第 16 卷第 6 期
1999 年 12 月

控制理论与应用
CONTROL THEORY AND APPLICATIONS

Vol.16, No.6
Dec., 1999

Delay-Dependent Robust Stabilizing Control Law Design for a Class of Uncertain Time-Delay Systems

Su Hongye, Jiang Peigang, Wang Jingcheng and Chu Jian
(Institute of Industrial Process Control, Zhejiang University·Hangzhou, 310027, P.R.China)

Abstract: This paper studies the delay-dependent memoryless robust stabilization problem for a class of uncertain linear system with time-varying delays in both state and control. The uncertain time-delay system under consideration include time-varying unknown-but-bounded uncertain parameters. A new sufficient condition for the existence of memoryless robust stabilizing control law is derived. The rusult is dependent on the size of delays and given in terms of linear matrix inequalities (LMIs).

Key words: uncertain linear time-delay systems; uncertain; memoryless state feedback; delay-dependent; linear mattrix inequality (LMI)

一类不确定时滞系统的时滞依赖型鲁棒控制器设计

苏宏业　蒋培刚　王景成　褚　健
(浙江大学工业控制技术研究所·杭州, 310027)

摘要: 本文研究了一类状态和控制同时存在滞后的不确定线性系统时滞依赖型鲁棒镇定问题.针对所有容许的时变未知但有界的不确定性,得到了一种新的确保系统可鲁棒镇定的充分条件和相应的鲁棒镇定控制器设计方法.所得到的结果是与时滞相关的,并且以线性矩阵不等式(LMI)形式给出.

关键词: 不确定线性时滞系统; 鲁棒镇定; 状态反馈; 时滞依赖; 线性矩阵不等式(LMI)

1 Introduction

The problem of stabilizing uncertain linear time-delay system using state feedback, which contains unknown-but-bounded parameters, has attracted a considerable interest in recent years. The results in [1,2] are given in terms of the solution of a Riccati equation which involves the tuning of scalars and/or positive definite symmetric matrices, so these methods are somehow difficult for practical use because no tuning procedure for such scalar and matrices is available. Moreover these reported results are independent of the size of the delays and thus, in general, are conservative, especially when practically existing time-delays are small.

Recently, increasing attention has been devoted to the development of methods for delay-dependent robust stability analysis and robust stabilization synthesis for linear time-delay system, for they are less conservative[3]. And very recently, the linear matrix inequality (LMI) approach which has the advantage that no tuning of scalars and/or positive definite symmetric matrices is involved is proposed to treat the problem of robust stability analysis and robust stabilization synthesis for uncertain linear time-delay systems, and less conservative results dependent on the size of delays have been obtained[4,5]. But the uncertain linear time-delay system under consideration is only with a single state delay.

In this paper, we extend the linear matrix inequality (LMI) method to time-varying uncertain linear dynamic system with time-varying time-delays in both state and control. A method for designing a memoryless state feedback stabilizing control law is obtained.

2 System description

Consider the following uncertain time-delay system described by

$$\dot{x}(t)=(A+\Delta A(t))x(t)+(A_1+\Delta A_1(t))x(t-d(t))+(B+\Delta B(t))u(t)+(B_1+\Delta B_1(t))u(t-h(t)), \tag{1}$$

$$x(t)=\phi(t),\quad t\in[-\tau,0],$$

where $x(t)\in\mathbb{R}^n$, $u(t)\in\mathbb{R}^m$; $A\in\mathbb{R}^{n\times n}$, $A_1\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times m}$, $B_1\in\mathbb{R}^{n\times m}$ are known constant matrices.

Manuscript received Sep.12, 1997, revised Feb.4, 1999.

The matrices $\Delta A(\cdot), \Delta A_1(\cdot), \Delta B(\cdot)$ and $\Delta B_1(\cdot)$ are real-valued continuous matrix functions with appropriate dimensions; It is assumed that there exist positive numbers d^*, h^* and τ such that $0 \leqslant d(t) \leqslant d^* \leqslant \tau$, $0 \leqslant h(t) \leqslant h^* \leqslant \tau$ hold for all t; $\phi(t)$ is initial function.

In this paper, the admissible uncertainties are assumed to be of the following form

$$\Delta A(t) = H_1 F_1(t) E_1, \quad \Delta A_1(t) = H_2 F_2(t) E_2,$$
$$\Delta B(t) = H_3 F_3(t) E_3, \quad \Delta B_1(t) = H_4 F_4(t) E_4,$$

where $F_i(t) \in \mathbb{R}^{s_i \times q_i}, i = 1,2,3,4, F_1(t) = F_3(t)$ are unknown real time varying matrices satisfying $F_i^{\mathrm{T}}(t) F_i(t) \leqslant I, i = 1,2,3,4$. $H_i, E_i, i = 1,2,3,4$, $H_1 = H_3$ are known real constant matrices with appropriate dimensions.

To facilitate further development, we introduce

$$A(t) = A + \Delta A(t), \quad A_1(t) = A_1 + \Delta A_1(t),$$
$$B(t) = B + \Delta B(t), \quad B_1(t) = B_1 + \Delta B_1(t).$$

3 Delay-dependent robust stabilizing control law synthesis

In this section, we will present a new robust stabilizability criteria and address the synthesis problem of robust stabilization for the system (1).

Theorem 1 For the uncertain linear time-delay systems (1), the given scalars d^*, h^* and τ satisfying the given condition, this uncertain linear time-delay system is robustly stabilizable if there exist positive definite symmetric matrices X, Z and $P_{ij}, i = 1,2, j = 1,2,3$; matrix Y, scalars $\alpha_i > 0, i = 1,2,3,4; \beta_{ij} > 0, i = 1,2, j = 1,2,3; \varepsilon_2 > 0$ and $\varepsilon_4 > 0$ satisfying the following linear matrix inequalities (LMIs),

$$\begin{bmatrix} S & M \\ M^{\mathrm{T}} & N \end{bmatrix} < 0,$$

$$\begin{bmatrix} X & XA^{\mathrm{T}} + Y^{\mathrm{T}}B^{\mathrm{T}} & XE_1{}^{\mathrm{T}} + Y^{\mathrm{T}}E_3^{\mathrm{T}} \\ AX + BY & P_{i1} - \beta_{i1}H_1H_1^{\mathrm{T}} & 0 \\ E_1X + E_3Y & 0 & \beta_{i1} \end{bmatrix} \geqslant 0,$$

$i = 1,2$,

$$\begin{bmatrix} X & XA_1^{\mathrm{T}} & XE_2^{\mathrm{T}} \\ A_1X & P_{i2} - \beta_{i2}H_2H_2^{\mathrm{T}} & 0 \\ E_2X & 0 & \beta_{i2} \end{bmatrix} \geqslant 0,$$

$$\begin{bmatrix} X & Y^{\mathrm{T}}B_1^{\mathrm{T}} & Y^{\mathrm{T}}E_4^{\mathrm{T}} \\ B_1Y & P_{i3} - \beta_{i3}H_4H_4^{\mathrm{T}} & 0 \\ E_4Y & 0 & \beta_{i3} \end{bmatrix} \geqslant 0, \quad i = 1,2, \tag{2}$$

where

$$M = [M_1 \quad M_2 \quad M_3 \quad M_4],$$
$$N = -\operatorname{diag}(N_1 \quad N_2 \quad N_3 \quad N_4),$$
$$S = AX + XA^{\mathrm{T}} + A_1X + XA_1^{\mathrm{T}} + BY + Y^{\mathrm{T}}B^{\mathrm{T}} + B_1Y + Y^{\mathrm{T}}B_1^{\mathrm{T}} + \sum_{i=1}^{4} \alpha_i H_i H_i^{\mathrm{T}} + d^* \varepsilon_2 H_2 H_2^{\mathrm{T}} + d^* A_1 P_1 A_1^{\mathrm{T}} + h^* \varepsilon_4 H_4 H_4^{\mathrm{T}} + h^* B_1 Z B_1^{\mathrm{T}} + 3q(d^* + h^*)X,$$
$$M_1 = [XE_1^{\mathrm{T}} \quad XE_2^{\mathrm{T}}], \quad N_1 = \operatorname{diag}(\alpha_1 I, \alpha_2 I),$$
$$M_2 = [Y^{\mathrm{T}}E_3^{\mathrm{T}} \quad Y^{\mathrm{T}}E_4^{\mathrm{T}}], \quad N_2 = \operatorname{diag}(\alpha_3 I, \alpha_4 I),$$
$$M_3 = d^* A_1 P_1 E_2^{\mathrm{T}}, \quad N_3 = d^*(\varepsilon_2 I - E_2 P_1 E_2^{\mathrm{T}}),$$
$$M_4 = h^* B_1 Z E_4^{\mathrm{T}}, \quad N_4 = h^*(\varepsilon_4 I - E_4 Z E_4^{\mathrm{T}}).$$

Moreover, a suitable robust stabilizing control law is given by $u(t) = YX^{-1}x(t)$.

Proof For system (1), introducing the control law $u(t) = Kx(t)$, and the close-loop system can be written as

$$\begin{aligned} \dot{x}(t) = {} & [A(t) + B(t)K + A_1(t) + B_1(t)K]x(t) - \\ & \int_{-d(t)}^{0} A_1(t)\{[A(t+s) + B(t+s)K]x(t+s) + \\ & A_1(t+s)x(t-d(t)+s) + \\ & B_1(t+s)Kx(t-h(t)+s)\}\mathrm{d}s - \\ & \int_{-h(t)}^{0} B_1(t)K\{[A(t+s) + B(t+s)K]x(t+s) + \\ & A_1(t+s)x(t-d(t)+s) + \\ & B_1(t+s)Kx(t-h(t)+s)\}\mathrm{d}s, \end{aligned} \tag{3}$$

and let the system's Lyapunov functional be $V(x(t), t) = x^{\mathrm{T}}(t)Px(t)$, where $P = P^{\mathrm{T}} > 0$, and using Lemma 1 in [4] we have

$$\begin{aligned} \dot{V}(x(t), t) = {} & A^{\mathrm{T}}P + PA + A_1^{\mathrm{T}}P + PA_1 + \\ & K^{\mathrm{T}}B^{\mathrm{T}}P + PBK + K^{\mathrm{T}}B_1^{\mathrm{T}}P + PB_1K + \\ & P[\sum_{i=1}^{4} \alpha_i H_i H_i^{\mathrm{T}}]P + \sum_{i=1}^{2} \alpha_i^{-1} E_i^{\mathrm{T}} E_i + \\ & \sum_{i=3}^{4} \alpha_i^{-1} K^{\mathrm{T}} E_i^{\mathrm{T}} E_i K + \\ & \sum_{j=1}^{3} d^* x^{\mathrm{T}}(t) P A_1(t) P_{1j} A_1^{\mathrm{T}}(t) P x(t) + \end{aligned}$$

$$\int_{-d(t)}^{0} x^{T}(t+s)[A(t+s)+B(t+s)K]^{T} \cdot P_{11}^{-1}[A(t+s)+B(t+s)K]x(t+s)ds + \int_{-d(t)}^{0} x^{T}(t-d(t)+s)A_1^{T}(t+s) \cdot P_{12}^{-1}A_1(t+s)x(t-d(t)+s)ds + \int_{-d(t)}^{0} x^{T}(t-h(t)+s)K^{T}B_1^{T}(t+s) \cdot P_{13}^{-1}B_1(t+s)Kx(t-h(t)+s)ds + \sum_{j=1}^{3} h^{*} x^{T}(t)PB_1(t)KP_{2j}K^{T}B_1^{T}(t)Px(t) + \int_{-h(t)}^{0} x^{T}(t+s)[A(t+s)+B(t+s)K]^{T} \cdot P_{21}^{-1}[A(t+s)+B(t+s)K]x(t+s)ds + \int_{-h(t)}^{0} x^{T}(t-d(t)+s)A_1^{T}(t+s) \cdot P_{22}^{-1}A_1(t+s)x(t-d(t)+s)ds + \int_{-h(t)}^{0} x^{T}(t-h(t)+s)K^{T}B_1^{T}(t+s) \cdot P_{23}^{-1}B_1(t+s)Kx(t-h(t)+s)ds. \tag{4}$$

Using Lemma 2(b) in [4], we have

$$\begin{aligned} &\sum_{j=1}^{3} d^{*} A_1(t)P_{1j}A_1^{T}(t) \leqslant W_2, \\ &\sum_{j=1}^{3} h^{*} B_1(t)KP_{2j}K^{T}B_1^{T}(t) \leqslant W_3, \end{aligned} \tag{5}$$

where

$$W_2 = d^{*}[A_1P_1A_1^{T} + A_1P_1E_2^{T}(\varepsilon_2 I - E_2P_1E_2^{T})^{-1}E_2P_1A_1^{T} + \varepsilon_2 H_2H_2^{T}],$$

$$W_3 = h^{*}[B_1KP_2K^{T}B_1^{T} + B_1KP_2K^{T}E_4^{T}(\varepsilon_4 I - E_4KP_2K^{T}E_4^{T})^{-1}E_4KP_2K^{T}B_1^{T} + \varepsilon_4 H_4H_4^{T}],$$

$P_1 = \sum_{j=1}^{3} P_{1j}, P_2 = \sum_{j=1}^{3} P_{2j}$ and $P_{1j}P_{2j}, j = 1,2,3$ are any $n \times n$ real positive definite symmetric matrices satisfying the following inequalities

$$\begin{aligned} &\varepsilon_2 I - E_2P_1E_2^{T} > 0, \quad \forall t \geqslant 0, \\ &\varepsilon_4 I - E_4KP_2K^{T}E_4^{T} > 0, \quad \forall t \geqslant 0, \end{aligned} \tag{6}$$

where $\varepsilon_2 > 0, \varepsilon_4 > 0$ are positive scalars to be chosen.

Assume there exist scalars $\beta_{ij} > 0, i = 1,2, j = 1,2,3$ satisfying the following inequalities

$$(A+BK)^{T}(P_{i1} - \beta_{i1}H_1H_1^{T})^{-1}(A+BK) + \beta_{i1}^{-1}(E_1+E_3K)^{T}(E_1+E_3K) \leqslant P, \tag{7}$$

$$A_1^{T}(P_{i2} - \beta_{i2}H_2H_2^{T})^{-1}A_1 + \beta_{i2}^{-1}E_2^{T}E_2 \leqslant P, \tag{8}$$

$$K^{T}B_1^{T}(P_{i3} - \beta_{i3}H_4H_4^{T})^{-1}B_1K + \beta_{i3}^{-1}K^{T}E_4^{T}E_4K \leqslant P, \tag{9}$$

and

$$\begin{cases} P_{i1} - \beta_{i1}H_1H_1^{T} > 0; \\ P_{i2} - \beta_{i2}H_2H_2^{T} > 0; \\ P_{i3} - \beta_{i3}H_4H_4^{T} > 0, \end{cases} \tag{10}$$

where $i = 1,2$.

Then using Lemma 2 (c) in [4], we have

$$[A(t+s)+B(t+s)K]^{T}P_{i1}^{-1}[A(t+s) + B(t+s)K] \leqslant P, \quad \forall t \geqslant 0, \tag{11}$$

$$A_1^{T}(t+s)P_{i2}^{-1}A_1(t+s) \leqslant P, \quad \forall t \geqslant 0, \tag{12}$$

$$K^{T}B_1^{T}(t+s)P_{i3}^{-1}B_1(t+s)K \leqslant P, \quad \forall t \geqslant 0. \tag{13}$$

Following the Razumikhin-Type theorem, we assume that for some real scalar $q > 1$, the inequality $V[x(s),s] < qV[x(t),t], t - 2\tau \leqslant s \leqslant t$ holds, then applying (5),(6) and (7) ~ (9) to (4), and let $X = P^{-1}$ we get

$$\dot{V}(x(t),t) \leqslant x^{T}(t)X^{-1}W_1X^{-1}x(t),$$

where

$$W_1 = A^{T}P + PA + A_1^{T}P + PA_1 + K^{T}B^{T}P + PBK + K^{T}B_1^{T}P + PB_1K + \sum_{i=1}^{2}\alpha_i^{-1}E_i^{T}E_i + \sum_{i=3}^{4}\alpha_i^{-1}K^{T}E_i^{T}E_iK + 3q(d^{*}+h^{*})P + P[\sum_{i=1}^{4}\alpha_i^{-1}H_iH_i^{T} + d^{*}(A_1P_1A_1^{T} + A_1P_1E_2^{T}(\varepsilon_2 I - E_2P_1E_2^{T})^{-1}E_2P_1A_1^{T} + \varepsilon_2H_2H_2^{T}) + h^{*}(B_1KP_2K^{T}B_1^{T} + B_1KP_2K^{T}E_4^{T}(\varepsilon_4 I - E_4KP_2K^{T}E_4^{T})^{-1}E_4KP_2K^{T}B_1^{T} + \varepsilon_4H_4H_4^{T})]P.$$

Obviously, W_1 is monotonely increasing with respect to d^{*}, h^{*} and q (in the sense of positive definiteness), if let $W = W_1$ as $q = 1$.

Then if there exist positive definite symmetric matrices X, P_{ij} and scalars $\beta_{ij} > 0, i = 1,2, j = 1,2,3; \alpha_i > 0, i = 1,2,3,4; \varepsilon_2 > 0$ and $\varepsilon_4 > 0$ satisfying the inequalities (6) ~ (10) and $W < 0$, then there must exist a suffciently small $q > 1$ such that $W_1 < 0$ for any $0 \leqslant d(t) \leqslant d^{*}$ and $0 \leqslant h(t) \leqslant h^{*}$, which implies $\dot{V}(x(t),t) \leqslant -\alpha \| x(t) \|^2$. Then, it follows from Razumikhin-Type theorem that the close-loop system is globally uniformly asymptotically stable.

Let $Y = KX, Z = KP_2K^{T}$ and using Schur complements, we obtain the result that inequalities (6) ~ (10) and $W < 0$ are equivalent to those LMIs in (2).

Remark 1 Theorem 1 provides a delay-dependent condition for robust stabilizability of the uncertain linear time-delay system (1) and the correspondent delay-dependent linear memoryless state feedback control law synthesis approach based on LMI technique. Since robustly stabilizing control law is dependent of the size of the time-delays, in general, it is expected to be less conservative than the delay-independent robust stabilizing control law design methods of [1,2]. In contrast with the result in [2], which developed delay-dependent robust stabilization method for uncertain time-delay systems in terms of the solution of Riccati equations, the obtained result in Theorem 1 is given in terms of the solutions of linear matrix inequalities and does not need any tuning of parameters, and it can be calculated very effectively by using interior point algorithms.

4 Illustrative example

In this section we present an example to illustrate the delay-dependent robustly stabilizing control law design procedure developed in this paper. Considering the linear uncertain time-delay system described by:

$$A = \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}, \quad A_1 = \begin{pmatrix} 0 & 0 \\ 0.1 & 0.1 \end{pmatrix},$$

$$B = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0.1 \\ 0.2 \end{pmatrix},$$

$$d^* = 0.6, \quad h^* = 0.3,$$

$$H_1 = H_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad H_2 = H_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

$$E_1 = E_3 = \begin{pmatrix} 0.1 \\ 0.1 \end{pmatrix}^{\mathrm{T}}, \quad E_2 = E_4 = 0.1,$$

with these parameter, the following solutions were obtained:

$$X = \begin{pmatrix} 0.4950 & -0.2794 \\ -0.2794 & 0.5410 \end{pmatrix}, \quad Y = \begin{pmatrix} -0.3924 \\ 0.0514 \end{pmatrix}^{\mathrm{T}}.$$

Furthermore, we obtained the memoryless state feedback stabilizing control law:

$$K = YX^{-1} = (-1.0432 \quad -0.4436).$$

5 Conclusion

This paper deals with the problem of robust stabilization synthesis for a class of uncertain linear time-delay system. Delay-dependent LMI based method of designing linear memoryless state feedback control law has been developed. A new criteria of delay-dependent robustly stabilizable for uncertain time-delay systems is given in terms of linear matrix inequality (LMI).

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本文作者简介

苏宏业 见本刊 1999 年第 3 期第 344 页.

蒋培刚 1976 年生. 1996 年毕业于西安交通大学自动控制系, 现为浙江大学工业控制技术研究所博士研究生. 主要研究兴趣是鲁棒控制, 时滞系统控制理论和应用研究.

王景成 见本刊 1999 年第 3 期第 344 页.

褚　健 见本刊 1999 年第 3 期第 344 页.