Identification of Nonparametric GFRF Model for a Class of Nonlinear Dynamic Systems *

Han Chongzhao, Wang Liqi, Tang Xiaoquan and Dang Yingnong
(School of Electronic and Information Engineering, Xi'an Jiaotong University·Xi'an, 710049, P.R.China)

Abstract: This paper presents the nonparametric identification algorithm of GFRF models for a class of nonlinear dynamic systems with polynomial form. The distinguishing feature of the algorithm is to desire a limited calculating quantity and store space, and the identification accuracy is high. The simulation results show that the identification algorithm is very effective, and the obtained model is of excellent generalization ability in general. So, it's an approach with important prospect for application.

Key words: polynomial nonlinear system; generalized frequency response function; nonparametric model identification

一类非线性动态系统的非参数 GFRF 模型辨识

韩崇昭　王立琦　唐晓泉　党映农
(西安交通大学电子与信息工程学院·西安, 710049)

摘要: 本文对一类用多项式描述的非线性动态系统提出使用 GFRF 模型类的非参数辨识算法. 这种算法的显著特点是需要很小的计算量和存储空间, 而且辨识精度较高. 仿真结果表明这种非参数模型辨识算法是有效的, 而且由辨识方法获得的模型一般具有很好的泛化能力, 因而是一种具有重要应用前景的实用方法.

关键词: 多项式非线性系统; 广义频率响应函数(GFRF); 非参数模糊辨识

1 Introduction

Recent reseaches provide the new train of thoughts for the nonlinear control system analysis, and have got some results on nonlinear system identification as well as practical application (Barrett[1], Schetzen[2], Billings and Tsang[3~5]), in which the behavior of a nonlinear dynamic system can be described by the generalized frequency response functions (GFRF'S). In the case of the frequency response function (FRF) of a linear system, the GFRF's belong to the nonparametric model and do not depend on the input signals, so they can describe the essential properties of the system completely. The GFRF's can reveal some typical frequency response properties of nonlinear systems, such as harmonics, intermodulation, gain compression/expansion, etc., and the figure showing of GFRF's can describe the essential properties more directly.

There are two ways to obtain GFRF model for a nonlinear dynamic system. One is to calculate it by using the recursive formula based on the known time-domain model, and another is to identify it from the input and output data. Up to now, the types of nonlinear time domain models by which GRFR model can be calculated directly are NARMAX, NIDE and NDE models etc.. The identification problem of GFRF's is more complicated because there are infinite terms of GFRF's for describing the system completely according to the Volterra functional series theory. And the scale of data needed in identification will increase exponentially with the system order and degree increasing.

Just for the reasons stated above, it is still a difficult problem to identify the GFRF's model for a nonlinear dynamic system, not as in the case of its time-domain model identification. Compared with the nonparametric model identification in linear system case, the identification of GFRF's model of nonlinear system is much more complicated. Therefore, it is of great value to solve the dimensional catastrophe problem in GFRF's identification.

Since the beginning of the 80's, the nonlinear spectral analysis theory based on Volterra functional series has been developed quickly. In 1980, the theoretical frame of

* This research is supported by the National Natural Science Foundation of China(69674009).
Manuscript received May 13, 1997, revised Jan.25, 1999.

plexity will not increase when the dimension of a system increase except the longer time of calculation. The disadvantage is that the way can only be used in one-machine system. How to expand it to multi-machine system is one of our future concerns with the field under discussion.

References

1 Kimemia J and Gershwin S B. An algorithm for the computer control of a flexible manufacturing system. II Transactions, 1983, 15(4): 353 – 362

2 Akella R and Kumar P R. Optimal control of production rate in a failure prone manufacturing system. IEEE Trans. Automat. Contr, 1986, 31 (2): 116 – 126

3 Caramanis M and Sharifnia A. Near optimal manufacturing flow controller design. The International Journal of FMS, 1991, 3(4): 321 – 336

4 Gershwin S B. Manufacturing systems engineering. Englewood Cliffs, New Jersey: PTR Prentice Hall, 1994

5 Lawrences ID (ed). Genetic algorithms and simulated annealing. London: Pitman, 1987

6 Michalewicz Z, Janikow C Z and Krawczyk J B. A modified Genetic Algorithm for optimal control problems. Computers Mathematics Application, 1992, 23(1): 83 – 94

7 Bielecki T and Kumar P R. Optimality of zero-inventory policies for unreliable manufacturing systems. Operations Research, 1987, 36(4): 532 – 541

8 Janikow C Z and Michalewicz Z. An experimental comparison of binary and floating point representations in Genetic Algorithms. Proceeding of the Fourth international conference on Genetic Algorithms, University of CaliFornia. San Diego, 1991

本文作者简介

张　鹏　1964年生．中科院自动化研究所博士，现为北京东方电子集团高级工程师．主要研究兴趣为控制理论的应用，计算机集成制造系统，物流控制．

郑应平　1941年生．现为中科院自动化研究所研究员、博士生导师．主要研究兴趣为复杂系统控制理论及其应用．

(Continued from page 819)

IEEE Trans. Signal Processing, 1992, 40(5): 1029 – 1040

9 Nam S W and Power E J. Application of higher order spectral analysis to cubically nonlinear system identification. IEEE Trans. Signal Processing, 1994, 42(7): 1746 – 1765

10 Ching-Hsiang Tseng. A mixed-domain method for identification of quadratically nonlinear Systems. IEEE Trans. Signal Processing, 1997, 45(4): 1013 – 1024

11 Sungbin Im and and Power E J. A fast method of discrete third-order Volterra filtering. IEEE Trans. Signal Processing, 1996, 44(9): 2195 – 2208

12 Bendat J S. Non-linear System Analysis and Identification from random-data John Wiley & Soms, 1990

本文作者简介

韩崇昭　见本刊1999年第3期第451页．

王立琦　1961年生．博士，讲师，1982年毕业于西北工业大学自动控制系，1985年在西安交通大学获硕士学位，1998年在西安交通大学获博士学位．主要研究领域是系统辨识、非线性系统分析等．

唐晓泉　博士生，1982年毕业于四川泸州化工学校，1995年在西安矿业学院获硕士学位，现在西安交通大学攻读博士学位．主要研究领域是非线性系统分析等．

党映农　1973年生．博士生，1991年毕业于西安交通大学自控系，1997年在西安交通大学获硕士学位，现在西安交通大学攻读博士学位．主要研究领域是非线性系统分析等．

Volterra kernel identification was proposed by Fakhouri[7]. Up to the 90's, the algorithm for the second-order spectral estimation was described by Cho[8], and the third-order spectral estimation by Nam[9]. Now, the new contributions in utility and efficiency of the estimation algorithms have been made[10,11].

In this paper, a new simplified nonparametric model identification algorithm of GFRF's will be presented based on the assumptions that the nonlinear dynamic system can be described by a three-order and three-degree model completely. The algorithm needs very little calculations and store spaces, but is of higher precision and excellent ability of model generalization.

The simulation results show that this identification algorithm is more effective, and therefore it has a great important prospect for application.

2 Description of generalised frequency response functions (GFRF's) model

The SISO polynomial nonlinear system can be represented by a nonlinear differential equation as

$$\sum_{n=1}^{N}\{\sum_{p_1=0}^{M}\cdots\sum_{p_n=0}^{M}[a_{n,p_1,\cdots,p_n}\prod_{i=1}^{n}D^{p_i}y(t)+ b_{n,p_1,\cdots,p_n}\prod_{i=1}^{m}\prod_{k=m+1}^{n}D^{p_i}y(t)\cdot D^{p_k}u(t)+ c_{n,p_1,\cdots,p_n}\prod_{i=1}^{n}D^{p_i}u(t)]\}=0, \tag{1}$$

where D is the differential operator, M is the maximum differential order and N is the maximum degree of the differential equation, $u(t)$ and $y(t)$ are the input and output of system separately, and a_n, b_n and c_n are the coefficients in the output terms, input terms and cross terms respectively.

If (1) has a Volterra series solution, then the output $y(t)$ in time domain can be represented as

$$y(t)=\sum_{n=1}^{\infty}y_n(t),$$

$$y_n(t)=\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}h_n(\tau_1,\cdots,\tau_n)\cdot \prod_{i=1}^{n}u(t-\tau_i)\mathrm{d}\tau_i,\ n\in\mathbb{N}, \tag{2}$$

where $\mathbb{N}$ is natural number set, $\{h_n(\tau_1,\tau_2,\cdots,\tau_n), n\in\mathbb{N}\}$ are the Volterra kernels or the generalized impulse response functions. Taking Fourier transform to above equations yields ones as follows

$$\hat{y}(\omega)=F(y(t))=\sum_{n=1}^{\infty}F(y_n(t))=\sum_{n=1}^{\infty}\hat{y}_n(\omega),$$

$$\hat{y}_n(\omega)=(2\pi)^{-(n-1)}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\hat{h}_n(\omega-\omega_2-\cdots-\omega_n,\omega_2,\cdots,\omega_n)\cdot\hat{u}(\omega-\omega_2-\cdots-\omega_n)\cdot \hat{u}(\omega_2)\cdots\hat{u}(\omega_n)\mathrm{d}\omega_2,\cdots,\mathrm{d}\omega_n,\quad n\in\mathbb{N}, \tag{3}$$

where $\hat{y}$, $\hat{y}_n$ and $\hat{u}$ are the Fourier transforms of y, y_n and u respectively. $\{\hat{h}_n, n\in\mathbb{N}\}$ is said to be the General Frequency Response Functions (GFRF's) of the system. And $\hat{h}_n$ are the n-dimensional Fourier transforms of the Volterra kernel h_n for $n\geqslant 2$, which are defined as

$$\hat{h}_n(\omega_1,\cdots,\omega_n)=F(h_n(\tau_1,\cdots,\tau_n))= \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}h_n(\tau_1,\cdots,\tau_n)\cdot \prod_{i=1}^{n}\mathrm{e}^{-\mathrm{j}\omega_i\tau_i}\mathrm{d}\tau_i,\quad n\geqslant 2. \tag{4}$$

It is clear that $\{h_n, n\in\mathbb{N}\}$ in (1) and (4) respectively are all nonparametric models. Just like the FRF in the case of linear system, they describe the essential properties of the nonlinear system, which do not depend on the choice of input signals. However, $\hat{h}_n$ is the n-dimensional complex functions, so the sampling points will be L^n if the sampling points for every frequency variable are L, and this is the so-called "dimensional catastrophe problem".

3 Identification algorithm

Firstly, the assumptions are made as below.

Assumption 1 The input signal is a linear combination of sine signals designed artificially, the frequency components are all times of the basic frequency. That is, assuming ω_0 is the basic ferquency, the other fuequency components are $m\omega_0$, $m\in\mathbb{N}$. At the same time, it is assumed that

$$\hat{u}(\omega(k))\neq 0,\ k=-Nr+1,\cdots,-1,0,1,\cdots,Nr-1, \tag{5}$$

where Nr is the frequency bend width of the input signal, and $Nr<Ns$, and Ns is the number of samples of the output in time field, which corresponds to the maximum frequency bend width of the response function.

Assumption 2 The input and output signals are sampled synchronously, and the selection of the sampling period and the sample ability Ns must satisfy the sampling law and Ns may be many times of the frequency bend width Nr of the input signal.

Assumption 3 The dynamic behavior of the system can be described approximately with the first three GFRF's.

Thus, the input and output data $\{\hat{u}(k\omega_0)\}$, $\{\hat{y}(k\omega_0)\}$, which have been transformed into frequency-domain by using FFT, should satisfy the following nonlinear functional equation

$$\begin{aligned}\hat{y}(k\omega_0) = {} & \hat{h}_1(k\omega_0)\hat{u}(k\omega_0) + \\ & \frac{1}{2Ns}\sum_{s=-Ns+1}^{Ns-1}\hat{h}_2((k-s)\omega_0, \\ & s\omega_0)\hat{u}((k-s)\omega_0)\hat{u}(s\omega_0) + \\ & \frac{1}{(2Ns)^2}\sum_{s=-Ns+1}^{Ns-1}\sum_{\tau=-Ns+1}^{Ns-1}\hat{h}_3((k-s-\tau)\omega_0, s\omega_0, \\ & \tau_0\omega)\hat{u}((k-s-\tau)\omega_0)\hat{u}(s\omega_0)\hat{u}(\tau\omega_0), \\ & k = -Ns+1,\cdots,-1,0,1,\cdots,Ns-2.\end{aligned} \tag{6}$$

According to the symmetric and conjugate symmetric properties, it is assumed that the values of $\hat{h}_n$ in the whole hyperplane $\omega_1+\omega_2+\cdots+\omega_n=\omega$ are the same, and therefore the functional Equation (6) may be simplified to be as (assuming $3Nr < Ns$)

$$\begin{aligned}\hat{y}(k\omega_0) = {} & \hat{h}_1(k\omega_0)\hat{u}(k\omega_0) + \frac{\hat{h}_2(k\omega_0,0)}{Ns}\cdot \\ & \sum_{s=0}^{Ns-1}\hat{u}((k-s)\omega_0)\hat{u}(s\omega_0) + \\ & \frac{\hat{h}_3(k\omega_0,0,0)}{(Ns)^2}\sum_{s=0}^{Ns-1}\sum_{\tau=0}^{Ns-1}\hat{u}((k- \\ & s-\tau)\omega_0)\hat{u}(s\omega_0)\hat{u}(\tau\omega_0) = \\ & \hat{h}_1(k\omega_0)\hat{u}(k\omega_0) + \frac{\hat{u}*\hat{u}(k\omega_0)}{Ns}\hat{h}_2(k\omega_0,0) + \\ & \frac{\hat{u}*\hat{u}*\hat{u}(k\omega_0)}{(Ns)^2}\hat{h}_3(k\omega_0,0,0). \\ & k = -3Nr+3,\cdots,-1,0,1,\cdots,3Nr-3.\end{aligned} \tag{7}$$

Thus, the steps of the identification algorithm can be described as

i) Design the input signal and apply it to the real process, and sample the input and output signals;

ii) Transform the sampled input and output data into the frequency-domain, the Fourier transform sequences $\{\hat{u}(k\omega_0)\}$ and $\{\hat{y}(k\omega_0)\}$ are obtained;

iii) By using (7), the estimates of sequences $\{\hat{h}_1(k\omega_0)\}$, $\{\hat{h}_2(k\omega_0,0)\}$ and $\{\hat{h}_3(k\omega_0,0,0)\}$ could be obtained according to the equation solution;

iv) Test the generalization ability for the obtained model; If the result reaches required precision, go to step v); If not, turn back to step i);

v) End.

4 Simulation example

Consider the nonlinear system as follows

$$\dot{y} = -15y + u + 0.1u^2 - 0.1y^2,$$

the result of identification is shown in Fig.1.

The generalization ability test is the same as the following: two groups of input and output data, which are not used in identification, are used to test the generalization ability of the model.

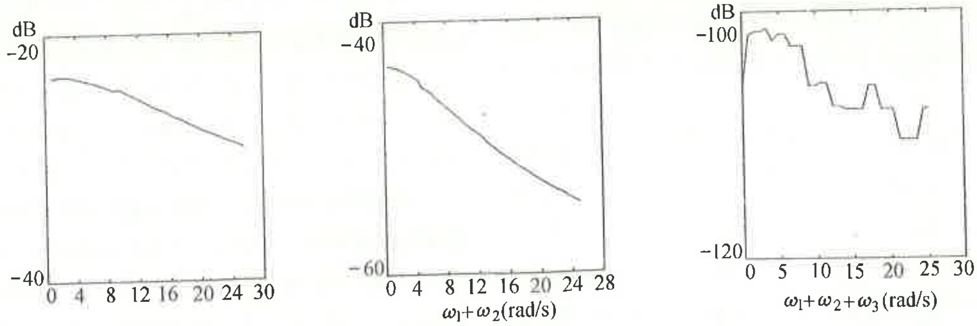
a) The case that the input is $u = 2\cos(2.45t)$. Comparing Fig.2 with Fig.3, we can see clearly that the GFRF model is of very good generalization ability, and the NN model is not good enough when the system is excited by a small sine signal.

b) The case that the input is $u = 20\cos(2.45t)$. Comparing Fig.4 with Fig.5, we can see clearly that the result is still the same as in case a) when the system is excited by a large sine signal.

c) The case that the input is

$$u = 2\mathrm{sgn}(\cos(2.45t)).$$

Comparing Fig.6 with Fig.7, we can see clearly that all two models are of excellent generalization ability when the system is excited by a sgn signal.



(a) $|h_1(\omega)|$ (b) $|h_2(\omega,0)|$ (c) $|h_3(\omega,0,0)|$

Fig. 1 GFRF model identificatied result

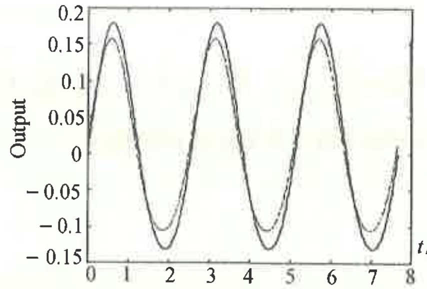


Fig. 2 —— output of the GFRF model
------ output of real system

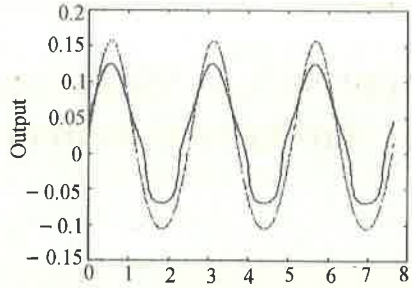


Fig. 3 —— output of the NN model
------ output of the real system

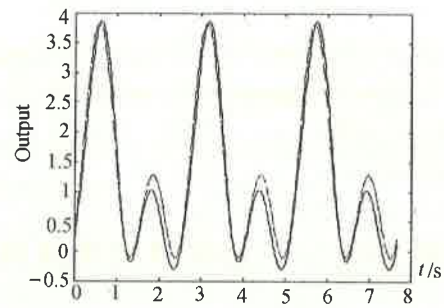


Fig. 4 ——output of the GFRF model
------output of real system

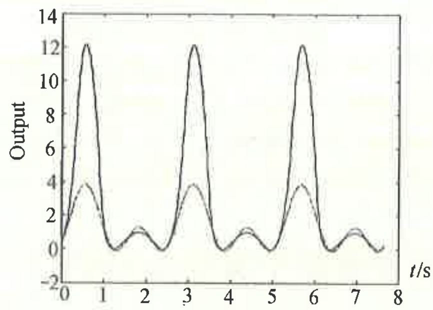


Fig. 5 —— output of the NN model
------output of the real system

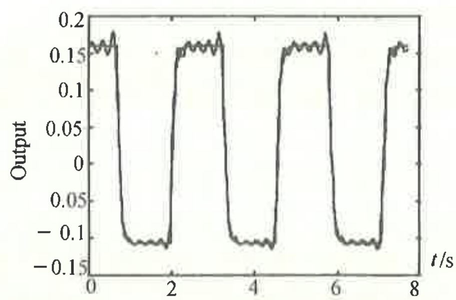


Fig. 6 —— output of the GFRF mode
—— output of practical system

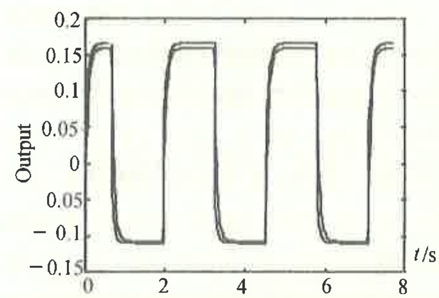


Fig. 7 ---- output of the NN mode
—— output of the real system

5 Conclusion

The identification algorithm of nonparametric GFRF model for a class of nonlinear systems are discussed in this paper. Because the GFRF model reflects the essential properties of the system, and is irrelative with the input signal, the generalization ability of the identified model can be guaranteed, and therefore the identified model is reliable. Although the GFRF model is limited to less than three-order, it can satisfy the engineering desire of system analysis and synthesis in general. Design of the optimal signals will be a main problem in the future research.

References

1 Arret J F. The use of functions in the analysis of nonlinear physical system. Journal of Electronics and Control, 1963, 15: 567 – 615

2 Schetzen M. The Volterra and Wiener Theories of Nonlinear System. New York: Wiley, 1980

3 Billings S A and Tsang K M. Spectral analysis for nonlinear systems, Part I: parametric nonlinear spectral analysis. Mechanical Systems and signal Processing, 1989, 3(4): 319 – 339

4 Billings S A and Tsang K M. Spectral analysis for nonlinear systems, Part Ⅱ: interpretation of nonlinear frequency response functions. Mechanical Systems and Signal Processing, 1989, 3(4): 341 – 359

5 Billings S A and Tsang K M. Spectral analysis for nonlinear systems, Part Ⅲ: case study examples. Mechanical Systems and Signal Processing, 1989, 4(1): 3 – 21

6 Billings S A and Peyton Jone J C. Mapping nonlinear integro-differential equation in the frequency domain. Int. J. Control, 1990, 52: 863 – 879

7 Fakhouri S Y. Identification of the Volterra kernels of nonlinear systems. IEE Proceedings-D Control Theory & Applications, 1980, 127(6): 296 – 304

8 Cho Y S, Kim S B, Hixson E L and Powers E J. A digital technique to estimate second-order distortion using higher order coherence spectra.

(Continued on page 825)