第7卷 第3期
1990年9月

控制理论与应用
CONTROL THEORY AND APPLICATIONS

Vol. 7, No. 3
Sept., 1990

隐式极点配置自校正解耦控制器

万 涛
(西南交通大学计算机系,四川,峨眉)

摘要 针对一类多变量系统,本文提出了一种新的极点配置自校正解耦控制算法.它能通过参数估计直接获得控制器参数,运算量很小.数字仿真表明本算法具有较好的性能.

关键词: 多变量系统;自校正控制;极点配置;解耦控制;隐式算法

1. 引 言

极点配置自校正解耦控制器具有以下优点:1)系统由于实现了闭环极点配置而具有良好的动态响应特性和稳定性;2)对多变量系统实现了动态解耦.现有算法[3,4]为了获取控制器参数必须求解 Diophantine 方程,运算量较大,甚至还可能出现数值不稳定.本文提出了一种新的极点配置自校正解耦控制算法,它通过参数最小二乘估计可直接获得控制器参数,因而克服了上述算法的缺点.

2. 极点配置解耦控制器的设计

设被控对象由下述差分方程描述:

$$A(q^{-1})Y(t) = q^{-k}B(q^{-1})U(t) + C(q^{-1})W(t), \tag{2.1}$$

其中 $Y\in R^{m\times 1}$、$U\in R^{m\times 1}$、$W\in R^{m\times 1}$分别为系统输出向量、输入向量和零均值高斯白噪声向量序列. $A\in R^{m\times m}(q^{-1})$、$B\in R^{m\times m}(q^{-1})$、$C\in R^{m\times m}(q^{-1})$为多项式矩阵. q^{-1}为后移算子,k 为系统延迟.

$$A(q^{-1}) = I + A_1q^{-1} + A_2q^{-2} + \ldots + A_{n_a}q^{-n_a}, \tag{2.2a}$$

$$B(q^{-1}) = B_0 + B_1q^{-1} + B_2q^{-2} + \ldots + B_{n_b}q^{-n_b}, \qquad \det B_0 \neq 0, \tag{2.2b}$$

$$C(q^{-1}) = I + C_1q^{-1} + C_2q^{-2} + \ldots + C_{n_c}q^{-n_c}. \tag{2.2c}$$

设 A、B 和 C 均互质,且 B 和 C 逆稳定,即:$\det B(q^{-1})$和 $\det C(q^{-1})$的零点均在 q^{-1}复平面的单位圆外.

设控制律具有如下形式:

$$Q(q^{-1})U(t) = R(q^{-1})Y_r(t) - P(q^{-1})Y(t), \tag{2.3}$$

式中 $Y_r\in R^{m\times 1}$为外部设定值. $Q\in R^{m\times m}(q^{-1})$、$R\in R^{m\times m}(q^{-1})$、$P\in R^{m\times m}(q^{-1})$为多项式矩阵.

结合(2.1)式和(2.3)式得闭环系统

$$(A + q^{-k}BQ^{-1}P)Y(t) = q^{-k}BQ^{-1}RY_r(t) + CW(t). \tag{2.4}$$

根据矩阵相乘的“伪互换性”,存在多项式矩阵 $\tilde{Q}(q^{-1})$和 $\tilde{B}(q^{-1})$,使得

本文于1988年10月19日收到. 1989年6月15日收到修改稿.

$$\widetilde{Q}B = \widetilde{B}Q, \tag{2.5}$$

且 $\det\widetilde{B}=\det B$，即 $\widetilde{B}$ 逆稳定.

把(2.5)式代入(2.4)式，整理得

$$(\widetilde{Q}A + q^{-k}\widetilde{B}P)Y(t) = q^{-k}\widetilde{B}RY_r(t) + \widetilde{Q}CW(t). \tag{2.6}$$

设 $T(q^{-1})=\mathrm{diag}[T_1(q^{-1})\quad T_2(q^{-1})\ldots T_m(q^{-1})]$，其中 $T_i(q^{-1})$ 的零点是闭环系统第 i 通道所希望的极点. T 由设计者根据控制系统的要求来确定. $T(0)=I$. 令

$$\widetilde{B}\widetilde{C}T = \widetilde{Q}A + q^{-k}\widetilde{B}P. \tag{2.7}$$

显然，$\widetilde{B}$ 是 $\widetilde{Q}$ 的左因式，故存在多项式矩阵 $\widetilde{D}(q^{-1})\in R^{m\times m}(q^{-1})$，使得

$$\widetilde{Q} = \widetilde{B}\widetilde{D}. \tag{2.8}$$

在(2.7)式中，$\widetilde{C}$ 满足

$$\widetilde{C}D = \widetilde{D}C, \tag{2.9}$$

式中

$$\det\widetilde{C} = \det C, \tag{2.10}$$

$$\deg D = \deg\widetilde{D}, \quad \deg\widetilde{C} = \deg C, \tag{2.11}$$

其中 $\widetilde{C}\in R^{m\times m}(q^{-1})$、$D\in R^{m\times m}(q^{-1})$.

把(2.7)－(2.9)式代入(2.6)式，故得

$$\widetilde{C}TY(t) = q^{-k}RY_r(t) + \widetilde{C}DW(t). \tag{2.12}$$

由于 R 根据需要由设计者选取. 为保证实现系统的极点配置和动态解耦，根据(2.12)式，故取

$$R = \widetilde{C}R', \tag{2.13}$$

式中 $R'\in R^{m\times m}(q^{-1})$ 是对角矩阵多项式. 且

$$R'(1) = T(1), \tag{2.14}$$

以实现系统对阶跃设定值的稳态无差跟踪. 在满足(2.14)式的前提下，R' 的具体形式仍由设计者来定，如实现闭环系统的零点配置.

至此，由(2.12)式和(2.13)式，得到所设计的系统为

$$TY(t) = q^{-k}R'Y_r(t) + DW(t), \tag{2.15}$$

满足预定的设计要求.

3. 极点配置自校正解耦控制算法

对于参数未知或缓慢时变的对象，通过参数辨识直接获取控制器参数，从而避免求解 Diophantine 方程(2.7)式，必然大大减少算法的运算量，增加其可靠性.

根据(2.7)式和(2.8)式，得

$$\widetilde{C}T = \widetilde{D}A + q^{-k}P. \tag{3.1}$$

上式有唯一解的条件为

$$\deg\widetilde{D} = k - 1, \tag{3.2a}$$

$$\deg P = n_a - 1. \tag{3.2b}$$

比较(2.5)式与(2.8)式，显然有

$$Q = \widetilde{D}B, \tag{3.3}$$

$$\deg Q = n_b + k - 1. \tag{3.4}$$

对(3.1)式两端右乘 $Y(t+k)$，再考虑到(2.1)式、(2.9)式、(2.10)式和(3.3)式，得

$$TY(t+k) = \widetilde{C}^{-1}[QU(t) + PY(t)] + DW(t+k). \tag{3.5}$$

下面证明，对(3.5)式作适当变换，利用前面的结果，可得到一个估计模型，通过递推最小二乘法即可估计出控制器参数 Q、P、R'.

设

$$\widetilde{C}^{-1} = \widetilde{C}_0^{-1} + \widetilde{C}'q^{-1}, \tag{3.6}$$

$$\widetilde{C} = \widetilde{C}_0 + \widetilde{C}''q^{-1}, \tag{3.7}$$

式中 $\widetilde{C}' \in R^{m\times m}(q^{-1})$，$\widetilde{C}'' \in R^{m\times m}(q^{-1})$. $\widetilde{C}'$通常是无穷阶矩阵多项式，而 $\widetilde{C}''(q^{-1})$是有限阶矩阵多项式.

$$\deg\widetilde{C}'' = \deg C - 1 \tag{3.8}$$

代入(3.5)式就有

$$TY(t+k) = \widetilde{C}_0^{-1}[QU(t) + PY(t)] + \widetilde{C}'[QU(t-1) + PY(t-1)] + DW(t+k). \tag{3.9}$$

对等式右端第二项，利用(2.3)式和(2.13)式作变换，得

$$TY(t+k) = \widetilde{C}_0^{-1}[QU(t) + PY(t)] + \widetilde{C}'\widetilde{C}[R'Y_r(t-1)] + DW(t+k). \tag{3.10}$$

把(3.6)式和(3.7)式代入，得

$$TY(t+k) = \widetilde{C}_0^{-1}[QU(t) + PY(t)] - \widetilde{C}_0^{-1}\widetilde{C}''[R'Y_r(t-1)] + DW(t+k). \tag{3.11}$$

令 $\overline{Q}=\widetilde{C}_0^{-1}Q$，$\overline{P}=\widetilde{C}_0^{-1}P$，$\overline{C}''=\widetilde{C}_0^{-1}\widetilde{C}''$，故有

$$TY(t+k) = \overline{Q}U(t) + \overline{P}Y(t) - \overline{C}''[R'Y_r(t-1)] + \xi(t+k), \tag{3.12}$$

式中 $\widetilde{C}_0^{-1}$是常数阵，故 $\deg\overline{Q}=\deg Q$，$\deg\overline{P}=\deg P$，$\deg\overline{C}''=\deg\widetilde{C}''$. 而

$$\xi(t+k) = DW(t+k). \tag{3.13}$$

因 $\deg D=\deg\widetilde{D}=k-1$，故 $\xi(t+k)$与(3.12)中等式右端出现的变量 $U(t-i)$、$Y(t-i)$、$R'Y_r(t-i-1)$，$i=0,1,2,\ldots$ 均不相关. 因此，用递推最小二乘法即可估计出(3.12)式的参数 $\overline{Q}$,$\overline{P}$ 和 $\overline{C}''$.(估计算法略)

对(2.3)式两端同左乘 $\widetilde{C}_0^{-1}$，即得控制律

$$\overline{Q}U(t) = [I + \overline{C}''q^{-1}]R'Y_r(t) - \overline{P}Y(t). \tag{3.14}$$

设 $\overline{Q}=\overline{Q}_0+\overline{Q}'q^{-1}$，$\overline{Q} \in R^{m\times m}(q^{-1})$，故得控制量为

$$U(t) = \overline{Q}_0^{-1}[(I + \overline{C}''q^{-1})R'Y_r(t) - \overline{P}Y(t) - \overline{Q}'U(t-1)]. \tag{3.15}$$

于是得到本自校正算法的计算步骤为

(1) 选定 T,R'；

(2) 根据(3.12)式组成数据向量，用递推最小二乘法估计控制器参数 $\overline{Q}$,$\overline{P}$,$\overline{C}''$；

(3) 由(3.15)式计算出控制量，对于下一采样时间，重复(1)至(3)的过程.

4. 数字仿真

下面通过一个具体的例子来说明本算法的性能.

设被控对象为

$$A(q^{-1})Y(t) = q^{-k}B(q^{-1})U(t) + C(q^{-1})W(t), \tag{4.1}$$

式中

$$k = 2,$$

$$A(q^{-1})=\begin{bmatrix}1-0.0693q^{-1}-0.1792q^{-2} & -0.1352q^{-1}-0.3793q^{-2}\\ -0.0824q^{-1}-0.1792q^{-2} & 1-0.1226q^{-1}-0.3789q^{-2}\end{bmatrix},$$

$$B(q^{-1})=\begin{bmatrix}0.4585+0.0917q^{-1} & 0.21+0.0997q^{-1}\\ 0.3563+0.0897q^{-1} & 0.311+0.1011q^{-1}\end{bmatrix},$$

$$C(q^{-1})=\mathrm{diag}(1+0.6q^{-1}\quad 1-0.7q^{-1}),$$

$$E[W(t)W^{\tau}(t)]=4I.$$

仿真时，取 $T(q^{-1})=\mathrm{diag}(1-0.5q^{-1}\quad 1-0.5q^{-1})$，$R'(q^{-1})=T(1)$，用 IBM/PC 进行数字仿真，其结果如图1和图2.

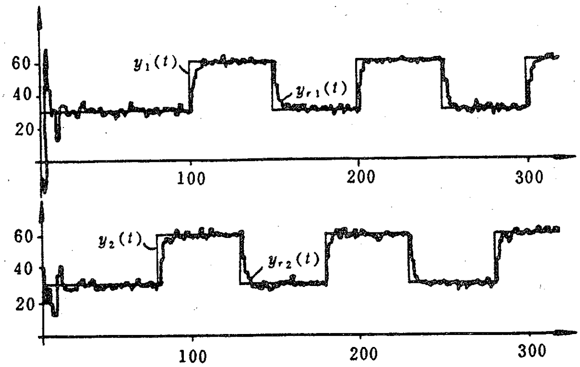

图 1　控制系统输出

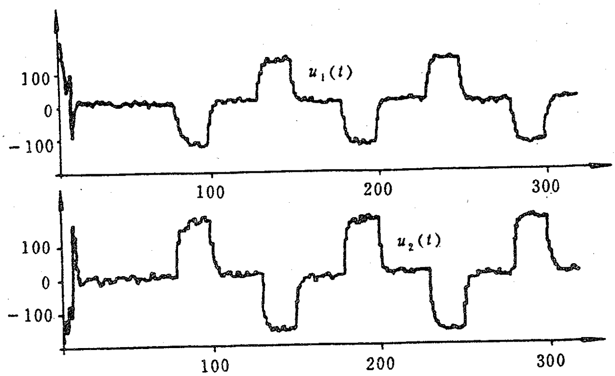

图 2　控制器输出

图1是控制系统对设定值的跟踪响应曲线. 它表明算法具有较好的动态特性、解耦特性及抗随机干扰特性，且对阶跃设定值实现了稳态无差跟踪. 图2反映了自校正控制器的输出情况.

值得一提的是，本算法具有较好的抗随机干扰特性，这主要是因为系统输出的随机分量是 $\xi(t)/T$，或 $DW(t)/T$，而文献[2—4]算法所设计系统的输出是 $QW(t)/T$. 而 $\deg D$ 显然小于 $\deg Q$.

5. 结　　论

理论推导和数字仿真表明：本文提出的自校正极点配置解耦控制算法性能良好且计算量

较小，很有实用价值.

参考文献

[1] Astrom,K. J. ,Wittenmark,B. ,Self—tuning Controllers Based on Pole—zero Placement,Proc. IEE,127(d),3,(1980),120—130.

[2] Prager,D. C. ,Wellstead,P. E. ,Multivariable Pole—assignment Self—tuning Regulator,Proc. IEE,128(d),1,(1981),9—18.

[3] 柴天佑、郎世俊、顾兴源，多变量自校正前馈控制器及应用，自动化学报，3,(1986),229—236.

[4] 钟慕良、苏春翌，多变量极点配置自校正动态解耦控制器，控制理论与应用，1,(1988),25—32.

Implicit Pole—assignment Self—tuning Decoupling Controller

Wan Tao

(Department of Computers,Southwestern Jiaotong University, Sichuan, Emei)

Abstract: This paper presents a new multivariable pole—assignment self—tuning decoupling control algorithm for a class of multivariable processes. The controller parameters of this algorithm can be obtained directly with the help of parameter identification. So, the algorithm has not nuch computation. Simulation indicates that this algorithm has a good characteristics.

Key words: martivariable system; self—tuning control; pole—assignment; decoupling control; implicit algorithm

(continued from page 55)

WORKING LANGUAGE

The working language of the Conference is English which will be used for printed material, presentation and discussion.

DEANLINES

November 30,1990	:Abstracts to be received by the Secretariat.
February 1,1991	:Notification of acceptance of papers.
April 15,1991	:Camera—ready full papers to be received by the Secretariat.

The abstract should be about 500 words, clearly showing the original contribution. It should include the title, the name (s) of the author(s), affiliation, address, telephone, Telex or Fax. In case of more than one author, the name of the author who is responsible for the preparation of the final version of the paper should be indicated.

Authors are requested to send 5 copies of their abstracts of papers to:

PROFESSOR HAN—SHU ZHONG

Secretariat, Asia—Pacific Conference on Measurement & Control

Laboratory for Control Theory & Applications

Department of Automation

South China University of Technology

Guangzhou

The People's Republic of China

PROGRAM AND REGISTRATION

The registration form will be mailed along with the notification of acceptance of papers to the authors. Detailed program will be included in the final announcement which will be published a few months before the event and will be sent to thosewho complete and return the registration form.

The submission of a full paper implies the commitment of the author to present the paper at the Conference.

The registration fee for each participant will be US¥200 which covers costs of the proceedings and social events.

POST—CONFERENCE TOUR

Post—conference tour to Beijing, Gui—lin, Xian and other places can be arranged.