A Semi–parametric Forecasting Method for ARMA Series*

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Abstract: A new semi–rarametric forecasting method for ARMA series is proposed in this paper. Without estimating MA parameters of the model,the mcthod can give the forecast directly just based on AR parameters and autocovariances of the series, so that the forecasting problem is solved much more simply. The method is specially suitable to adaptive forecast and slowly time–varying case,Its application in short–term water load forecasting is also presented in the paper.

Key words: time series analysis; ARMA model; semi–parametric forecasting; innovation; load forecasting

1. Introduction

As a kind of powerful forecasting techniques, time series analysis method based on ARMA (or ARIMA) model has been extensively applied in many fields. In the past decades there have been a lot of approaches to the modeling and forecasting of ARMA series. However,as is known to all, the estimation of MA parameters is much more difficult than that of AR parameters .It means that, the modelling work may become rather complex if the model of the series contains MA part .A well–known solution to this difficulty is to approximate a high order pure AR model to the ARMA series.In this paper,this difficulty is dealt with in another approach. It is worth notice that, if the question of interest is forecasting only rather than control or otherwise,the procedure for modelling is by no means absolutely necessary. Bearing this idea in mind, we present in this paper a semi–parametric forecasting algorithm which gives the prediction directely without estimating the total parameters of ARMA model. In the proposed algorithm,only AR parameters are estimated while the tedious work of estimating MA parameters is avoided,so that the forecasting problem is solved mucn more simply.

2. Innovation Theorem

As the preliminaries, some theoretic results which are important to our problem are cited

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here.

Let $\{x_t\}$ be a stationary ARMA(p,q) series, i.e.

$$x_t - \sum_{i=1}^{p} \varphi_i x_{t-i} = a_t - \sum_{i=1}^{q} \theta_i a_{t-i} \tag{1}$$

where {at} is a white Gaussian noise series with mean zero and variance σ_a^2.

Denote

$$X_k \hat{=} \left\{ x : x = \sum_{i=-\infty}^{k} c_i x_i, \quad c_i \in R_n \sum_{i=-\infty}^{k} c_i^2 < \infty \right\}. \tag{2}$$

$$X_k^1 \hat{=} \left\{ x : x = \sum_{i=1}^{k} c_i x_i, \quad c_i \in R \right\} \tag{3}$$

$$\hat{X}_k(m) \hat{=} E(x_{k+m} | X_k) \tag{4}$$

$$\hat{X}_{k+m,k} \hat{=} E(x_{k+m} | X_k^1) \tag{5}$$

With these denotions, $\hat{x}_k(m)$ and $\hat{x}_{k+m,k}$ represent the linear minimum mean square error estimates of $\hat{x}_{k+m}$ based on the infinite sample series $\{x_k, x_{k-1}, \cdots\}$and finite sample series $\{x_k, x_{k-1}, \cdots, x_1\}$respectively. The one step ahead forecast error for $\hat{x}_k(m)$ is obviously the driving white noise, i.e.

$$a_k = x_k - \hat{x}_{k-1}(1) \tag{6}$$

whils the one step ahead forecast error for $\hat{x}_{k+m,k}$ which we define as "finite sample innovation" is denoted as

$$\varepsilon_k \hat{=} x_k - \hat{x}_{k,k-1}. \tag{7}$$

For short, $\{\varepsilon_k\}$is called "innovation" in what follows in this paper.

Then, the following result is interesting.

Innovation theorem [3]:

Let

$$y_k = \begin{cases} x_k & k \leqslant Q \\ x_k - \sum_{i=1}^{p} \varphi_i x_{k-i} & k > Q \end{cases} \tag{8}$$

where Q = max{p,q},then

$$\varepsilon_k = \begin{cases} y_1 = x_1 & k = 1 \\ y_k - \sum_{j=1}^{k-1} J_{kj}\varepsilon_j & 1 < k \leqslant Q \\ y_k - \sum_{j=k-p}^{k-1} J_{kj}\varepsilon_j & k > Q \end{cases} \tag{9}$$

with

$$J_{kj} = \begin{cases} \left(E(y_k y_j) - \sum_{i=1}^{j-1} J_{kj} J_{ji} E(\varepsilon_i^2)\right) / E(\varepsilon_j^2) & 1 < k \leqslant Q, \quad 1 \leqslant j \leqslant k-1 \\ \left(E(y_k y_j) - \sum_{i=k-q}^{j-1} J_{ki} J_{ji} E(\varepsilon_i^2)\right) / E(\varepsilon_j^2) & k > Q, \quad k-q \leqslant j \leqslant k-1 \end{cases} \tag{10}$$

and

$$E(\varepsilon_k^2)=\begin{cases}E(y_1^2) & k=1\\ E(y_k^2)-\sum_{j=1}^{k-1}J_{ki}^2E(\varepsilon_j^2) & 1<k\leqslant Q\\ E(y_k^2)-\sum_{j=k-q}^{k-1}J_{kj}^2E(\varepsilon_j^2) & k>Q\end{cases}\tag{11}$$

The proof for above theorem can be found in [4]. A straight corollary of this theorem is the innovation forecasting formula as follows [4]:

$$\hat{x}_{k+m,k}=\begin{cases}\sum_{j=1}^{k}J_{k+m,j}\varepsilon_j, & k+m\leqslant Q,\\ \sum_{j=1}^{p}\varphi_j\hat{x}_{k+m-j,k}+\sum_{j=k+m-q}^{k}J_{k+m,j}\varepsilon_j, & k+m>Q\end{cases}\tag{12}$$

Again, it is worth pointing out that the term "innovation" here is concerned with the case for finite samples and so are the related results. The innovation forecasting formula provides a way for obtaining the strict linear minimum mean square error prediction based on the finite sample series instead of the infinite sample series.

3. Semi–parametric Forecasting Algorithm

The above–mentioned innovation forecasting formula is usually applied with the following procedure: Firstly,estimate the parameters $\{\varphi_i\}$,(θ_i), σ_a^2 and calculate $E(y_k,y_j)$ based on these parameters, then calculate J_{kj}, $E(\varepsilon_k^2)$ and $\{\varepsilon_k\}$ through eq(9)–(11), finally, obtain the forecast by eq (12). Clearly, the availableness of total parameters in eq(1) is essential to above forecasting procedure,so it still belongs in the catalogue of parametric forecasting technique.

It has come to our notice that ,in order to obtain the prediction by eq(12), all we have to know are $\{\varphi_i\}$ and $\{\varepsilon_k\}$ only.To estimate $\{\varphi_i\}$, the well–known modified Yule–walker (MYW) estimator is available. As to $\{\varepsilon_k)$, the innovation theorem shows that the key point is $E(y_ky_j)$ which is usually estimated based on $\{\varphi_i\}$, $\{\theta_i\}$ and σ_a^2 as mentioned above. However, another expression of $E(y_ky_j)$ can be found if eq(8) is concerned in . $E(y_ky_j)$ can be expressed as follows:

$$E(y_ky_j)=\begin{cases}r(k-j) & j\leqslant k\leqslant Q\\ r(k-j)-\sum_{s=1}^{p}\varphi_s r(k-j-s) & j\leqslant Q,\ k>Q\\ \sum_{s=0}^{p}\sum_{t=0}^{p}\varphi_s\varphi_t r(k-j-(s-t)) & k\geqslant j>Q\end{cases}\tag{13}$$

where $\varphi_0=-1$ and $r(h)=E(x_nx_{n+h})$. It means that $E(y_ky_j)$ can be estimated just based on $\{\varphi_i\}$ and $r(h)$. Obviously, the estimation for $r(h)$is much simpler than that for $\{\theta_i\}$.

Therefor, the above anslysis suggests a new forecasting procedure for which only $\{\varphi_i\}$ and $r(h)$ are required, the tedious work of estimating MA parameters $\{\theta_i\}$ is ingeniously avoided.

The program of the new forecasting algorithm is described as follows:

Step 1. Given time series $\{x_t\}$, calculate the sample auto–covariance of $\{x_t\}$by

$$\hat{r}(h)=\frac{1}{N}\sum_{t=1}^{N-h}x_t x_{t+h} \qquad h=1,2,\cdots,H; \tag{14}$$

Step 2. Estimate AR parameters $\{\varphi_i\}$ through MYW equation;

Step 3. Substitute $\{\hat{\varphi}_i\}$ and $\hat{r}$(h)into eq (13) and estimate $E(y_k y_j)$;

Step 4. Estimate $\{\varepsilon_k\}$ through eq (9)–(11);

Step 5. Obtain the forecast by eq(12).

In this paper the new algorithm is called the semi–parametric forecasting algorithm because that only a part of parameters are estimated in the forecasting procedure.

4. Short–term Water Load Forecasting

The algorthm proposed was is fact motivated in our research work of water load forecasting problem .In this section,the short–term water load forecasting for a city in China is described as an application example of the algorithm.

The need for water load forecasting arises in the computer–aided optimal control problem of urban water supply system.In order to made a decision on the dispatching scheme of water network, it is necessary to have some knowlodge of the water load demand in the near future time at certain nodes of the network previously. That is ,the short–term load forecasting is necessary for the optimal management.

There are numbers of load forecasting methods. One of them is based on time series analysis. Depending on the water load behaviour, the load forecasting model may be different form city to city. We had established a composite load model for a city in Jiangsu ,China as follows [5] :

$$\begin{cases} w_t = p_t + x_t \\ x_t - \sum_{i=1}^{p}\varphi_i x_{t-i} = a_t - \sum_{i=1}^{q}\theta_i a_{t-1} \end{cases} \tag{15}$$

where $p_t = p_{t+24}$ is the determinstic periodic component determined by

$$p_i = \frac{1}{[N/24]}\sum_{k=1}^{[N/24]} w_{N+i-24k} \qquad i=1,2,\cdots,24 \tag{16}$$

and x_t is the random residual component described by ARMA(p,q). The model was established in off–line way based on different periods of historical load data; According to the actual water lood characteristic, the structures of the mode (i.e. the orders (p,q)) are time–invariant while its parameters are slowly time–varying. The slowly time–varying parameters reflect the influence of trend and seasonal variation of water load which isn't explicitly included in the model .To deal with this slowly time–varying component the mi-parametric forecasting method was used, Based on the samples $w_1, w_2, \cdots, w_{312}$ which dicate that city's water load from 1:00 Aug .30 to 24:00, Sept. 11, the load forecasts with 24-hour ahead time were obtained .Some results are listed in Tab.1. The forecasting curve r $\hat{w}_{k+m,k}$ $(k=261,\ m=1,2,\cdots,24)$ and correspondent real load curve are shown in Fig.1. can be seen that the accuracy of the new algorithm can satisfactorily meet the requirement optimal dispatching for water supply system.

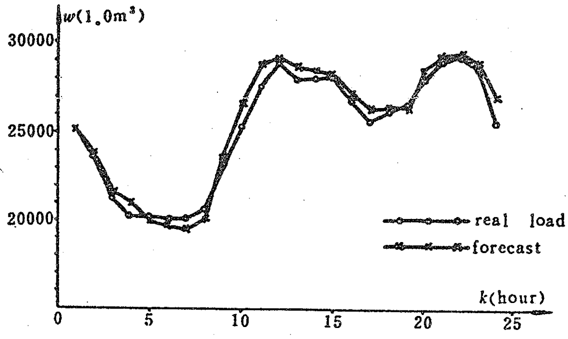

Fig.1 Forecasting Curve and Real Load Curve

Tab.1 (unit: 1.0 M^3)

k	w_{k+1}	$\hat{w}_{k+1,k}$	$\lvert w-\hat{w}\rvert / w$	w_{k+24}	$\hat{w}_{k+24,k}$	$\lvert w-\hat{w}\rvert / w$
260	26156	25982	0.00666	28629	28724	0.00330
261	25166	25158	0.00033	25593	26801	0.04720
262	23618	23669	0.00217	24819	25700	0.03551
263	21286	21538	0.01184	23215	24089	0.03765
264	20269	20859	0.02910	20481	21895	0.06903
265	20142	19553	0.02926	20273	21279	0.04961
266	20282	19704	0.02848	20188	20215	0.00131
267	20005	19845	0.00802	20147	19891	0.01269
268	20609	20319	0.01408	20086	19666	0.02092
269	23308	23977	0.02870	20068	20089	0.00105
270	25370	26658	0.05078	22247	23633	0.06228
271	27573	28009	0.01583	26132	26763	0.02416
272	28852	28286	0.01963	28058	28816	0.02703
273	27888	28316	0.01534	28737	29164	0.01486
274	27955	27925	0.00109	28982	28683	0.01032
275	28101	27821	0.00995	28811	28453	0.01244
276	26686	27082	0.01485	27491	28218	0.02644
277	25648	26022	0.01457	26107	27234	0.04318
278	26208	25928	0.01069	26842	26363	0.01784
279	26498	26281	0.00817	25749	26406	0.02551

5. Conclusions and Remarks

In this paper a semi–parametric forecasting method for ARNA series is proposed and it has applied to the short–term water load forecasting problem.

The novel point of the method lies in that only AR parameters of the model are required for forecasting using ARMA model and the tedious work of estimating MA parameters is ingeniously avoided, so that the forecasting problem is solved much more simply. In fact, all calculations involed in the algoruthm are linear or simple recursive operations, Besides, due to the stationarity of the series,the computation for estimating $E(y_k y_j)$ depends only on q and is independent of k.

As to the forecast accuracy of the algorithm, in spite of many favourable results obtained in simulation and practice, the further theoretic analysis is still to be done .A clear fact is that the forecast accuracy depends to a great extent on the estimation of AR parameters $\{\varphi_j)$. It is well known, the MYW estimator for $\{\varphi_j\}$ is quite simple in compitation and has fairly good bebaviour. It is consistent, asymptotic unbiased and asymptotic Gaussian, but not asymptotic efficient (Gersch, 1970; Stoica, et al, 1985; Cao, et al, 1986). Recently many efforts have been made in attempt to improve MYW estimator. Several extensions and variations of the estimator have appeared. Among them the work of stoica et al (986) is distinctive. Noticing that MYW is in fact a special case of instrumental variable (IV) method of system identification technique (Young, 1972), Stoica et al proposed an extension of MYW—optimal instrumental variable (OIV) algorithm which is asymptotic efficient. Hence, when OIV algorithm is adopted in the semi–parametric forecasting program, the improvement on forecast accuracy can naturally be expected. On the other hand, there are also some occasions in which the forecasting speed is far more important than other requirements. In this case, to estimate $\{\varphi_j\}$, another method based on 0–1 binary time series analysis can be chosen (Kedem,1980). This method is extremely simple with the accuracy slightly decreased. Therefore,depending on the concrete problem, we can choose different method to estimate $\{\varphi_i\}$, which makes the semi–parametric forecasting algorithm quite flexible.

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ARMA序列的半参数预报*

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摘要 在关于ARMA序列新息定理的基础上，本文给出了ARMA序列的一种半参数预报方法。该方法避开对 ARMA 模型中 MA 参数作估计这一计算复杂的工作，在只须知道 AR 参数的情况下直接得到序列的预报值，从而使计算量大为简化同时又保证有一定的精度。本文还介绍了半参数预报方法在城市自来水负荷预报中的成功应用。

关键词：时间序列分析；ARMA 模型；半参数预报；新息负荷预报

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