Elimination of Position-Dependent Disturbances in Constant-Speed-Rotation Control Systems
—A Nonlinear Repetitive Control Approach*

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Abstract: To eliminate position-dependent disturbances in a constant-speed-rotation system, we have proposed a very effective approach that introduces a new concept called the "position domain". However, because the position domain model of a linear plant is in fact nonlinear, a linear control strategy may not meet the design requirements. To solve this problem, this paper proposes a nonlinear repetitive control approach. The procedure for designing a control system comprises two steps. First, an input-output linearization approach is used to linearize the input-output characteristics of the nonlinear plant in the position domain. Then, a repetitive controller is designed for the linearized plant to ensure good performance.

Key words: constant-speed-rotation system; position-dependent disturbances; position domain; input-output linearization; repetitive control; H_∞ control

恒速旋转控制系统中位置相关扰动的抑制
——一种非线性重复控制方法

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摘要: 为了抑制恒速旋转控制系统中的位置相关扰动,我们提出了一种非常有效的方法,它引入一种称为"位置域"的新概念.但是,由于一个线性对象的位置域模型实际上是非线性的,因而线性控制策略有时不能满足设计要求.本文提出一种非线性重复控制方法来解决这个问题.设计控制系统的过程分为两步,首先是利用输入-输出线性化方法把位置域非线性对象的输入-输出特性线性化,然后针对线性化对象设计一种重复控制器,以保证好的控制性能.

关键词: 恒速旋转控制系统; 位置相关扰动; 位置域; 输入-输出线性化; 重复控制; H_∞控制

1 Introduction

In constant-speed-rotation systems, the rotational speed must track the reference speed rapidly and precisely. However, fluctuations in the rotational speed, which are frequently caused by position-dependent disturbances, have hindered all efforts to improve the precision of such systems. These fluctuations are caused by such things as the non-uniformity of the magnetic flux in DC motors and eccentricity in the structure of the rotation systems. We have already proposed a method to eliminate this kind of disturbance[1]. Focusing on the fact that this kind of disturbance constitutes a periodic function of the rotational angle, we have defined a new concept called the "position domain", which is a set in which every element is a function of the rotational angle; and have carried out the design of the proposed constant-speed-rotation control system in this new domain so as to eliminate such disturbances completely regardless of the rotational speed.

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A linear plant in the time domain, however, is nonlinear in the position domain. The proposed design method for repetitive control systems was based on a linearized model at a standard rotational speed. So, a difference arises between the nominal plant and the real plant when the rotational speed differs from the standard speed. This degrades the transient response, and the control system may even become unstable if the difference in speed is too big.

As one solution of this problem, this paper proposes a design method based on a nonlinear model of the plant in the position domain. The proposed control system contains a double control loop. The inner control loop exactly linearizes the input-output characteristics of the plant, and the outer control loop ensures good performance for the linearized plant.

Notation and definitions λ: delay operator; $\mathbb{RH}_\infty$: set of real-rational functions in λ which have no poles in the closed unit circle; $\mathbb{R}[\lambda]$: ring of polynomials in λ ($\subset \mathbb{RH}_\infty$); $\|G(\lambda)\|_\infty := \sup_{0\leqslant\phi\leqslant 2\pi} |G(e^{j\phi})|$ ($G(\lambda) \in \mathbb{RH}_\infty$).

2 Discrete model of the rotation system in the position domain

Consider a three-input, one-output stabilizable and detectable rotation system in the time domain

$$\begin{aligned} dx(t)/dt &= Ax(t) + Bu(t) + \Gamma v_1(t), \\ \omega(t) &= \Psi x(t) + v_2(t), \end{aligned} \tag{1}$$

where $x(t)$ is the state, $u(t)$ is the control input, $\omega(t)$ is the rotational speed, and $v_1(t)$ and $v_2(t)$ are position-dependent disturbances. The following assumptions are made in the time domain.

A1) All zeros of the transfer function from $u(t)$ to $\omega(t)$ are stable.

A2) The relative degree of the transfer function from $u(t)$ to $\omega(t)$ is equal to one.

A3) The direction of rotation is unchanged. Whithout loss of generality, the direction of rotation is designated to be the positive direction.

A1) is generally satisfied for a rotation system. A2) is used for simplicity. It is well known that a multi-sampling technique[2] can be used to eliminate A2). A3) guarantees the existence of the inverse function $t := t(\theta)$ of $\theta = \theta(t)$. This assumption guarantees that the stability defined in the position domain is the same as that defined in the time domain.

The solution of the state equation in (1) is

$$x(t_{i+1}) = e^{A(t_{i+1}-t_i)}x(t_i) + \int_{t_i}^{t_{i+1}} e^{A(t_{i+1}-t)}Bu(t)dt + \int_{t_i}^{t_{i+1}} e^{A(t_{i+1}-t)}\Gamma v_1(t)dt. \tag{2}$$

Considering the relationship between time and the rotational angle

$$\frac{d\theta}{dt} = \omega(t) = \omega(t(\theta)) := \omega(\theta), \quad t = \int_0^\theta \frac{d\theta}{\omega(\theta)}, \tag{3}$$

and setting the sampling period with respect to the rotational angle to be $\Delta\theta := \theta_{i+1} - \theta_i = \theta(t_{i+1}) - \theta(t_i) = \text{const.}$, we obtain the following discrete state equation in the position domain for a zero-order hold:

$$\begin{aligned} x_{i+1} = {} & \exp\left(A\int_{\theta_i}^{\theta_{i+1}} \frac{d\theta}{\omega(\theta)}\right)x_i + \\ & \left[\int_{\theta_i}^{\theta_{i+1}} \exp\left(A\int_{\theta}^{\theta_{i+1}} \frac{d\theta}{\omega(\theta)}\right)B\frac{d\theta}{\omega(\theta)}\right]u_i + \\ & \int_{\theta_i}^{\theta_{i+1}} \exp\left(A\int_{\theta}^{\theta_{i+1}} \frac{d\theta}{\omega(\theta)}\right)\Gamma v_1(\theta)\frac{d\theta}{\omega(\theta)}. \end{aligned} \tag{4}$$

Sampling the output equation in (1) in the position domain allows the discrete output equation to be

$$\omega_i = \Psi x_i + v_{2i} \tag{5}$$

(4) and (5) are the discrete form of the rotation system in the position domain. Clearly, this transformation makes a linear plant nonlinear. The advantage of performing the transformation is that the disturbances v_1 and v_2 become periodic functions in the position domain. So, their effect on the rotational speed can be eliminated by a repetitive controller[3] with the same period as that of the disturbances.

3 Control system design in the position domain

The configuration of the constant-speed-rotation control system is shown in Fig. 1, where the controller

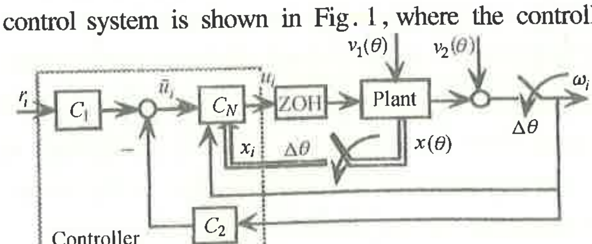

Fig. 1 Configuration of constant-speed-rotation control system

C_N is designed so as to linearize the input-output characteristics of the plant exactly, and the controller $[C_1 \quad C_2]$ is designed so as to eliminate the disturbances and provide the desired performance.

3.1 Input-output linearization—inner loop design

In order to eliminate disturbances over a wide range of speeds, the nonlinearity of the plant has to be taken into account. The input-output linearization approach[4] is employed to deal with the nonlinearity. The condition for the input-output exact linearization is given in [4]. For our case, the condition becomes

$$\Psi\int_{\theta_i}^{\theta_{i+1}}\exp\left(A\int_{\theta}^{\theta_{i+1}}\frac{\mathrm{d}\theta}{\omega(\theta)}\right)\frac{\mathrm{d}\theta}{\omega(\theta)}B \neq 0.$$

That holds for almost all sampling periods.

Choosing a $\Phi(x_i)$ that satisfies

$$\left|\partial\begin{bmatrix}\Psi x_i\\ \Phi(x_i)\end{bmatrix}/\partial x_i\right| \neq 0, \tag{6}$$

and carrying out the coordinate transformation and the input transformation

$$\begin{bmatrix}\xi_i\\ \eta_i\end{bmatrix} = \begin{bmatrix}\Psi x_i\\ \Phi(x_i)\end{bmatrix},$$

$$u_i = \frac{\tilde{u}_i - \Psi\exp\left(A\int_{\theta_i}^{\theta_{i+1}}\frac{\mathrm{d}\theta}{\omega(\theta)}\right)x_i}{\Psi\int_{\theta_i}^{\theta_{i+1}}\exp\left(A\int_{\theta}^{\theta_{i+1}}\frac{\mathrm{d}\theta}{\omega(\theta)}\right)\frac{\mathrm{d}\theta}{\omega(\theta)}B}, \tag{7}$$

yields

$$\begin{bmatrix}\xi_{i+1}\\ \eta_{i+1}\end{bmatrix}=\begin{bmatrix}\tilde{u}_i+\Psi\int_{\theta_i}^{\theta_{i+1}}\exp\left(A\int_{\theta}^{\theta_{i+1}}\frac{\mathrm{d}\theta}{\omega(\theta)}\right)\Gamma v_1(\theta)\frac{\mathrm{d}\theta}{\omega(\theta)}\\ \bar{q}(\xi_i,\eta_i,\tilde{u}_i,v_1)\end{bmatrix},$$

$$\omega_i = \xi_i + v_{2i}. \tag{8}$$

The input-output characteristics of (8) are given by

$$\omega_{i+1} = \tilde{u}_i+\Psi\int_{\theta_i}^{\theta_{i+1}}\exp\left(A\int_{\theta}^{\theta_{i+1}}\frac{\mathrm{d}\theta}{\omega(\theta)}\right)\Gamma v_1(\theta)\frac{\mathrm{d}\theta}{\omega(\theta)}+v_{2(i+1)}. \tag{9}$$

They are clearly linear from the new control input $\tilde{u}_i$ to the output ω_i.

On the other hand, the state η_i is unobservable. To stabilize the linearized plant, the state η_i must be stable. That is, the zero dynamics $\eta_{i+1} = Q(\xi_0, \eta_i, u_0, 0, 0)$ must be stable at the equlibrium point $(\xi_i, \eta_i, u_i, v_{1i}, v_{2i}) = (\xi_0, \eta_0, u_0, 0, 0)$. The following result is useful for the stability test.

Lemma[5] With a zero-order hold and a small enough sampling interval, the zero dynamics of a sampled system is stable if and only if the original continuous zero dynamics is stable.

We use this lemma to verify the stability of the linearized plant. A simple calculation (details are given in [8]) shows that the eigenvalues of the linear approximate zero dynamics of the continuous plant in the position domain are the zeros of the plant in the time domain scaled by the scaling factor $1/\omega_0\ (>0)$. Due to A1), all the eigenvalues are stable. Thus, the zero dynamics is asymptotically stable. According to the Lemma, a small sampling interval ensures that the zero dynamics of the discrete plant is stable. So, (4) and (5) is stablilizable.

3.2 Repetitive control—outer loop design

Let T_1 and T_2 be the periods of v_1 and v_2, respectively, then, T can be chosen as the least common multiple of T_1 and T_2. For a suitable choice of the sampling period $\Delta\theta$, the number of steps of the repetitive controller is $L = T/\Delta\theta$, and the repetitive controller is $1/(1-\lambda^L)$. According to the internal model principle, the outer loop controller $C = [C_1, C_2]$ has to contain the factor $(1-\lambda^L)$ in its denominator in order to eliminate the disturbances and achieve zero tracking error.

Carrying out a coprime factorization on the pulse transfer function of the linearized plant gives

$$P = ND^{-1},\quad N = \lambda,\quad D = 1,\quad N, D \in \mathbb{R}\mathrm{H}_\infty. \tag{10}$$

If $X, Y' \in \mathbb{R}\mathrm{H}_\infty$ are chosen to be $X = \lambda^{L-1}$ and $Y' = 1$, then the Bezout equation $XN + [(1-\lambda^L)Y']D = 1$ is satisfied. Therefore, the repetitive controller can be parametrized as

$$\begin{aligned}C &= [C_1 \quad C_2] = \\ &\{(1-\lambda^L)Y'-(1-\lambda^L)K_2N\}^{-1}[K_1 \quad X+(1-\lambda^L)K_2D] = \\ &\{(1-\lambda^L)(1-\lambda K_2)\}^{-1}[K_1 \quad \lambda^{L-1}+(1-\lambda^L)K_2];\\ &K_1, K_2 \in \mathbb{R}\mathrm{H}_\infty.\end{aligned} \tag{11}$$

3.2.1 Design of parameter K_1

K_1 is determined by the method proposed by [6] and [7] to achieve dead-beat control that moderately restricts the input-output error and the control input within the settling time. It can be summarized as follows.

For step-type command input, the ripple-free conditions are, the tracking error $e(\lambda) := r(\lambda) - y(\lambda)$ is a finite polynomial of λ; and the pulse transfer function from $r(\lambda)$ to $\tilde{u}(\lambda)$ is a finite polynomial of λ.

The parameter K_1 which satisfies these conditions can be parametrized as

$$K_1 = 1 + (1-\lambda)\bar{K}_1; \quad \bar{K}_1 \in \mathbb{R}[\lambda], \tag{12}$$

where $\bar{K}_1$ can be any polynomial. Thus, we can choose an appropriate polynomial $\bar{K}_1 \neq 0$ to optimize the transient response. Let

$$\bar{K}_1 = \bar{k}_0 + \bar{k}_1\lambda + \cdots + \bar{k}_q\lambda^q = \sum_{i=0}^{q} \bar{k}_i\lambda^i. \tag{13}$$

Then we have

$$\begin{aligned} e &= r - y = \frac{1 - NK_1}{1-\lambda} = 1 - \lambda\bar{K}_1 := \sum_{i=0}^{q+1} e_i\lambda^i, \\ \Delta\tilde{u} &:= (1-\lambda)\tilde{u} = 1 - \lambda + (1-\lambda)^2\bar{K}_1 := \sum_{i=0}^{q+2} \Delta\tilde{u}_i\lambda^i. \end{aligned} \tag{14}$$

Here, $\bar{K}_1$ is chosen to minimize the following transient response performance index:

$$J_1 = \sum_{i=0}^{q+2}\{|e_i|^2 + \rho^2|\Delta\tilde{u}_i|^2\}. \tag{15}$$

3.2.2 Design of parameter K_2

$$v_i := \Psi\int_{\theta_i}^{\theta_{i+1}} \exp\left(A\int_{\theta}^{\theta_{i+1}} \frac{\mathrm{d}\theta}{\omega(\theta)}\right)\Gamma v_1(\theta)\frac{\mathrm{d}\theta}{\omega(\theta)} + v_{2(i+1)} \tag{16}$$

represents the equivalent position-dependent disturbance. A simple calculation shows that its effect on the rotational speed is

$$\omega_v = (1 + PC_2)^{-1}v = Sv. \tag{17}$$

Since the disturbances are periodic, a weighting function W is chosen to be

$$W = (1 - \lambda^L)^{-1}\tilde{W}, \tag{18}$$

and the goal of elimination disturbances can be achieved by minimizing the following performance index:

$$J_2 = \|WS\|_\infty = \|W(1+PC_2)^{-1}\|_\infty = \|\tilde{W}(1-\lambda K_2)\|_\infty. \tag{19}$$

4 Simulation

Consider the following plant

$$\begin{aligned} &J\mathrm{d}\omega/\mathrm{d}t + D\omega = B_u u + B_v v_1, \\ &J = 0.1, \quad D = 1, \quad B_u = B_v = 1. \end{aligned} \tag{20}$$

The position-dependent disturbance is assumed to be

$$v_1(\theta) = 2\sin(\theta(t)) + \sin(2\theta(t)) + 0.5\sin(3\theta(t)). \tag{21}$$

The number of steps of the repetitive controller and the sampling interval are $L = 36$ and $\Delta\theta = 2\pi/L = 2\pi/36 = 0.008727\mathrm{rad}$, respectively. The state space representation of this plant is

$$A = -10, \quad B = 10, \quad \Gamma = 10, \quad \Psi = 1. \tag{22}$$

Following the design procedure in Section 3, we obtain the inner loop linearization controller

$$u_i = \frac{\tilde{u}_i - \Psi\exp\left(\int_{\theta_i}^{\theta_{i+1}}\frac{\mathrm{d}\theta}{\omega(\theta)}\right)x_i}{\Psi\int_{\theta_i}^{\theta_{i+1}}\exp\left(A\int_{\theta}^{\theta_{i+1}}\frac{\mathrm{d}\theta}{\omega(\theta)}\right)\frac{\mathrm{d}\theta}{\omega(\theta)B}} \approx \frac{\tilde{u}_i - \exp\left(-\frac{10\Delta\theta}{\omega_i}\right)\omega_i}{1 - \exp\left(-\frac{10\Delta\theta}{\omega_i}\right)}, \tag{23}$$

and the parameters $\rho = 1$ and $q = 11$, and $\tilde{W} = 1$ are used to design $\bar{K}_1$ and K_2, respectively.

The simulation results are presented in Fig. 2 and Fig. 5. In addition, the simulation results obtained by using the design method for linear control[1] are shown in Fig. 3 and Fig. 4 (Standard speed: 20rad/s).

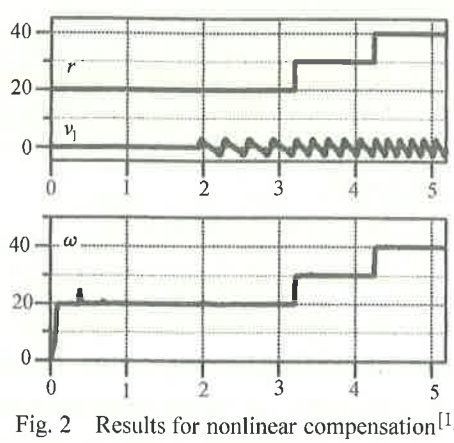

Fig. 2 Results for nonlinear compensation[1]

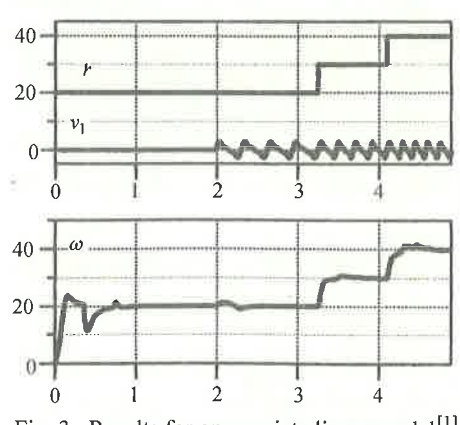

Fig. 3 Results for appropriate linear model[1]

In Fig. 2, the rotational speed was first increased from 0 to 20 rad/s. After the system reached the steady-state, the position-dependent disturbances (21) were input. Then, the rotational speed was further increased from 20 rad/s to 30 rad/s, and from 30 rad/s to 40 rad/s. For comparison, the response for the linear control method is shown in Fig. 3. Clearly, the control system using the nonlinear compensation method has both smaller overshoots during the transient response and also a shorter response time.

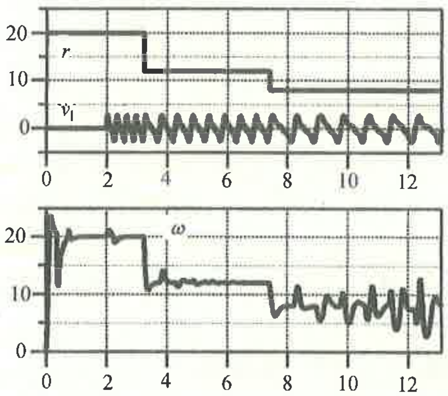

Fig. 4 Results for appropriate linear model[2]

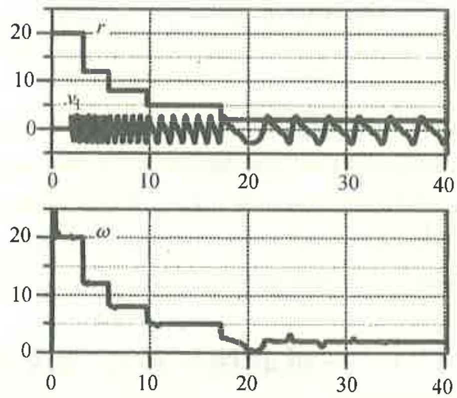

Fig. 5 Results for nonlinear compensation [2]

For the linear control method, the system loses stability at low rotational speeds. It is clear from Fig. 4 that the system cannot track a reference input of 8 rad/s stably. In contrast, the nonlinear compensation method can even track a reference input of 2 rad/s stably (Fig. 5).

From the simulation results, we can see that nonlinear compensation improves not only the stability but also the transient response of the system.

5 Conclusion

This paper presents a design method combining input-output linearization and repetitive control techniques for constant-speed-rotation control systems. The validity of the proposed method has been demonstrated through simulations.

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