A Globally Convergent Multivariable Stochastic Adaptive Control Algorithm Based on CARIMA Model*

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Abstract: In this paper a novel stochastic direct adaptive control algorithm for multivariable linear systems described by a Controlled Autoregressive Integrating Moving Average (CARIMA) model is proposed. This scheme not only ensures the robust offset rejection for any constant load disturbance acting on the plant but also has globally convergent properties even for nonminimum systems. In particular, it requires only knowledge of the integer valued parameters of the system interactor matrix.

Key words: stachastic multivariable systems; system interactor; global convergence; load disturbance

1. Introduction

A multivariable self-tuning controller for CARIMA systems was proposed [1]. This algorithm ensures robust offset rejection, but requires a priori knowledge of the system interactor matrix and its global convergence analysis is not established. The global convergence of a self-tuning controller for general multivariable systems was established in [2], the leading assumption is that the system interactor matrix is known a priori. This paper follows the idea in [3] to eliminate a priori knowledge of the noninteger valued parameters of interactor matrix. The plant controlled by the adaptive scheme proposed is assumed to be described by a CARIMA model. The global convergence proof is carried out by resorting to a modified least square identification algorithm.

2. System Model and Control Law

Let the system be described by the following matrix polynomial CARIMA model:

$$A(z^{-1})y(t) = B(z^{-1})u(t) + C(z^{-1})\delta(z^{-1})^{-1}\xi(t), \tag{2.1}$$

where $u(t)\in R^n$ and $y(t)\in R^n$ are the control and output vectors, respectively, $\delta(z^{-1})=1-z^{-1}$ and $A(z^{-1}), B(z^{-1}), C(z^{-1})$ are polynomial matrices in the backward shift operator z^{-1} such that $A(0)=C(0)=I$, I being the $n\times n$ identity matrix and det $C(z^{-1})\neq 0$ for $|z|\geqslant 1$. Rank $B(z^{-1})=n$. $B(z^{-1})$ is of the form

$$B(z^{-1}) = [z^{-k_{ij}}B_{ij}(z^{-1})], \quad B_{ij}(z^{-1}) = \sum_{l=0}^{n_{bij}} b_{ijl}z^{-1}, \quad b_{ij0}\neq 0, \tag{2.2}$$

$\xi(t)\in R^n$ is a noise vector characterized by

$$E[\xi(t)/F_{t-1}] = 0 \quad \text{a.s.} \quad E[\xi(t)\xi(t)^T/F_{t-1}] = Q \quad \text{a.s.} \quad \text{with trace } Q<\infty. \tag{2.3}$$

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$$\limsup_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}\|\xi(t)\|^2<\infty \quad \text{a. s.} \tag{2.4}$$

F_t denotes the δ—algebra generated by data up to and including time t. As shown in [3], if rank $B(z^{-1})=n$, there exist diagonal matrices $D(z^{-1})$ and $K(z^{-1})$ such that

$$\lim_{z\to\infty} D(z^{-1})^{-1}B(z^{-1})K(z^{-1}) = K_b, \quad \det K_b \neq 0, \tag{2.5}$$

where

$$D(z^{-1}) = \operatorname{diag}(z^{-k_1}) \quad k_1 \geqslant 1,\ K(z^{-1}) = \operatorname{diag}(z^{-d_j}) \quad d_j \geqslant 0.$$

If K_{ij} are known, the polynomial matrices $D(z^{-1})$ and $K(z^{-1})$ can be determined by the method [3]. Because of spectial limitations we do not described this method here. If $D(z^{-1})$ and $K(z^{-1})$ are determined, then define

$$B_d(z^{-1}) = D(z^{-1})^{-1}B(z^{-1})K(z^{-1}), \quad B_d(z^{-1}) = [z^{-\bar{k}_{ij}}B_{ij}(z^{-1})], \quad B_d(0) = K_b, \tag{2.6}$$

$$\bar{k}_{ij} = k_{ij} - k_i - d_j \qquad (i,j = 1,\cdots,n).$$

Then the system model (2. 1) can be transformed into

$$A(z^{-1})y(t) = D(z^{-1})B_d(z^{-1})\bar{u}(t) + C(z^{-1})\delta(z^{-1})^{-1}\xi(t), \tag{2.7}$$

where

$$u(t) = K(z^{-1})\bar{u}(t). \tag{2.8}$$

Here we first derive the optimal control law minimizing with respect to $\bar{u}(t)$ the following performance index:

$$J = E[\|e(t+k)\|^2/F_t], \tag{2.9}$$

where

$$e(t+k) = P(z^{-1})D(z)y(t) - R(z^{-1})w(t) + Q(z^{-1})\delta(z^{-1})\bar{u}(t), \tag{2.10}$$

and $w(t)\in R^n$ is a bounded reference vector, and P,R,Q are weighting polynomial matrices in z^{-1} such that $P(0)=I$, and $k=\max\limits_{1\leqslant i\leqslant n} k_i$. The control law is deduced by using the optimal prediction $\varphi^*(t+k/t)$ for $\varphi(t+k)=P(z-1)D(z)y(t)$ given by the following lemma.

Lemma 2. 1 The optimal prediction $\varphi^*(t+k/t)$ of $\varphi(t+k)$ satisfies the following equations.

$$\varphi^*(t+k/t) = \alpha(z^{-1})y(t) + \beta(z^{-1})\bar{u}(t) + \bar{\bar{G}}(z^{-1})\varphi^*(t/t-k), \tag{2.11}$$

$$\varphi^*(t+k/t) = \varphi(t+k) - v(t+k), \tag{2.12}$$

where $\alpha(z^{-1})$, $\beta(z^{-1})$ and $\bar{\bar{G}}(z^{-1})$ are polynomial matrices in z^{-1} whose orders are n_1, n_2 and n_3, respectively.

$$v(t+k) = F(z^{-1})\xi(t+k), \quad \text{degree } F(z^{-1}) = k-1. \tag{2.13}$$

Proof Define the polynomial matrices $\bar{A}(z^{-1})$, $\bar{C}(z^{-1})$, $\tilde{C}(z^{-1})$, $F(z^{-1})$, $\tilde{F}(z^{-1})$, $\bar{F}(z^{-1})$, $G(z^{-1})$, $\tilde{G}(z^{-1})$, $\bar{\bar{G}}(z^{-1})$ satisfying the following set of relations:

$$C(z^{-1})\bar{A}(z^{-1}) = A(z^{-1})\bar{C}(z^{-1}),\ \det C(z^{-1}) = \det\bar{C}(z^{-1}),\quad \bar{C}(0) = I, \tag{2.14}$$

$$P(z^{-1})\bar{D}(z^{-1})\bar{C}(z^{-1}) = F(z^{-1})\delta(z^{-1})\bar{A}(z^{-1}) + z^{-k}G(z^{-1}).$$

$$\text{degree}\quad F(z^{-1}) = k-1, \quad \bar{D}(z^{-1}) = z^{-k}D(z). \tag{2.15}$$

$$\tilde{C}(z^{-1})F(z^{-1}) = \tilde{F}(z^{-1})C(z^{-1}), \quad \det\tilde{C}(z^{-1}) = \det C(z^{-1}), \quad \tilde{C}(0) = I, \tag{2.16}$$

$$\tilde{C}(z^{-1})G(z^{-1}) = \tilde{G}(z^{-1})\bar{C}(z^{-1}). \tag{2.17}$$

$$I = \bar{F}(z^{-1})\tilde{C}(z^{-1}) + z^{-k}\bar{\bar{G}}(z^{-1}). \tag{2.18}$$

Along the lines depicted in [2], (2.11) and (2.12) can be established.

The optimal control law is now described by the following theorem.

Theorem 2.1 The optimal control law and the minimum possible value of the performance index are given by

$$\varphi^*(t+k/t) = R(z^{-1})w(t) - Q(z^{-1})\delta(z^{-1})\bar{u}(t), \tag{2.19}$$

$$J = E[\|e(t+k)\|^2/F_t] = E[\|v(t+k)\|^2/F_t] = \gamma^2. \tag{2.20}$$

Proof Using the same argument as is used in [2] results in (2.19) and (2.20).

3. Adaptive Algorithm and Global Convergence Analysis

In order to derive the direct adaptive algorithm, first we use (2.11) and (2.12) to obtain the estimation equation of controller parameters described by

$$\varphi(t) = \alpha(z^{-1})y(t-k) + \beta(z^{-1})\delta(z^{-1})\bar{u}(t-k) + \bar{\bar{G}}(z^{-1})\varphi^*(t-k/t-2k) + F(z^{-1})\xi(t). \tag{3.1}$$

Using (2.11) and (2.19) results in

$$\alpha(z^{-1})y(t) + \beta(t)\delta(z^{-1})\bar{u}(t) + \bar{\bar{G}}(z^{-1})\varphi^*(t/t-k) = y^*(t+k), \tag{3.2}$$

where

$$y^*(t+k) = R(z^{-1})w(t) - Q(z^{-1})\delta(z^{-1})\bar{u}(t). \tag{3.3}$$

To ensure that the adaptive control algorithm has global convergence properties we use a slightly modified least squares algorithm to estimate the controller parameters. Define parameter matrix θ and data vector $X(t)$ as follows

$$\theta = [\alpha_0, \alpha_1, \cdots; \beta_0, \beta_1, \cdots;]$$

and

$$X(t) = [y(t)^T, y(t-1)^T, \cdots; \delta(z^{-1})u(t)^T, \delta(z^{-1})u(t-1)^T, \cdots; \bar{y}(t)^T, \bar{y}(t-1)^T, \cdots]^T,$$

where $\bar{y}(t)$ is the posteriori prediction replacing $\varphi^*(t/t-k)$ and given by

$$\bar{y}(t) = \hat{\theta}(t)X(t-k), \tag{3.4}$$

with $\bar{y}(\tau)=0$ for $\tau\leqslant k-1$ and where $\hat{\theta}(t)$ is the estimate at t for θ. Thus direct adaptive control algorithm can be described by

$$\hat{\theta}(t) = \hat{\theta}(t-k) + a(t-k)[\bar{\varphi}(t) - \hat{\theta}(t-k)X(t-k)]X(t-k)^T P(t-k). \tag{3.5}$$

$$\hat{\theta}(t)X(t) = y^*(t+k). \tag{3.6}$$

$$P(t-k) = P(t-2k) - \frac{P(t-2k)X(t-k)X(t-k)^T P(t-k)}{1 + X(t-k)^T P(t-2k)X(t-k)}, \tag{3.7}$$

with $P(-1)=\cdots=P(-k)\delta I \ \delta>0$

$$a(t-k) = 1. \tag{3.8}$$

Condition A

$$r(t-k)\mathrm{tr}P(t-k) \leqslant K_1 < \infty \quad \text{and } X(t-k)^T P(t-2k)X(t-k) \leqslant K_2 < \infty, \tag{3.9}$$

where

$$r(t-k) = r(t-k-1) + X(t-k)^T X(t-k), \tag{3.10}$$

with $r(-k) = \cdots = r(-1) = n(n1+n2+n3)$.

If condition A is not satisfied, then

$$P(t-k)=\frac{r(t-2k)}{r(t-k)}P(t-2k), \tag{3.11}$$

$$a(t-k)=\frac{1}{1+X(t-k)^TP(t-k)X(t-k)}. \tag{3.12}$$

The global convergence result for this adaptive algorithm can be stated in the following theorem.

Theorem 3.1 Assume that $(A1)$ the time delay k_{ij} are known; $(A2)$ an upper bound for $n1, n2, n3$ is known; $(A3)$ the off—line choices of P and Q are such that

$$\det[Q(z^{-1})\delta(z^{-1})\bar{\bar{A}}(z^{-1})+P(z^{-1})D(z)\bar{\bar{B}}(z^{-1})]\neq 0 \ for \ |z|\geqslant 1,$$

where $\bar{\bar{A}}(z^{-1})$ and $\bar{\bar{B}}(z^{-1})$ are defined as follows:

$$D(z^{-1})B_d(z^{-1})\bar{\bar{A}}(z^{-1})=A(z^{-1})\bar{\bar{B}}(z^{-1}) \qquad \det\bar{\bar{A}}(z^{-1})=\det A(z^{-1}), \tag{3.13}$$

$(A4)$ $\bar{\bar{C}}(z^{-1})^{-1}-0.5I$ is strictly positive real and defined in (2.18) a.s.

$$\bar{\bar{C}}(z^{-1})=\bar{F}(z^{-1})\tilde{C}(z^{-1})=I-z^{-k}\bar{\bar{G}}(z^{-1}). \tag{3.14}$$

Then the algorithm (3.5)—(3.12) when applied to system (2.1) has the following properties with probability one

$$1)\ \limsup_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}\|y(t)\|^2<\infty \quad \limsup_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}\|\delta(z^{-1})u(t)\|^2<\infty, \tag{3.15}$$

$$2)\ \lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}E\|e(t+k)\|^2/F_t)=\gamma^2. \tag{3.16}$$

Proof part 1. Define the following quadratic form in $\tilde{\theta}(t)(\tilde{\theta}(t)=\hat{\theta}(t)-\theta)$

$$V(t)=\frac{\mathrm{tr}[\tilde{\theta}(t)P(t-k)^{-1}\tilde{\theta}(t)^T]}{r(t-k)}. \tag{3.17}$$

Then along the lines depicted in [2,4] and Kronecker's Lemma, it can easily be concluded that

$$\lim_{N\to\infty}\frac{N}{r(N)}\frac{1}{N}\sum_{t=1}^{N}\|b(t)\|^2=0 \quad \text{a.s.} \tag{3.18}$$

Part 2. Introduce the matrices $\bar{B}(z^{-1}), \bar{Q}(z^{-1}), A^*(z^{-1}), P^*(z^{-1})$ such that

$$\bar{B}(z^{-1})Q(z^{-1})=\bar{Q}(z^{-1})D(z^{-1})B_d(z^{-1}), \quad \det D(z^{-1})B_d(z^{-1})=\det\bar{B}(z^{-1}), \tag{3.19}$$

$$A^*(z^{-1})P(z^{-1})D(z)=P^*(z^{-1})A(z^{-1}), \quad \det A(z^{-1})=\det A^*(z^{-1}). \tag{3.20}$$

Premultiplying (2.10) by A^* or $\bar{B}$, respectively and combining (2.7), (3.19) and (3.20) the following output and input dynamics hold:

$$(P^*DB_d+A^*Q\delta)\delta\bar{u}(t)=\delta A^*e(t+k)-P^*C\xi(t)+A^*R\delta w(t), \tag{3.21}$$

$$(\bar{B}PD(z)+\bar{Q}A\delta)y(t)=\bar{B}e(t+k)+\bar{B}Rw(t)+\bar{Q}C\xi(t). \tag{3.22}$$

From (3.13), (3.19) and (3.20)

$$\begin{aligned}\det[p^*DB_d+A^*Q\delta]&=\det\{A^*[PD(z)A^{-1}DB_d+Q\delta]\}\\&=\det\{A^*[PD(z)\bar{\bar{B}}+Q\delta\bar{\bar{A}}]\bar{\bar{A}}^{-1}\}=\det[PD(z)\bar{\bar{B}}+Q\delta\bar{\bar{A}}]\end{aligned}$$

$$\det[\bar{B}PD(z)+\bar{Q}A\delta]=\det\{\bar{B}[PD(z)+Q(DB_d)^{-1}A\delta]\}=\det[PD(z)\bar{\bar{B}}+Q\bar{\bar{A}}\delta]. \tag{3.23}$$

Using the same method as is used in [4], and from (3.21), (3.22) and (3.23) we have

$$r(N)/N\leqslant(C_1/N)\sum_{t=1}^{N}\|z(t)\|^2+C_2\leqslant(C_3/N)\sum_{t=1}^{N}\|b(t)\|^2+C_4. \tag{3.24}$$

Thus from (3. 18) and stability of $\overline{C}(z^{-1})$ we have

$$\lim_{N\to\infty}(1/N)\sum_{t=1}^{N}\|b(t)\|^2=0 \quad \text{a. s.} \quad \text{and} \lim_{N\to\infty}(1/N)\sum_{t=1}^{N}\|z(t)\|^2=0 \quad \text{a. s.} \tag{3. 25}$$

(3. 15) now follows from (3. 10), (3. 24) and (3. 25). Using the similar argument used in [2] yields

$$\lim_{N\to\infty}(1/N)\sum_{t=k}^{N}\|e(t)-v(t)\|^2=0.$$

Noting $[e(t)-v(t)]=[\varphi(t)-y^*(t)-v(t)]=[\varphi^*(t/t-k)-y^*(t)]$ is F_{t-k} measurable and then we have

$$\lim_{N\to\infty}(1/N)\sum_{t+1}^{N}E(\|e(t-k)\|^2/F_t)$$

$$=\lim_{N\to\infty}(1/N)\sum_{t=1}^{N}E\|e(t+k)-v(t+k)+v(t+k)\|^2=\gamma^2 \quad \text{a. s.}$$

Remark 3. 1 For simplicity Q and P can be chosen such that $Q=\lambda I$ and $P=I$, λ satisfies

$$\det[\lambda(1-z^{-1})A(z^{-1})+D(z^{-1})+D(z^{-1})B_t(z^{-1})]\neq 0 \quad \text{for } |z|\geqslant 1. \tag{3. 26}$$

4. Simulation Results

Consider a nonminimum—phase double—input double—output system described by

$$\begin{vmatrix} 1-0.95z^{-1} & 0 \\ 0 & 1-0.1z^{-1} \end{vmatrix} y(t)=\begin{vmatrix} 0.2z^{-1}+0.268z^{-2} & 0.1z^{-1} \\ -z^{-2} & 10z^{-2} \end{vmatrix} u(t)+\xi(t)\delta(z^{-1})^{-1}+d,$$

where $\xi(t)=[\xi_1(t)\ \xi_2(t)]^T$ is a vector of zero—mean random variable with covariance matrix equal to 0. 1I and a vector of constant additive disturbance $d=[3\quad 2]^T$. The reference signals $w_1(t)$ and $w_2(t)$ are two square waves of period 80 samples and ranges ± 5. Simulation results show that the algorithm proposed has the robust offset rejection properties.

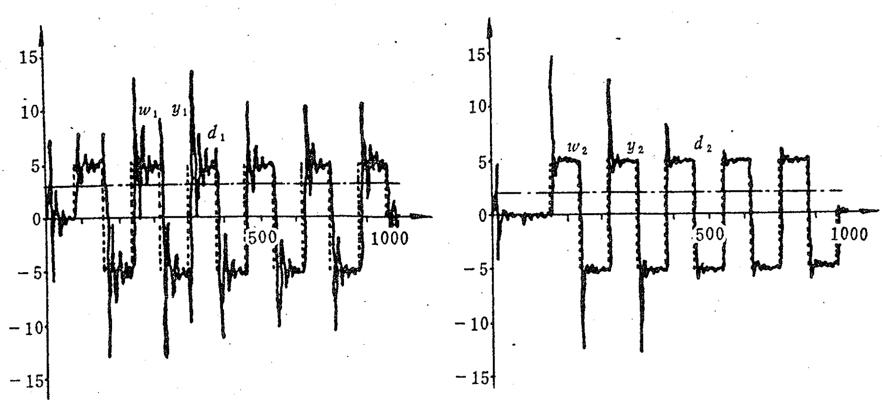

Fig 1 Response of nonminimum phase system using the controller proposed

5. Conclusions

This paper has shown that, in multivariable stochastic adaptive control, it is possible to replace the usual assumption of known interactor matrix by a weaker assumption that the integer valued parameters are known. Because systems under control are described by CARIMA the resulting adaptive control algorithm has effective integrating properties, thus ensuring the robust zero—error regulation for any constant set point and load disturbance. A global convergence proof for this adaptive scheme has been presented without assuming that systems are minimum phase.

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基于CARIMA模型的全局收敛的多变量随机自适应控制算法

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摘要 本文针对多变量受控自回归积分滑动平均模型(CARIMA)提出了新的直接随机自适应控制算法.该算法不仅能鲁棒抑制任何定常负荷干扰而且应用于非最小相位系统仍具有全局收敛性.该算法不要求事先知道系统的关联矩阵只要求知道关联矩阵的整值参数.

关键词: 随机多变量系统;系统的关联矩阵;全局收敛性;负荷干扰