

where
$$A_q(k)=\hat{q}(k|k)v(0)\sigma_r^2\ \eta_q^{-1}(k). \tag{26}$$

Proof: When $q(k)$ is known, the maximum a posteriori (or minimum-variance*) estimate of $\mu(k)$, $\hat{\mu}(k|k)$, is [2,10]

$$\hat{\mu}(k|k)=\begin{cases}0 & : \ q(k)=0\\ v(0)\sigma_r^2\eta^{-1}(k)\,\tilde{z}(k|k-1) & : \ q(k)=1\end{cases}$$

where $\eta(k)=V_{ar}\{\tilde{z}(k|k-1)\}$. When $q(k)$ is unknown, however, according to the separation principle of maximum a posteriori estimation of Bernoulli-Gaussian sequence [2, 11], we first detect the maximum a posteriori estimate of $q(k)$, $\hat{q}(k|k)$, then compute the maximum a posteriori estimate of $\mu(k)$, $\hat{\mu}(k|k)$, as

$$\hat{\mu}(k|k)=\hat{q}(k|k)v(0)\sigma_r^2\ \eta_q^{-1}(k)\,\tilde{z}(k|k-1).$$

We can easily find that our detection-oriented Kalman filter is an adaptive filter.

For state-variable model (7) and (8), following the same procedure, we can obtain the same results by using $v(1)$, $\hat{q}(k-1|k)$ and $\hat{\mu}(k-1|k)$ instead of $v(0)$, $\hat{q}(k|k)$ and $\hat{\mu}(k|k)$, respectively, for the state-variable model (4) and (5). In fact, we have

Corollary 1 For state-variable model (7) and (8), a real-time maximum a posteriori threshold detector is

$$\tilde{z}^2(k|k-1)\ \underset{\hat{q}(k-1|k)=0}{\overset{\hat{q}(k-1|k)=1}{\gtrless}}\ T(k), \tag{27}$$

*When sequence $q(k)$ is known, under the Gaussian assumptions described in section 2, our case is the Gaussian and linear model. Hence, the minimum-variance estimator of $\mu(k)$ leads to the maximum a posteriori estimator of $\mu(k)$.

where threshold $T(k)$ is

$$T(k) = v^2(1)\sigma_r^2\ \xi(k)[1+\xi(k)] \left\{ \ln\left[1+\frac{1}{\xi(k)}\right] - 2\ln\left(\frac{\lambda}{1-\lambda}\right)\right\} \quad (28)$$

in which

$$\xi(k) \triangleq \frac{\eta_0(k)}{v^2(1)\sigma_r^2} = \frac{h'\Phi P(k-1|k-1)\Phi' h + \sigma_n^2}{v^2(1)\sigma_r^2}, \quad (29)$$

and, the real-time maximum a posteriori estimate of $\mu(k-1)$, $\hat{\mu}(k-1|k)$, is

$$\hat{\mu}(k-1|k) = A_q(k)\tilde{z}(k|k-1) = \hat{q}(k-1|k)v(1)\sigma_r^2\ \eta_q^{-1}(k)\tilde{z}(k|k-1). \quad (30)$$

4. Simulation

As an example, we use a pseudorandom number generator program to generate the Bernoulli-Gaussian sequence (for which $\lambda = 0.1$, $\sigma_r = 0.25$ and $N = 150$). This signal is convolved with the fourth-order ARMA system described by

$$V(z) = \frac{0.91003 - 2.4203z^{-1} + 2.3408z^{-2} - 0.82997z^{-3}}{1 - 3.3198z^{-1} + 4.5146z^{-2} - 2.9511z^{-3} + 0.79699z^{-4}},$$

to which zero-mean, white and Gaussian noise with $\sigma_n = 0.03$ is added to produce the observation data.

Our detection-oriented Kalman filter is applied to these data. The simulation result is depicted in Fig. 1, in which circles mark the true impulse amplitudes and bars depict the corresponding estimates. As one can see, there are five missed detections and one false alarm in Fig. 1.

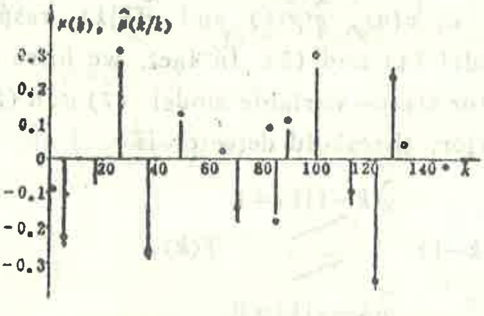

Fig. 1 Simulation Result

5. Conclusions

The problem of detecting and estimating a Bernoulli-Gaussian sequence is an unusual and rather difficult problem. The detectonoriented Kalman filter presented in this paper is a suboptimal on-line algorithm. Obtaining more effective on-line estimators is still an open problem.

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检测与估计 Bernoulli-Gaussian 序列的 Kalman 滤波方法

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摘　要

本文采用一种“面向检测”的 Kalman 滤波器，对 Bernoulli-Gaussian 序列的脉冲出现时刻和脉冲高度进行在线估计，而该 Bernoulli-Gaussian 序列是在有噪声情况下量测的。所获之结果由数字仿真进行了验证。