第 16 卷第 6 期 控制理论与应用 Vol.16, No.6
1999 年 12 月 CONTROL THEORY AND APPLICATIONS Dec., 1999

Nonlinear Controller Design for Absolute Stabilization Control Problems *

Guo Lei
(Research Institute of Automation, Southeast University·Nanjing, 210096, P.R.China)
Zang Minglei
(Department of Mathematics, Fudan University·Shanghai, 200437, P.R.China)
Feng Chunbo
(Research Institute of Automation, Southeast University·Nanjing, 210096, P.R.China)

Abstract: This paper discusses the absolute stabilization problem for Lur'e systems with multiple nonlinear loops in terms of state-space approach. Solvability conditions are presented to design nonlinear controllers such that the closed-loop system is absolutely stable via the algebraic matrix inequality (AMI) approach. It is shown that feedback controllers exist if and only if a class of special multilinear matrix inequalities (MLMIs) are solvable. Also, the AMI-based design method obtained in this paper is simplified so as to be computationally feasible and tractable. The approach can be generalized to deal with other problems such as H_2, H_∞ and dissipation control problem.

Key words: nonlinear system; stabilization; Lur'e system; absolute stability; algebraic matrix inequality

Lur'e 系统镇定问题的非线性控制器设计

郭 雷
(东南大学自动化所·南京, 210096)
臧明磊 冯纯伯
(复旦大学数学系·上海, 200437) (东南大学自动化所·南京, 210096)

摘要: 本文研究了用 Lur'e 多非线性系统描述的被控对象的镇定问题. 把问题的可解性归结到特殊的多线性矩阵不等式的可解性. 非线性状态反馈和输出反馈控制器的设计分别依赖于一个双线性和三个三线性矩阵不等式的解. 给出了基于线性矩阵不等式的交替寻优算法的设计步骤.

关键词: 非线性系统; Lur'e 系统; 镇定; 线性矩阵不等式

1 Introduction

Although the importance of the stabilization control problem for nonlinear systems has been recognized for a long time, researches are not very sufficient even for the Lur'e system, which is a kind of typical nonlinear systems (See, e.g., [1, 2] and references therein). Recently, in [3] Savkin and Petersen considered an extension of Popov criterion in state space. A necessary and sufficient condition involved in two parameter-dependent algebraic Riccati equation (ARE) (with a coupled constraints) was presented for this property. However, the Lur'e system under consideration in [3] only includes one nonlinear loop and some assumptions made on the Lur'e system were redundant.

In this paper we consider the absolute stabilization control problems for the generalized plants described by the Lur'e system with multiple nonlinearities. The objective is to design controllers such that the closed-loop systems are absolutely stable. The Lur'e control vector is considered as a measurable output. Thus, unlike the linear feedback case, the control law given in this paper is nonlinear and the former case can be dealt with as a special one. Based on LMI approach in control theory[4], both state feedback and output feedback cases are discussed and the solvability conditions are reduced to multlinear matrix inequalities (MLMIs). If the number

* Project supported by the National Natural Science Foundation of China under Grant 69704004 and the Chinese Post-Doctoral Science Foundation.
Manuscript received May 2, 1997, revised Jun. 22, 1998.

of the nonlinear loops turns to be one, results obtained in this note is sufficient and necessary in the sense of existence of Lyapunov function.

The controlled plant G is described as

$$\begin{cases} \dot{x} = Ax + B_p p + B_u u, \\ y = C_y x, \\ q = C_q x, \end{cases} \tag{1}$$

where

$$p^{\mathrm{T}} = (p_1(t), \cdots, p_{n_p}(t)),$$

$$q^{\mathrm{T}} = (q_1(t), \cdots, q_{n_p}(t)),$$

$$p_i(t) = \phi_i(q_i(t)), \quad i = 1, 2, \cdots, n_p.$$

$\phi_i(\sigma)$ satisfies the follwing sector condition

$$0 \leqslant \sigma\phi_i(\sigma) \leqslant l_i\sigma^2. \tag{2}$$

And $x \in \mathbb{R}^n$, $u \in \mathbb{R}^{n_u}$ and $y \in \mathbb{R}^{n_y}$, represent state, control input, and measurable output, respectively. $p \in \mathbb{R}^{n_p}$ is Lur'e control vector, while $q \in \mathbb{R}^{n_p}$ is the signal for control, which relates p and x via a sector bound function. Suppose coefficient matrices in (1) have appropriate dimensions.

Denote $L: = \mathrm{diag}\{l_1, \cdots, l_{n_p}\}$ to describe the bound of the nonlinear sector.

Consider the following free system

$$\begin{cases} \dot{x} = Ax + B_p p, \\ q = C_q x. \end{cases} \tag{1$'$}$$

The absolute stability of (1)$'$ can be converted to the existence of the Lyapunov function in the following form

$$V(x) = x^{\mathrm{T}}Px + 2\sum_{i=1}^{n_p}\lambda_i\int_0^{C_{i,q}x}\phi_i(\sigma)\mathrm{d}\sigma, \tag{3}$$

where

$$P > 0, \Lambda: = \mathrm{diag}\{\lambda_i, \cdots, \lambda_{n_p}\} \geqslant 0,$$

and

$$C_q^{\mathrm{T}}: = (C_{1,q}^{\mathrm{T}}, \cdots, C_{n_p,q}^{\mathrm{T}}).$$

It is well known that the existence of $V(x)$ in the form of (3) is equivalent to the famous Popov criterion[3].

The following lemma reduces the absolute stability for Lur'e system to the solvability condition of a certain LMI.

Lemma 1 (Absolutely stable condition) For Lur'e system (1)$'$, if there exist $P > 0, \Lambda \geqslant 0$, and $T: = \mathrm{diag}\{t_1, \cdots, t_{n_p}\} \geqslant 0$ satisfying

$$\begin{bmatrix} A^{\mathrm{T}}P + PA & PB_p + A^{\mathrm{T}}C_q^{\mathrm{T}}\Lambda + C_q^{\mathrm{T}}LT \\ B_p^{\mathrm{T}}P + \Lambda C_q A + TLC_q & \Lambda C_q B_p + B_p^{\mathrm{T}}C_q^{\mathrm{T}}\Lambda - 2T \end{bmatrix} < 0, \tag{4}$$

then (1)$'$ is absolutely stable.

Lemma 1 follows from the statement of § 8.2 of [4] with a little difference.

Remark 1 When $n_p = 1$, the conditions in the above lemma are equivalent to the existence of Lyapunov function. When $n_p > 1$, the results are conservative.

Absolute stabilization problem: To design controllers such that the closed-loop Lur'e system is absolutely stable.

2 Main results

2.1 State feedback case

Suppose the Lur'e control signal p is measurable and consider feedback law $u = Kx + K_p p$. Thus, u is nonlinear since it is related to x via a nonlinear function. When p is not available, its gain K_p can be set to be 0 and the corresponding results for linear feedback controllers can be given.

The closed-loop system becomes

$$\begin{cases} \dot{x} = \bar{A}x + \bar{B}_p p, \\ q = C_q x, \end{cases} \tag{5}$$

where

$$\bar{A} = A + B_u K, \quad \bar{B}_p = B_p + B_u K_p.$$

Our goal is to find K and K_p such that (5) is absolutely stable.

Denote $Q: = P^{-1}$ and $KQ = R$. Applying Lemma 1 to the closed-loop system, the inequality (4) corresponding to the closed loop system (5) can be rewritten to the following form via matrix transformation.

$$\begin{bmatrix} R^{\mathrm{T}}B_u^{\mathrm{T}} + QA^{\mathrm{T}} + AQ + B_u R & B_p + B_u K_p + R^{\mathrm{T}}B_u^{\mathrm{T}}C_q^{\mathrm{T}}\Lambda + QA^{\mathrm{T}}C_q^{\mathrm{T}}\Lambda + QC_q^{\mathrm{T}}LT \\ B_p^{\mathrm{T}} + K_p^{\mathrm{T}}B_u^{\mathrm{T}} + \Lambda C_q AQ + \Lambda C_q B_u R + TLC_q Q & \Lambda C_q(B_p + B_u K_p) + (B_p + B_u K_p)^{\mathrm{T}}C_q^{\mathrm{T}}\Lambda - 2T \end{bmatrix} < 0. \tag{6}$$

Hence, we have the following result for absolute stabilization.

Theorem 1 If there exist Λ, T, $Q(>0)$, R and K_p satisfying (6), then there exists a state feedback nonlinear controller $u = Kx + K_p p (K = RQ^{-1})$ such that the closed-loop system is absolutely stable.

Proof is omitted for brevity.

Remark 2 (6) is a bilinear matrix inequality (BLMI) of K_p, Q, R, T and Λ. For BLMI, Goh, et al., presented local and global optimization methods in [5] and [6]. If the parameter T (related to S-procedure) and Λ of the Lyapunov function (3) is given, the condition is exactly a linear matrix inequality (LMI) for K_p, Q and R.

2.2 Dynamic output feedback case

Suppose that Σ_C is a nonlinear dynamical output feedback controller with order of n_c, whose state equation has the following form

$$\begin{cases} \dot{x}_c = A_c x_c + B_c y + B_{cp} p, \\ u = C_c x_c + D_c y + D_{cp} p. \end{cases} \tag{7}$$

Combine (7) with the plant (1), then the closed-loop system Σ_{cl} is described by

$$\begin{cases} \dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}_p p, \\ q = \bar{C}_q \bar{x}, \end{cases} \tag{8}$$

where

$$\bar{A} := \hat{A} + \hat{B}_u G \hat{C}_y, \quad \bar{C}_q = [C_q \quad 0],$$
$$\bar{B}_p = \hat{B}_p + \hat{B}_u \hat{D}_p = \begin{bmatrix} B_p + B_u D_{cp} \\ B_{cp} \end{bmatrix}, \tag{9}$$

and

$$\begin{bmatrix} \hat{A} & \hat{B}_u \\ \hat{C}_y & G^{\mathrm{T}} \end{bmatrix} := \left[\begin{array}{cc|cc} A & 0 & B_u & 0 \\ 0 & 0 & 0 & I_{n_c} \\ \hline C_y & 0 & D_c^{\mathrm{T}} & B_c^{\mathrm{T}} \\ 0 & I_{n_c} & C_c^{\mathrm{T}} & A_c^{\mathrm{T}} \end{array}\right],$$

$$\hat{B}_p = \begin{bmatrix} B_p \\ 0 \end{bmatrix}, \quad \hat{D}_p = \begin{bmatrix} D_{cp} \\ B_{cp} \end{bmatrix}. \tag{10}$$

Our objective is to find G and $\hat{D}_p$ such that (8) is absolutely stable.

Substitute the coefficient matrices of $\bar{G}$ for those in (4) correspondingly. Hence, (4) implies

$$\Omega + BGC + (BGC)^{\mathrm{T}} < 0, \tag{11}$$

where

$$B := \begin{bmatrix} P\hat{B}_u \\ \Lambda\bar{C}_q\hat{B}_u \end{bmatrix}, \quad C := [\hat{C}_y \quad 0],$$

$$\Omega := \begin{bmatrix} \hat{A}^{\mathrm{T}}P + P\hat{A} & \hat{M}_1 \\ \hat{M}_1^{\mathrm{T}} & \hat{M}_2 \end{bmatrix},$$

and

$$\hat{M}_1 = P\hat{B}_p + \hat{A}^{\mathrm{T}}\hat{C}_q^{\mathrm{T}}\Lambda + \Lambda\hat{B}_u\hat{D}_p + \hat{C}_q^{\mathrm{T}}LT,$$
$$\hat{M}_2 = \Lambda\hat{C}_q\hat{B}_p + \hat{B}_p^{\mathrm{T}}\hat{C}_q^{\mathrm{T}}\Lambda + \Lambda\hat{C}_p\hat{B}_u\hat{D}_p + \hat{D}_p^{\mathrm{T}}\hat{B}_u^{\mathrm{T}}\hat{C}_p^{\mathrm{T}}\Lambda - 2T.$$

Set $Q := P^{-1}$ and note that

$$B^{\perp} = \begin{bmatrix} \hat{B}_u \\ 0 \end{bmatrix}^{\perp} \begin{bmatrix} Q & 0 \\ -\Lambda\bar{C}_q Q & I \end{bmatrix},$$

and

$$\begin{bmatrix} \hat{C}_y^{\mathrm{T}} \\ 0 \end{bmatrix}^{\perp} = \begin{bmatrix} C_y^{\mathrm{T}\perp} & 0 & 0 \\ 0 & 0 & I \end{bmatrix},$$
$$\begin{bmatrix} \hat{B}_u \\ 0 \end{bmatrix}^{\perp} = \begin{bmatrix} B_u^{\perp} & 0 & 0 \\ 0 & 0 & I \end{bmatrix}, \tag{12}$$

where $B^{\perp}$ is defined as $B^{\perp}B = 0$ and $B^{\perp}B^{\perp\mathrm{T}} > I$. Partition P to be a block matrix compatible with (10),

$$P := \begin{bmatrix} X & P_{12} \\ P_{12}^{\mathrm{T}} & P_{22} \end{bmatrix}, \quad Y := (X - P_{12}P_{22}^{-1}P_{12}^{\mathrm{T}})^{-1}. \tag{13}$$

In this case

$$Q = P^{-1} = \begin{bmatrix} Y & * \\ * & * \end{bmatrix}$$

and

$$X - Y^{-1} = P_{12}P_{22}^{-1}P_{12}^{\mathrm{T}}. \tag{14}$$

It can be shown that Σ_{cl} is absolutely stable if and only if the following two formulas hold (see [4] for details)

$$\begin{bmatrix} C_y^{\mathrm{T}\perp} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A^{\mathrm{T}}X + XA & M_1 \\ M_1^{\mathrm{T}} & M_2 \end{bmatrix} \begin{bmatrix} C_y^{\mathrm{T}\perp\mathrm{T}} & 0 \\ 0 & I \end{bmatrix} < 0, \tag{15}$$

$$\begin{bmatrix} B_u^{\perp} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} YA^{\mathrm{T}} + AY & N_1 \\ N_1^{\mathrm{T}} & N_2 \end{bmatrix} \begin{bmatrix} B_u^{\perp\mathrm{T}} & 0 \\ 0 & I \end{bmatrix} < 0. \tag{16}$$

where

$$M_1 = XB_p + A^{\mathrm{T}}C_q^{\mathrm{T}}\Lambda + C_q^{\mathrm{T}}LT + XB_uD_{cp} + P_{12}B_{cp},$$
$$M_2 = \Lambda C_qB_p + B_p^{\mathrm{T}}C_q^{\mathrm{T}}\Lambda + \Lambda C_qB_uD_{cp} + D_{cp}^{\mathrm{T}}B_u^{\mathrm{T}}C_q^{\mathrm{T}}\Lambda - 2T,$$
$$N_1 = B_p - A^{\mathrm{T}}YC_q^{\mathrm{T}}\Lambda + YC_q^{\mathrm{T}}LT + B_uD_{cp},$$
$$N_2 = -\Lambda C_qYC_q^{\mathrm{T}}LT - TLC_qYC_q^{\mathrm{T}}\Lambda - 2T. \tag{17}$$

The following condition results from the decomposition of (13)

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geqslant 0. \tag{18}$$

Theorem 2 If there exist $\Lambda, T, X > 0, Y > 0, P_{12}, B_{cp}$ and D_{cp} satisfying (15), (16), (18) and the constraint (14) for an arbitrary $P_{22} > 0$, then there exists a nonlinear output feedback controller Σ_C such that the closed-loop system is absolutely stable. In this case, if we suppose the column dimension of P_{22} is r, the order of Σ_C is $n_c = r$.

Proof Based on the above derivation, it is shown that the conditions in Theorem 2 hold if and only if (4) (in Lemma 1) holds when the coefficient matrices of Σ_{cl} are substituted for (4). Hence, Σ_{cl} is absolutely stable. According to (10), (13) and (14), it is easy to show that n_c is equal to the dimension of P_{22}, i.e., r. Q.E.D.

2.3 On computation and design procedure

As to the constraint (14), we present a theorem in the following to show that this constraint can be removed without loss of generality.

Noting that P_{12} and B_{cp} which appear in (15) have the form of $P_{12}B_{cp}$, if we define $P_0 = P_{12}B_{cp}$, Theorem 2 can be simplified as follows.

Theorem 3 If there exist $\Lambda, T, X > 0, Y > 0$, P_0 and D_{cp} satisfying (15), (16) and (18), then there exists an output feedback controller Σ_C such that the closed-loop system is absolutely stable.

(Proof is omitted for brevity.)

Although the constraint is removed by Theorem 3, (15) is bilinear for the controller parameter D_{cp} and X. In the following, we consider a more tractable case, where D_{cp} in (7) is set to 0.

Thus, the submatrices in (15) and (16) can be simplified and (15) becomes linear for X, Λ, T and P_0. In addition, a static condition that should be satisfied is

$$\Lambda C_q B_p + B_p^{\mathrm{T}} C_q^{\mathrm{T}} \Lambda - 2T < 0. \tag{19}$$

The design method based on iteration algorithm can be presented correspondingly.

i) Solve LMI (19) to get a pair of Λ and T;

ii) Solve LMI (15) to get a pair of $X > 0$ and P_0, if (15) has no solutions for X and P_0, turn to step i);

iii) Solve LMI (16) and (18) to get $Y > 0$, if (16) and (18) have no solutions for Y, turn to step i);

iv) If $X - Y^{-1} > 0$ (otherwise, choose an appropriate e such that $(X + eI, Y)$ satisfies (15), (16) and (18) as well as $X + eI - Y_{-1} > 0$), denote $B_{cp} = P_0$, $P_{12} = I$ and $P_{22} = (X - Y^{-1})^{-1}$. Thus, P is obtained;

v) Solve G via LMI (11). Thus, the nonlinear controller is obtained.

Since each step in the above procedure is only involved in convex optimization of LMIs, this design method is feasible and traceable by use of LMI-toolbox in MATLAB.

In fact, the absolute stabilization problem stated in this paper is similar to the problem discussed by Savkin and Petersen (in [3]), if only linear feedback controllers are take into account. Three improvements have been made in this paper: First, the Lur'e system considered here has multiple nonlinear loops instead of single one. Second, the Lur'e control vector p is used for the feedback law and hence the controllers become nonlinear. Third, some assumptions such as controllability of (A, B_p), observability of (A, C_q) and $B_u^{\mathrm{T}} C_q^{\mathrm{T}} C_q B_u > 0$ supposed in [3] are removed here. If we confined the controllers to be linear, the results can be simplified as follows.

It is well known that Lur'e systems can be considered as linear systems with nonlinear uncertainties. Some robust control methods (for example, the approach discussed in [7]) can deal with Lur'e systems. However, a direct treatment on Lur'e systems may describe their properties much more explicitly. Moreover, when the Lur'e control variable p is needed for feedback controllers, the robust results for linear systems may be unavailable.

3 Conclusion

This paper discusses the absolute stabilization problem for Lur'e systems with multiple nonlinearities. Solvability conditions are presented in terms of multilinear matrix inequalities (MLMIs) for both state feedback and output feedback cases. Feasible design algorithms obtained in the paper are involved in optimization of LMI or BLMI, which have been studied in [4] and [5, 6] more sufficiently. The approach presented in this paper can be used to deal with other problems for Lur'e systems related to LMIs, such as L_2-gain control, passive control and other dissipative control problems.

References

1 Knobloch H W, Isidori A and Flockerzi D. Topics in Control Theory,

Switzerland, Birkhauser Verlag, 1993

2 Narendra K S and Taylor J H. Frequency Domain Criteria for Absolute Stability. New York: Academic Press, 1973

3 Savkin A V and Petersen I R. A method for robust stabilization related to the Popov stability criterion. Int. J. Control, 1995, 62(5): 1105 – 1115

4 Boyd S, EL Ghaoui L, Feron E and Balakrishnan V. Linear matrix inequality in system and control theory. Philadelphia, SIAM Studies in Applied Mathematics, 15: 1994

5 Goh K C, Turan L, Safonov M G et al. Bilinear matrix inequality properties and computational methods. Proc. of Conf. on ACC, Baltimore, 1994, 850 – 855

6 Goh K C, Safonov M G and Papavassilopoulos G P. A global optimization approach for the BMI problem. Proc. of Conf. Decision and Control, Tokyo, 1994, 2009 – 2014

7 Xie L, Fu M and de Souza C E. H^{∞} control and quadratic stabilization of systems with parameter uncertainties via output feedback. IEEE Trans. Automat. Contr., 1992, AC-37: 1253 – 1260

本文作者简介

郭 雷 见本刊 1999 年第 4 期第 624 页.

臧明磊 1965 年生. 现为复旦大学数学系研究生. 研究方向为优化方法与计算.

冯纯伯 见本刊 1999 年第 1 期第 126 页.

(Continued from page 787)

7 Gao Lin, Wang Chengyao, Wang Dingwei, Yin Zhisong and Wang Shuning. A production scheduling system for parallel machines in an electrical appliance plant. Computers & Industrial Engineering, 1998, 35 (1 – 2): 105 – 108

本文作者简介

高 林 1973 年生. 1993 年, 1996 年, 1999 年于东北大学自动控制系本科、硕士、博士毕业. 现为清华大学自动化系博士后. 主要研究方向为生产计划与调度系统的建模、优化与仿真, 智能优化方法, 计算机应用.

汪定伟 1948 年生. 1984 年华中理工大学自动控制系硕士毕业, 1993 年东北大学自动控制系博士毕业. 现为东北大学自动控制系教授、博士生导师. 主要研究方向为生产计划与调度理论, 建模与决策, 智能优化方法.

王书宁 1956 年生. 1982 年湖南大学电气工程系本科毕业, 1984 年, 1988 年华中理工大学自动控制系硕士、博士毕业. 现为清华大学自动化系教授、博士生导师. 主要研究方向为系统辨识, 参数估计, 优化理论和算法以及决策分析.