LQG Self-tuning Controller Using Input-output Model

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Abstract

Using input-output plant model, a new LQG self-tuning controller is derived, where the solution of the Riccati equation and spectral factorization are not required.

The design of optimal controllers for the stochastic LQG (Linear-Quadratic-Gaussian) problem is well established, using frequency and time-domain theory [1,2,3]. The treatment in the frequency-domain using the Wiener-Newton approach requires a time consuming spectral factorization. The algorithms using time-domain theory rest on the so-called certainty equivalence hypothesis and the solution of the Riccati equation is necessary. LQG controllers are wildly used elsewhere but have not been applied in self-tuning systems till now, except in minimum variance and generalized minimum variance forms which are based on only single-stage cost function minimization. This may be due to the large computational load of solving the Riccati equation and performing the spectral factorization for LQG controllers. In this paper, a new version of a LQG self tunner is presented, in which numerical problem of solution of the Riccati equation and spectral factorization are avoided.

An input-output model of the plant may be represented in discrete time as

$$A(z^{-1})y(t)=z^{-K}B(z^{-1})u(t)+C(z^{-1})\xi(t) \tag{1}$$

where y(t) is the measured variable at time t, $u(t)$ the control Signal, $A(z^{-1})$ and $B(z^{-1})$ are polynomials of the backward shift operator z^{-1}, k is

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the integral time delay of the plant and $\xi(t)$ is a random variables with zero mean; here the noise polynomial $c(z^{-1})$ is assumed to unity, i.e. $c(z^{-1})=1$.

By using the equation

$$1 = AE + z^{-k}F \tag{2}$$

Astrom obtained the result that the minimum variance predictor over k steps is given by

$$\hat{y}(t+k/t) = BEu(t) + Fy(t) \tag{3}$$

The minimum variance control law was derived by minimizing the criterion.

$$J = E[y(t+k)^2]$$

Therefore, minimum variance control can be considered as one-stage cost function stochastic optimal control.

In order to derive a more than one-stage cost function control law, the authors extend (2) to the following form [5]

$$1 = AE_N + z^{-K-N}F_N \tag{4}$$

where polynomials E_N and F_N are of degree $k+N-1$ and $n-1$, respectively.

Using (1) and(4), the optimal predictor over k+N steps is given by

$$\hat{y}(t+k+N/t) = BE_N u(t+N) + F_N y(t) \tag{5}$$

An auxiliary output $\phi(t) = P(z^{-1})y(t)$ where $P(z^{-1}) = P_n(z^{-1})/P_d$ and $P(1)=1$ is introduced here and then it will be included in the cost function.

Suppose $w(t+k)$ and $w(t+k+1)$ are the future setpoints, moreover, define $e(t+j) = w(t+j) - \hat{\phi}(t+j)$ to be the predicted future error. Then the quadratic cost function for two-stage case is established in the following form

$$J = \underline{e}'R\underline{e} + \underline{u}'Q\underline{u} \tag{6}$$

where diagonal matrices R and Q are the weighting matrices on $\underline{e}$ and $\underline{u}$. The vectors $\underline{e}$ and $\underline{u}$ are of dimension 2 in the two-stage case,

$$\underline{e} = [e(t+k),\ e(t+k+1)]' = \underline{w} - \underline{\hat{\phi}}\ ,\ \underline{u} = [u(t),\ u(t+1)]' \tag{7}$$

when the auxiliary output $\phi(t)$ is used to replace $y(t)$, the equation (4) becomes

$$\frac{P}{A} = E_N + z^{-K-N}\frac{F_N}{AP_d}, \qquad N = 0,\ 1,\ 2,\ \cdots \tag{8}$$

The polynomials E_N and F_N are of degrees $k+N-1$ and $\max\{n+\delta(P_d)-1,$

$\delta(P_d)-k-N\}$, respectively. Multiplying (1) (assumed $C(z^{-1})=1$) by $E_N(z^{-1})$ and combining (1) and (8) give

$$\phi(t+k+N)=BE_N u(t+N)+F_N y(t)/P_d+E_N\xi(t+k+N) \quad N=0,\ 1,\ 2,\ \cdots \tag{9}$$

For the two-stage cost case, $N=0$ and $N=1$ should be considered. Obviously, the optimal predictors are

$$\hat{\phi}(t+k/t)=G_0\ u(t)+F_0\ y(t)/P_d \tag{10}$$

$$\hat{\phi}(t+k+1/t)=G_1\ u(t+1)+F_1\ y(t)/P_d \tag{11}$$

where $G=BE_N$ is a polynomial in z^{-1}, it may be written as

$$G_0=g_{00}+G_0', \qquad G_1=g_{10}+g_{11}z^{-1}+G_1'z^{-2} \tag{12}$$

Combining (7), (10) and (11) gives the predicted future error vector $\underline{e}$ as

$$\underline{e}=H-G\underline{u} \tag{13}$$

where

$$H=\begin{bmatrix} w(t+k)-G_0'\ u(t-1)-F_0 y(t)/P_d \\ w(t+k+1)-G_1'\ u(t-1)-F_1 y(t)/P_d \end{bmatrix}=\begin{bmatrix} h_1 \\ h_2 \end{bmatrix},\quad G=\begin{bmatrix} g_{00} & 0 \\ g_{11} & g_{10} \end{bmatrix}$$

Substituting (13) into (6), the cost function can be written as

$$J=(H-G\underline{u})'R(H-G\underline{u})'+\underline{u}'Q\underline{u} \tag{14}$$

The controller is chosen to minimize the quadratic cost function J and the optimal control is given by

$$\underline{u}^*=(G'RG+Q)^{-1}G'RH \tag{15}$$

Although the optimal control vector $\underline{u}^*$ is obtained here, it is worth mentioning that this control law must be used in the receding-horizen sense (i.e. an N-stage cost is minimized at every sample instat, and only the first control signal is calculated and implemented) to ensure that the same control law applied for all time[4].

Define the weighting matrices

$$R=\begin{Bmatrix} r_1 & 0 \\ 0 & r_2 \end{Bmatrix},\qquad Q=\begin{Bmatrix} q_1 & 0 \\ 0 & q_2 \end{Bmatrix}$$

then, solving for the control Signal $u^*(t)$ from (15) yields

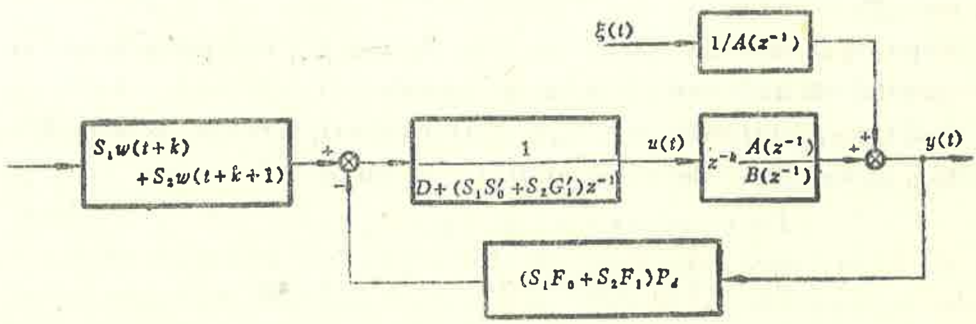
$$u^*(t)=\frac{S_1 w(t+k)+S_2 w(t+k+1)}{D+(S_1G_0'+S_2G_1')z^{-1}}-\frac{(S_1F_0+S_2F_1)y(t)/P_d}{D+(S_1G_0'+S_2G_1')z^{-1}} \tag{16}$$

where D, S_1 and S_2 are given by

$$D=\det(G'RG+Q)=(r_1\,g_{00}^2+q_1)(r_2 g_{10}^2+q_2)+r_2 q_2 g_{11}^2$$

$$S_1=r_1 g_{00}(r_2\,g_{10}^2+q_2),\qquad S_2=r_2\,q_2\,g_{11}$$

The controller obtained above takes a feedback form; the block-diagram of the feed-back system can be drawn as Fig. 1 according to (1) and (16).



Note: In this figure, $D+(S_1S_0'+S_2G_1')Z^{-1}$ should read $D+(S_1G_0'+S_2G_1')Z^{-1}$

$(S_1F_0+S_2F_1)Pd$ should read $(S_1F_0+S_2F_1)/Pd$

Fig.1

The closed-loop transfer function may be derived by combining (9) with the control law (16) and the plant, (10) and (11). The system output $y(t)$ is given by

$$y(t)=\{z^{-K}(S_1\,w(t+k)+S_2\,w\,(t+k+1)\,B+[D+(S_1\,G_0'+S_2G_1')z^{-1}]\,\xi(t)\}/T(z^{-1})$$

where T is the closed-loop characteristic equation,

$$T(z^{-1})=[q_1\,(r_2\,g_{10}^2+q_2)-r_2\,q_2\,g_{10}g_{11}z]\,A+(S_1+S_2z)PB. \tag{17}$$

To calculate the steadystate value of the output, consider the case $\xi(t)=0$. Using the final-value theorem, the steadystate output is then given by

$$y(\infty)=(S_1+S_2)B(1)w(\infty)/T(1),$$

$$T(1)=(S_1+S_2)P(1)B(1)+[\,q_1(r_2g_{10}^2+q_2)-r_2q_2g_{10}g_{11}]A(1).$$

Considering $P(1)=1$, zero steady-state offset for step input will require that the elements of R and Q must satisfy the following relationship

$$q_1\,(\,r_2\,g_{10}^2+q_2)=r_2\,q_2\,g_{10}g_{11}. \tag{18}$$

(18) means that three elements q_1, q_2 and r_2 can not all be chosen a priori; in practice, two of them are selected in advance and the other is determined by (18).

Self-tuning approaches are applicable to system where the structure is known but the parameters unknown. It is nonethless possible to use a recursive parameter estimator to obtain estimates of th epredictor parameters. Here the recursive instrumental variable method is used, where the instrumental variable is selected as $z(t)=[u(t),\cdots,u(t-i),\cdots]'$, and the algorithm of the LQG self-tuning controller is given as follows

Data: Choose P, elements r_1 and two among r_2, q_1 q_2 of the weighting matrices R and Q.

Step 1: Estimate the parameters G_0, F_0 and G_1, F_1 using recursive instrumental variable and the predictor models

$Py(t+k)=G_0\ u(t)+F_0\ y(t)/P_d$, $\qquad Py(t+k)=G_1\ u(t)+F_1\ y(t-1)/P_d$

Step 2: Calculate the value of D, S_1, S_2 using

$$D=(r_1 g_{00}^2+q_1)(r_2\ g_{10}^2+q_2)+r_2\ q_2 g_{11}^2,$$

$$S_1=r_1\ g_{00}(r_2\ g_{10}^2+q_2), \qquad S_2=r_2 q_2 g_{11},$$

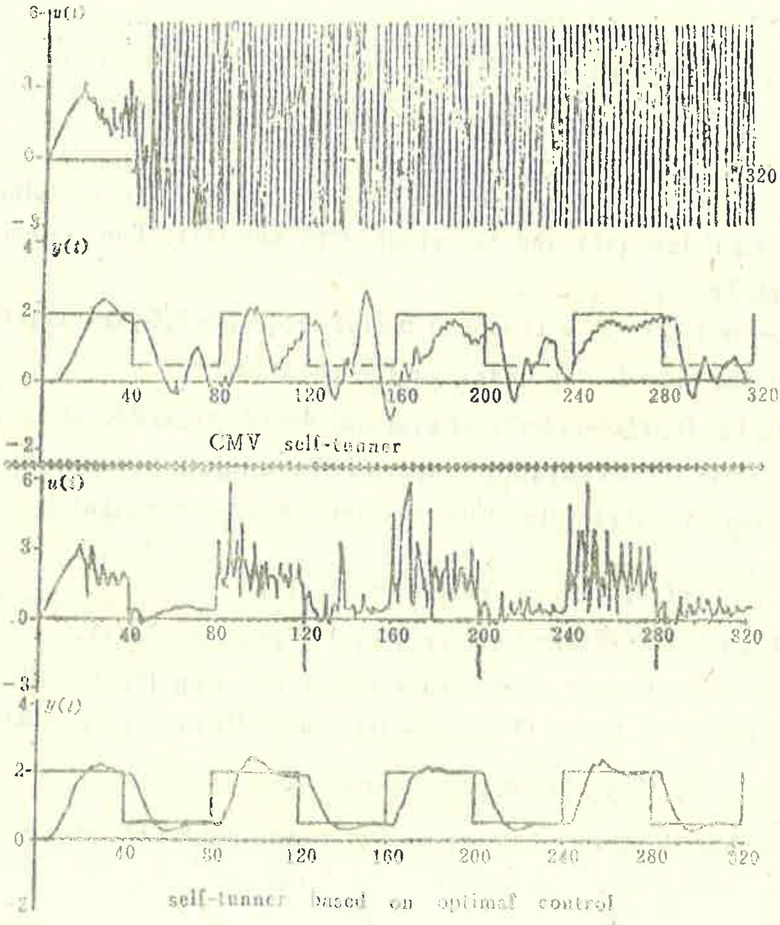


Fig.2 comparative performance for underestimated time-delay where one of elements r_2, q_1, g_2, which has not been selected in advance, is determined by (18).

Step 3: Calculate the control singal from (16), where G_0', G_1' can be easily obtained from G_0 and G_1. Return to Step 1.

A comparative studyfor the case of underestimated time-delay is presented in Fig.2, the plant model was described as

$$G(s)=e^{-9s}/(1+10s)(1+5s)$$

which was sampled at 2-second interval. Both LQG and GMV controllers assumed a dead-time 4 instead of the actual value of 9. It is found that the LQG algorithm presented in this paper provides good response in the case of underestimated dead time but the GMV self-tunner perform poorly.

After extending the equation (2) to the N-stage case, a new LQG self tuning algorithm based on the two-stage cost function minimization and the input-output plant model has been presented. The solution of the Riccati equation or spectral factorization is not required for this approach. The result derived above can be easily extended to the N-stage cost case.

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用输入输出模型的LQG自校正控制器

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摘　要

本文提出了一种新的LQG自校正控制器，它是基于最优控制理论和对象的输入输出模型设计的。该算法无需求解Riccati方程或进行谱因子分解，故计算量大为减少，易于实现。