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H_∞ Robust Control for Multi-Time Delay Systems

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Abstract: In this paper, we first proposes a sufficient condition for multi-time delay systems to satisfy H_∞ design indices, and a necessary and sufficient condition for a Riccati inequarlity with uncertainties to have robustness. And then, we attain a sufficient condition for multi-time delay systems with uncertainties to satisfy H_∞ state feedback robust control design indices.

Key words: multi-time delay; uncertainty; state feedback; H_∞ robust control

多时滞系统 H_∞ 鲁棒控制

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摘要: 文章首先提出了多时滞系统满足 H_∞ 性能指标的充分条件及带不确定性 Riccati 不等式具有鲁棒性的充分必要条件, 然后提出了带不确定性多时滞被控对象经状态反馈后满足 H_∞ 鲁棒设计指标的充分条件.

关键词: 多时滞; 不确定性; 状态反馈; H_∞ 鲁棒控制

1 Introduction

Much research has been done[1~3] on the multi-time delay systems and the closed loop systems which are constructed by combining them with feedback controllers. In the case of some simple time delay systems, applying H_∞ optimal control, Navantimua-pick interpolational theory and infinite-dimentional technique, [1] give H_∞ optimal design with weighted sensitivity. Applying Razumikhim-type theorem, [2] proposes a criterion of stability for time delay systems with uncertainties to be dependently stable. The condition given is dependent on time delays. However, the paper does not touch upon the problem of H_∞ performance indices. Applying the method of using state feedback to resolve the problem of H_∞ optimal control to the time delay systems, [3] proposes a method for design of the H_∞ state feedback control of single and multi-time delay systems. The method is to attain a state feedback matrix which ensures that the H_∞ norm of the closed loop transfer function is smaller than a given number, by resolving an algebric Riccati equation in reiteration. However, the paper does not deal with the problem of the H_∞ robust control for multi-time delay systems with uncertainties, there has not been research in the paper, and to our knowledge, no sufficient research has been done upon it so far. For this reason, we will, in this paper, apply the theory on stability described by characteristic equation of frequencial region and state space to prove a sufficient condition for multi-time delay systems to satisfy H_∞ design indices, and give a necessary and sufficient condition for a Riccati inequality with uncertainties to have robustness. And then, on the basis of the two conditions, we will propose a design technique of a state feedback control for multi-time delay systems with structure parameter uncertainties, and give the sufficient conditions for the closed loop system controllers to satisfy the H_∞ robust performance criterion.

In advance, we let a matrix X which will be frenquently used in this paper be always symmetric.

2 H_∞ design criterion

Consider a multi-time delay system

$$x(t) = A_0 x(t) + \sum_{i=1}^{m} A_i x(t-h_i) + Bw(t), \tag{2.1a}$$

$$z(t) = Cx(t), \tag{2.1b}$$

where, $x(t) \in \mathbb{R}^n$ is a state vector of the system; $w(t)$

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$\in \mathbb{R}^{ml}$ is the disturbance vector; $z(t) \in \mathbb{R}^{pl}$ is the appraise output vector; $A_i(i = 0,1,2,\cdots,m)$, B and C are appropriate dimention matrices, and $h_i(i = 1,2,\cdots,m)$ are the delay constants of states. The transfer function from w to z is

$$G_{zw}(s) = C[sI - (A_0 + \sum_{i=1}^{m} A_i e^{-h_i s})]^{-1}B = CG^{-1}(S)B, \tag{2.2}$$

where

$$G(s) = [sI - (A_0 + \sum_{i=1}^{m} A_i e^{-h_i s})]. \tag{2.3}$$

Obviously, we can let

$$P(s) = \det G(s) = \det[sI - A_0 - \sum_{i=1}^{m} A_i e^{-h_i s})] \tag{2.4}$$

be a characteristic polynomial of system (2.1).

Lemma 2.1[4] For the characteristic polynomial (2.4), the system (2.1) is asymptotically stable, if and only if

$$P(s) \neq 0, \quad \mathrm{Re}\ s \geqslant 0. \tag{2.5}$$

Theorem 2.1 For a given index, $\gamma > 0$, the multi-time delay system (2.1) is

1) asymptotically stable;

2) $\| G_{zw}(s) \|_\infty < \gamma$, if there exists a positive definite matrix X satisfing Riccati inequality

$$A_0^T X + XA_0 + \gamma^{-2} XBB^T X + C^T C + \sum_{i=1}^{m} (XA_i A_i^T X + I_i) < 0, \tag{2.6}$$

where, $I_i(i = 1,2,\cdots,m) = I$ are n -dimentional unit matrices.

Proof Let $w(t) = 0$. Take Lyapunov function as

$$V(t) = x^T(t)Xx(t) + \sum_{i=1}^{m} \int_{t-h_i}^{t} x^T(\tau)x(\tau)d\tau. \tag{2.7}$$

Then, the derivative of $V(t)$, with respect to time, along the track of system (2.1) is

$$\begin{aligned}\dot{V}(t) = {} & x^T(t)A_0^T Xx(t) + \sum_{i=1}^{m} x^T(t-h_i)A_i^T Xx(t) + \\ & x^T(t)XA_0 x(t) + \sum_{i=1}^{m} x^T(t)XA_i x(t - h_i) + \\ & \sum_{i=1}^{m} x^T(t)I_i x(t) - \sum_{i=1}^{m} x^T(t-h_i)x(t-h_i) = \\ & x^T(t)[A_0^T X + XA_0]x(t) + \\ & \sum_{i=1}^{m} [x^T(t-h_i)A_i^T Xx(t) + x^T(t)XA_i x(t-h_i) + \\ & x^T(t)I_i x(t) - x^T(t-h_i)x(t-h_i)].\end{aligned} \tag{2.8}$$

Let

$$u_i = x(t - h_i), v_i = A_i^T Xx(t), i = 1,2,\cdots,m.$$

Then, because there is

$$u^T u + v^T v \geqslant u^T v + v^T u,$$

we have

$$\begin{aligned}& x^T(t - h_i)A_i^T Xx(t) + x^T(t)XA_i x(t - h_i) \leqslant \\ & x^T(t)XA_i A_i^T Xx(t) + x^T(t - h_i)x(t - h_i), \\ & i = 1,2,\cdots,m.\end{aligned}$$

Introducing it into (2.8) and putting it in order, we get

$$\dot{V}(t) < x^T(t)[A_0^T X + XA_0 + \sum_{i=1}^{m} (XA_i A_i^T X + I_i)]x(t). \tag{2.9}$$

Noticing

$$\gamma^{-2} XBB^T X + C^T C \geqslant 0$$

and from (2.6), we have

$$A_0^T X + XA_0 + \sum_{i=1}^{m} (XA_i A_i^T X + I_i) < 0. \tag{2.10}$$

So, by (2.9), we get

$$\dot{V}(t) < 0.$$

i.e., system (2.1) is asymptotically stable.

For (2.6), we have

$$\begin{aligned}C^T C < {} & -A_0^T X - XA_0 - \sum_{i=1}^{m} (XA_i A_i^T X + I_i) - \gamma^{-2} XBB^T X = \\ & [sI - (A_0 + \sum_{i=1}^{m} A_i e^{h_i s})^T]X + \\ & X[sI - (A_0 + \sum_{i=1}^{m} A_i e^{-h_i s}] - \\ & \sum_{i=1}^{m} (XA_i e^{-h_i s} - I_i)(A_i^T X e^{h_i s} - I_i) - \\ & \gamma^{-2} XBB^T X.\end{aligned} \tag{2.11}$$

Letting

$$T(s) = \sum_{i=1}^{m} (A_i^T X e^{-h_i s} - I_i)$$

and from (2.3), (2.11) can become

$$C^T C < G^T(-s)X + XG(s) - T^T(-s)T(s) - \gamma^{-2} XBB^T X. \tag{2.12}$$

From asymptotic stability and (2.5), we know, $G^{-1}(s)$ exists for $\forall\ \mathrm{Re}\ s \geqslant 0$ and $G^{-1}(-s)$ exist for $\forall\ \mathrm{Re}\ s \leqslant 0$. Thus, from (2.2) and (2.12), we have

$$\gamma^2 I - G_{zw}^{\mathrm{T}}(-s)G_{zw}(s) =$$
$$\gamma^2 I - B_1^{\mathrm{T}}G^{-\mathrm{T}}G^{-\mathrm{T}}(-s)C^{\mathrm{T}}CG^{-1}(s)B_1 >$$
$$\gamma^2 I - B_1^{\mathrm{T}}G^{-\mathrm{T}}(-s)[G^{\mathrm{T}}(-s)X + XG(s) -$$
$$T^{\mathrm{T}}(-s)T(S) - \gamma^{-2}XB_1B_1^{\mathrm{T}}X]G^{-1}(s)B_1 =$$
$$\gamma^2 - B_1^{\mathrm{T}}XG^{-1}(s)B_1 - B_1^{\mathrm{T}}G^{\mathrm{T}}(-s)XB_1 +$$
$$B_1^{\mathrm{T}}G^{-\mathrm{T}}(-s)T^{\mathrm{T}}(-s)T(s)G^{-1}(s)B_1 +$$
$$\gamma^{-2}B_1^{\mathrm{T}}G^{-\mathrm{T}}(-s)XB_1B_1^{\mathrm{T}}XG^{-1}(s)B_1 =$$
$$[\gamma I - \gamma^{-1}B_1^{\mathrm{T}}G^{-\mathrm{T}}(-s)XB_1][\gamma I - \gamma^{-1}B_1^{\mathrm{T}}XG^{-1}(s)B_1] +$$
$$B_1^{\mathrm{T}}G^{-\mathrm{T}}(-s)T^{\mathrm{T}}(-s)T(s)G^{-1}(s)B_1 \geqslant 0,$$
$$\forall s = \mathrm{j}\omega. \tag{2.13}$$

So, we get

$$\| G_{zw}(s) \|_\infty < \gamma. \tag{2.14}$$

3 Robustness theorem

Assume that perturbation matrix $\Delta A_i (i = 0,1,2,\cdots,m)$ satisfies

$$\Delta A_i = E_i \Sigma F_i, \quad i = 0,1,2,\cdots,m, \tag{3.1}$$

where, E_i, F_i are known constant matrices and

$$F_i^{\mathrm{T}}F_i = I, \quad i = 1,2,\cdots,m, \tag{3.2}$$

Σ is unknown matrix and

$$\Sigma = \Sigma^{\mathrm{T}} \in \Omega = \{\Sigma : \Sigma^{\mathrm{T}}\Sigma \leqslant I\}. \tag{3.3}$$

Then, we have two lemmas and our result.

Lemma 3.1[5] For any matrix $E \in \mathbb{R}^{n\times r}$, $F \in \mathbb{R}^{q\times n}$ and positive definite matrix X, and $\forall \xi \in \mathbb{R}^n$, there exists $\Sigma_m \in \Omega \in \mathbb{R}^{r\times q}$, such that

$$\max_{\Sigma\in\Omega}(\xi^{\mathrm{T}}XE\Sigma F\xi)^2 = (\xi^{\mathrm{T}}XE\Sigma_m F\xi)^2 = \xi^{\mathrm{T}}XEE^{\mathrm{T}}X\xi\xi^{\mathrm{T}}F^{\mathrm{T}}F\xi.$$

Lemma 3.2[5] Assume that X, Y, Z are given n-dementional square matrices, and $X \geqslant 0, Y < 0, Z \geqslant 0$. If for any vector $\xi \in \mathbb{R}^n, \xi \neq 0$, there is

$$(\xi^{\mathrm{T}}Y\xi)^2 > 4\xi^{\mathrm{T}}X\xi\xi^{\mathrm{T}}Z\xi,$$

then, there exists an appropriate scalar $\alpha > 0$, such that

$$a^2X + aY + Z < 0.$$

Theorem 3.1 There is a positive matrix X such that

$$(A_0+\Delta A_0)^{\mathrm{T}}X + X(A_0+\Delta A_0) + \gamma^{-2}XBB^{\mathrm{T}}X + C^{\mathrm{T}}C +$$
$$\sum_{i=1}^{m}[X(A_i + \Delta A_i)(A_i+\Delta A_i)^{\mathrm{T}}X + I_i] < 0,$$
$$\forall \Sigma \in \Omega, \tag{3.4}$$

if

$$A_0^{\mathrm{T}}X + XA_0 + X(\gamma^{-2}BB^{\mathrm{T}} + \lambda^2E_0E_0^{\mathrm{T}})X +$$
$$C^{\mathrm{T}}C + \lambda^{-2}F_0^{\mathrm{T}}F_0 + \sum_{i=1}^{m}[(1+\lambda^{-2})XA_iA_i^{\mathrm{T}}X +$$
$$(1+\lambda^2)XE_iE_i^{\mathrm{T}}X + I_i] < 0 \tag{3.5}$$

holds for any scalar $\lambda > 0$.

Proof Due to (3.1) and (3.2), (3.4) is equivalent to

$$(A_0 + E_0\Sigma F_0)^{\mathrm{T}}X + X(A_0 + E_0\Sigma F_0) +$$
$$\gamma^{-2}XBB^{\mathrm{T}}X + C^{\mathrm{T}}C +$$
$$\sum_{i=1}^{m}[X(A_i + E_i\Sigma F_i)(A_i + E_i\Sigma F_i)^{\mathrm{T}}X + I_i] =$$
$$A_0^{\mathrm{T}}X + XA_0 + F_0^{\mathrm{T}}\Sigma^{\mathrm{T}}E_0^{\mathrm{T}}X +$$
$$XE_0\Sigma F_0 + \gamma^{-2}XBB^{\mathrm{T}}X + C^{\mathrm{T}}C +$$
$$\sum_{i=1}^{m}[XA_iA_i^{\mathrm{T}}X + XA_iF_i^{\mathrm{T}}\Sigma^{\mathrm{T}}E_i^{\mathrm{T}}X +$$
$$XE_i\Sigma F_iA_i^{\mathrm{T}}X + XE_i\Sigma\Sigma^{\mathrm{T}}E_i^{\mathrm{T}}X + I_i] < 0, \quad \forall \Sigma \in \Omega. \tag{3.6}$$

For any matrix U, V and scalar $\lambda \neq 0$, we have

$$\lambda^2UU^{\mathrm{T}} + \lambda^{-2}VV^{\mathrm{T}} > UV^{\mathrm{T}} + VU^{\mathrm{T}},$$

and taking into consideration (3.2), we get

$$A_0^{\mathrm{T}}X + XA_0 + F_0^{\mathrm{T}}\Sigma^{\mathrm{T}}E_0^{\mathrm{T}}X +$$
$$XE_0\Sigma F_0 + \gamma^{-2}XBB^{\mathrm{T}}X + C^{\mathrm{T}}C +$$
$$\sum_{i=1}^{m}[XA_iA_i^{\mathrm{T}}X + XA_iF_i^{\mathrm{T}}\Sigma^{\mathrm{T}}E_i^{\mathrm{T}}X +$$
$$XE_i\Sigma F_iA_i^{\mathrm{T}}X + XE_i\Sigma\Sigma^{\mathrm{T}}E_i^{\mathrm{T}}X + I_i] \leqslant$$
$$A_0^{\mathrm{T}}X + XA_0 + \lambda^2XE_0E_0^{\mathrm{T}}X + \lambda^{-2}F_0^{\mathrm{T}}\Sigma^{\mathrm{T}}\Sigma F_0 +$$
$$\gamma^{-2}XBB^{\mathrm{T}}X + C^{\mathrm{T}}C +$$
$$\sum_{i=1}^{m}[XA_iA_i^{\mathrm{T}}X + \lambda^{-2}XA_iA_i^{\mathrm{T}}X +$$
$$\lambda^2XE_i\Sigma\Sigma^{\mathrm{T}}E_i^{\mathrm{T}}X + XE_i\Sigma\Sigma^{\mathrm{T}}E_i^{\mathrm{T}}X + I_i] \leqslant$$
$$A_0^{\mathrm{T}}X + XA_0 + \lambda^2XE_0E_0^{\mathrm{T}}X + \lambda^{-2}F_0^{\mathrm{T}}F_0 +$$
$$\gamma^{-2}XBB^{\mathrm{T}}X + C^{\mathrm{T}}C +$$
$$\sum_{i=1}^{m}[(1+\lambda^2)XE_iE_i^{\mathrm{T}}X + (1+\lambda^{-2})XA_iA_i^{\mathrm{T}}X + I_i],$$
$$\forall \Sigma \in \Omega. \tag{3.7}$$

Obviously, if (3.7) is smaller than zero, i.e., (3.5) holds, then (3.6) holds, i.e., (3.4) holds.

In Theorem 3.1, if we can let

$$E_i\Sigma F_j = 0, \quad E_i^{\mathrm{T}}F_j = 0, \quad F_i^{\mathrm{T}}F_j = 0,$$
$$i \neq j, \quad i,j = 0,1,2,\cdots,m, \quad \forall \Sigma \in \Omega. \tag{3.8}$$

anyway, the condition (3.5) will be necessary as well. In fact, if (3.4) holds, from (3.6), we have

$$S + F_0^{\mathrm{T}}\Sigma^{\mathrm{T}}E_0X + XE_0\Sigma F_0 + \sum_{i=1}^{m}[XA_iF_i^{\mathrm{T}}\Sigma^{\mathrm{T}}E_i^{\mathrm{T}}X + XE_i\Sigma F_iA_i^{\mathrm{T}}X] < - \sum_{i=1}^{m} XE_i\Sigma\Sigma^{\mathrm{T}}E_i^{\mathrm{T}}X, \quad \forall \Sigma \in \Omega,$$

where

$$S = A_0^{\mathrm{T}}X + XA_0 + \gamma^{-2}XBB^{\mathrm{T}}X + C^{\mathrm{T}}C + \sum_{i=1}^{m}[XA_iA_i^{\mathrm{T}}X + I_i].$$

For $\forall \xi \in \mathbb{R}^n, (\xi \neq 0)$, , there is

$$\xi^{\mathrm{T}}S\xi + 2\xi^{\mathrm{T}}XE_0\Sigma F_0\xi + 2\sum_{i=1}^{m}\xi^{\mathrm{T}}XE_i\Sigma F_iA_i^{\mathrm{T}}X\xi < - \sum_{i=1}^{m}\xi^{\mathrm{T}}XE_i\Sigma\Sigma^{\mathrm{T}}E_i^{\mathrm{T}}X\xi \leqslant - \sum_{i=1}^{m}\xi^{\mathrm{T}}XE_iE_i^{\mathrm{T}}X\xi, \quad \forall \Sigma \in \Omega. \tag{3.9}$$

From Lemma 3.1, we know there exists $\Sigma_m \in \Omega$, such that

$$\max \mid \xi^{\mathrm{T}}XE_0\Sigma F_0\xi \mid = \xi^{\mathrm{T}}XE_0\Sigma_mF_0\xi > 0, \tag{3.10}$$

$$\max \mid \xi^{\mathrm{T}}XE_i\Sigma F_iA_i^{\mathrm{T}}X\xi \mid = \xi^{\mathrm{T}}XE_i\Sigma_mF_iA_i^{\mathrm{T}}X\xi > 0, \quad i = 1,2,3,\cdots,m. \tag{3.11}$$

So, from (3.9), we get

$$\xi^{\mathrm{T}}S\xi + 2\xi^{\mathrm{T}}XE_0\Sigma_mF_0\xi + 2\sum_{i=1}^{m}\xi^{\mathrm{T}}XE_i\Sigma_mF_iA_i^{\mathrm{T}}X\xi < - \sum_{i=1}^{m}\xi^{\mathrm{T}}XE_iE^{\mathrm{T}}X\xi.$$

The above inequality can be rewritten as

$$\xi^{\mathrm{T}}S\xi + \sum_{i=1}^{m}\xi^{\mathrm{T}}XE_iE_i^{\mathrm{T}}X\xi = \xi^{\mathrm{T}}[S + \sum_{i=1}^{m}XE_iE_i^{\mathrm{T}}X]\xi < - 2\xi^{\mathrm{T}}XE_0\Sigma_mF_0\xi + 2\sum_{i=1}^{m}\xi^{\mathrm{T}}XE_i\Sigma_mF_iA_i^{\mathrm{T}}X\xi = - 2\xi^{\mathrm{T}}[XE_0\Sigma_mF_0 + \sum_{i=1}^{m}XE_i\Sigma_mF_iA_i^{\mathrm{T}}X]\xi. \tag{3.12}$$

From (3.10), (3.11) and (3.12), and taking (3.8) into account, we get

$$\{\xi^{\mathrm{T}}[S + \sum_{i=1}^{m}XE_iE_i^{\mathrm{T}}X]\xi\}^2 > 4\{\xi^{\mathrm{T}}[XE\Sigma_mF_0 + \sum_{i=1}^{m}XE_i\Sigma_mF_iA_i^{\mathrm{T}}X]\xi\}^2 = 4\{\xi^{\mathrm{T}}[XE_0 + \sum_{i=1}^{m}XE_i]\Sigma_m[F_0 + \sum_{i=1}^{m}F_iA_i^{\mathrm{T}}X]\xi\}^2 =$$
$$4\xi^{\mathrm{T}}[XE_0 + \sum_{i=1}^{m}XE_i][XE_0 + \sum_{i=1}^{m}XE_i]^{\mathrm{T}}\xi\xi^{\mathrm{T}} \cdot [F_0 + \sum_{i=1}^{m}F_iA_i^{\mathrm{T}}X]^{\mathrm{T}}[F_0 + \sum_{i=1}^{m}F_iA_i^{\mathrm{T}}X]\xi = 4\xi^{\mathrm{T}}[XE_0E_0^{\mathrm{T}}X + \sum_{i=1}^{m}XE_iE_i^{\mathrm{T}}X]\xi\xi^{\mathrm{T}} \cdot [F_0^{\mathrm{T}}F_0 + \sum_{i=1}^{m}XA_iA_i^{\mathrm{T}}X]. \tag{3.13}$$

From (3.12) and (3.10), (3.11) we know

$$S + \sum_{i=1}^{m}XE_iE_i^{\mathrm{T}}X < 0.$$

According to Lemma 3.2 and (3.13), we know there exists an $\alpha^2 > 0$, such that

$$\alpha^2 + [XE_0E_0^{\mathrm{T}}X + \sum_{i=1}^{m}XE_iE_i^{\mathrm{T}}X] + \alpha^2[S + \sum_{i=1}^{m}XE_iE_i^{\mathrm{T}}X] + F_0^{\mathrm{T}}F_0 + \sum_{i=1}^{m}XA_iA_i^{\mathrm{T}}X < 0,$$

i.e.,

$$\alpha^2XE_0E^{\mathrm{T}}X + \alpha^{-2}F_0^{\mathrm{T}}F_0 + \sum_{i=1}^{m}[\alpha^2XE_iE_i^{\mathrm{T}}X + \alpha^{-2}XA_iA_i^{\mathrm{T}}X] + A_0^{\mathrm{T}}X + XA_0 + \gamma^{-2}XBB^{\mathrm{T}}X + C^{\mathrm{T}}C + \sum_{i=1}^{m}[XA_iA_i^{\mathrm{T}}X + XE_iE_i^{\mathrm{T}}X + I_i] < 0.$$

Comparing the above inequality with (3.5), and by the arbitrarility of λ , we immediately prove the necessity of Theorem 3.1.

4 Robust H$_\infty$ control

Consider the multi-time delay system with parameter perturbation

$$x(t) = (A_0+\Delta A_0)x(t) + \sum_{i=1}^{m}(A_i+\Delta A_i)x(t - h_i) + B_1w(t) + (B_2 + \Delta B)u(t), \tag{4.1a}$$

$$z(t) = Cx(t) + Du(t), \tag{4.1b}$$

where $x(t) \in \mathbb{R}^n$ is a state vector of the system; $w(t)$

$\in \mathbb{R}^{m1}$ is the disturbance vector; $u(t) \in \mathbb{R}^{m2}$ is the control vector; $z(t) \in \mathbb{R}^{p1}$ is the appraise output vector; $h_i(i = 1,2,\cdots,m)$ is time delay constant of state; $A_i(i = 0,1,2,\cdots,m), B_1, B_2, C, D$ are known appropriate dimension matrices;

$$[\Delta A_i \quad \Delta B] = [E_i\Sigma F_{ai} \quad E_0\Sigma F_b], \quad i = 0,1,2,\cdots,m; \tag{4.2}$$

E_i, F_{ai}, F_b are known constant matrices satisfying

$$F_{ai}^{\mathrm{T}}F_{ai} = I, \quad i = 1,2,\cdots,m; \tag{4.3}$$

Σ is unknown matrix described by (3.3).

Assume that rank $D = m_2$, (A_0, B_2) is a pair stabilizable, and that state feedback control

$$u(t) = Kx(t) \tag{4.4}$$

is adopted for system (4.1), where, $K \in \mathbb{R}^{m2\times n}$ is the gain matrix of feedback controller. Then, a closed loop system constructed by the system (4.1) and controller (4.4) is

$$x(t) = (A_f + \Delta A_f)x(t) + \sum_{i=1}^{m}(A_i + \Delta A_i)x(t - h_i) + B_1 w(t), \tag{4.5a}$$

$$z(t) = C_f x(t), \tag{4.5b}$$

where

$$\begin{aligned} A_f &= A_0 + B_2K, \quad C_f = C + DK, \\ \Delta A_f &= E_0\Sigma F_f, \quad F_f = F_{a0} + F_bK. \end{aligned} \tag{4.6}$$

A transfer function from w to z can be denoted by

$$G_{zw}(s) = C_f[sI - (A_f + \Delta A_f) - \sum_{i=1}^{m}(A_i + \Delta A_i)\mathrm{e}^{-h_i s}]^{-1}B_1. \tag{4.7}$$

Comparing the closed system (4.5) with the system (2.1), and referring to Theorem 2.1, we know that if there exist appropriate positive definite matrix X and feedback matrix K, such that

$$(A_f + \Delta A_f)^{\mathrm{T}}X + X(A_f + \Delta A_f) + \gamma^{-2}XB_1B_1^{\mathrm{T}}X + C_f^{\mathrm{T}}C_f + \sum_{i=1}^{m}[X(A_i + \Delta A_i)(A_i + \Delta A_i)^{\mathrm{T}}X + I_i] < 0 \tag{4.8}$$

holds for $\forall \Sigma \in \Omega$, then the closed system satisfies the robust H_∞ performance criteria

1) The closed loop system is asymptotically stable for $\forall \Sigma \in \Omega$;

2) $\| G_{zw}(s) \|_\infty < \gamma, \quad \forall \Sigma \in \Omega.$ (4.9)

For the sake of simplisity, we assume that

$$D^{\mathrm{T}}[D \quad C] = [I \quad 0]. \tag{4.10}$$

Theorem 4.1 For the given $\lambda > 0$, there exists a feedback matrix K, such that the closed system (4.5) constructed by (4.1), (4.2) and (4.3) satisfies (4.8), $\forall \Sigma \in \Omega$, if there exists a scalar $\lambda > 0$, such that Riccati inequality

$$A_0^{\mathrm{T}}X + XA_0 + X(\lambda^2E_0E_0^{\mathrm{T}} + \gamma^{-2}B_1B_1^{\mathrm{T}})X + C^{\mathrm{T}}C + \lambda^{-2}F_{a0}^{\mathrm{T}}F_{a0} - (XB_2 + \lambda^{-2}F_{a0}^{\mathrm{T}}F_b)R^{-2}(B_2^{\mathrm{T}}X + \lambda^{-2}F_b^{\mathrm{T}}F_{a0} + \sum_{i=1}^{m}[(1 + \lambda^{-2})XA_iA_i^{\mathrm{T}}X + (1 + \lambda^2)XE_iE_i^{\mathrm{T}}X + I_i] < 0 \tag{4.11}$$

has a positive solution X, where

$$R^2 = I + \lambda^{-2}F_b^{\mathrm{T}}F_b.$$

If such solution exists, the state feedback controller for the closed system to satisfy H_∞ performance criteria 1) and 2) can be given as

$$K = -R^{-2}(B_2^{\mathrm{T}}X + \lambda^{-2}F_b^{\mathrm{T}}F_{ao}). \tag{4.12}$$

Proof Comparing (4.8) with (3.4) and the corresponding conditions of them, and referring to Theorem 3.1, we know that if there exists a $\lambda > 0$, such that Riccati inequality

$$A_f^{\mathrm{T}}X + XA_f + X(\gamma^{-2}B_1B_1^{\mathrm{T}} + \lambda^2E_0E_0^{\mathrm{T}})X + C_f^{\mathrm{T}}C_f + \lambda^{-2}F_f^{\mathrm{T}}F_f + \sum_{i=1}^{m}[(1 + \lambda^{-2})XA_iA_i^{\mathrm{T}}X + (1 + \lambda^2)XE_iE_i^{\mathrm{T}}X + I_i] < 0 \tag{4.13}$$

holds, then there exist positive matrix X and feedback matrix K, such that (4.8) holds.

Applying (4.6) to develop (4.13), we get

$$A_0^{\mathrm{T}}X + XA_0 + X(\gamma^{-2}B_1B_1^{\mathrm{T}} + \lambda^2E_0E_0^{\mathrm{T}})X + C^{\mathrm{T}}C + \lambda^{-2}F_{a0}^{\mathrm{T}}F_{a0} + (XB_2 + \lambda^{-2}F_{a0}^{\mathrm{T}}F_{a0})K + K^{\mathrm{T}}(B_2^{\mathrm{T}}X + \lambda^{-2}F_b^{\mathrm{T}}F_{a0}) + K^{\mathrm{T}}(I + \lambda^{-2}F_b^{\mathrm{T}}F_b)K + \sum_{i=1}^{m}[(1 + \lambda^{-2})XA_iA_i^{\mathrm{T}}X + (1 + \lambda^2)XE_iE_i^{\mathrm{T}}X + I_i] < 0. \tag{4.14}$$

It can be rewritten as

$$A_0^{\mathrm{T}}X + XA_0 + X(\gamma^{-2}B_1B_1^{\mathrm{T}} + \lambda^2E_0E_0^{\mathrm{T}})X + C^{\mathrm{T}}C + \lambda^{-2}F_{a0}^{\mathrm{T}}F_{a0} - (XB_2 + \lambda^{-2}F_{a0}^{\mathrm{T}}F_b)R^{-2}(B_2^{\mathrm{T}}X + \lambda^{-2}F_b^{\mathrm{T}}F_{a0}) + \sum_{i=1}^{m}[(1 + \lambda^{-2})XA_iA_i^{\mathrm{T}}X + (1 + \lambda^2)XE_iE_i^{\mathrm{T}}X + I_i] < - [K + R^{-2}(B_2^{\mathrm{T}}X + \lambda^{-2}F_b^{\mathrm{T}}F_{a0})]^{\mathrm{T}}R^2[K +$$

$$R^{-2}(B_2^{\mathrm{T}}X + \lambda^{-2}F_b^{\mathrm{T}}F_{a0})] < 0. \qquad (4.15)$$

Assuming there exists a $\lambda > 0$, that Riccati inequality (4.11) has a positive solution, and applying the X and to construct K as (4.12), then, from (4.11) and (4.12), we have

$$A_0^{\mathrm{T}}X + XA_0 + X(\gamma^{-2}B_1B_1^{\mathrm{T}} + \lambda^2E_0E_0^{\mathrm{T}})X + C^{\mathrm{T}}C + \lambda^{-2}F_{a0}^{\mathrm{T}}F_{a0} - (XB_2 + \lambda^{-2}F_{a0}^{\mathrm{T}}F_b)R^{-2}(B_2^{\mathrm{T}}X + \lambda^{-2}F_b^{\mathrm{T}}F_{a0}) + [K + R^{-2}(B_2^{\mathrm{T}}X + \lambda^{-2}F_b^{\mathrm{T}}F_{a0})]^{\mathrm{T}}R^2[K + R^{-2}(B_2^{\mathrm{T}}X + \lambda^{-2}F_b^{\mathrm{T}}F_{a0})] + \sum_{i=1}^{m}[(1+\lambda^{-2})XA_iA_i^{\mathrm{T}}X+(1+\lambda^2)XE_iE_i^{\mathrm{T}}X+I_i] < 0.$$

The above inequality is equivalent to (4.15). Thus, we know that (4.13) holds, furthermore, (4.8) holds.

If $\Delta B = 0$, we can take $F_b = 0$. Then Theorem 4.1 can have a simple form.

Corollary 4.1 For the given plant (4.1) with $\Delta B = 0$, if there exists a $\lambda > 0$, such that Riccati inequality

$$A_0^{\mathrm{T}}X + XA_0 + X(\gamma^{-2}B_1B_1^{\mathrm{T}} + \lambda^2E_0E_0^{\mathrm{T}} - B_2B_2^{\mathrm{T}})X + C^{\mathrm{T}}C + \lambda^{-2}F_{a0}^{\mathrm{T}}F_{a0} + \sum_{i=1}^{m}[(1+\lambda^{-2})XA_iA_i^{\mathrm{T}}X+(1+\lambda^2)XE_iE_i^{\mathrm{T}}X+I_i] < 0 \qquad (4.16)$$

has a positive solution X then the state feedback control matrix for the closed loop system to satisfy robust H$_\infty$ performance criteria 1) and 2) is

$$K = -B_2^{\mathrm{T}}X.$$

In the Theorem 4.1 and the Corollary 4.1, if we can let

$$E_i\Sigma F_{aj} = 0, \quad E_i^{\mathrm{T}}E_j = 0, \quad F_{ai}^{\mathrm{T}}F_{aj} = 0,$$
$$i \neq j, \quad i,j = 0,1,2,\cdots,m, \quad \forall\Sigma \in \Omega,$$

it can be easily proved that the conditions (4.11) and (4,16) enables the conditions to have less conservative form for the corresponding conclusion, respectively.

5 Conclusion

We have got a sufficient condition for multi-time delay systems to satisfy H$_\infty$ design indices, and a necessary and sufficient condition for the Riccati inequality with parameter perturbances to have robustness. On the basis of them, we have attained the sufficient and less conservative conditions for the robust H$_\infty$ state feedback control of multi-time delay systems with uncertainties. Resolving an algebraic matrix Riccati inequality, we can determine a state feedback controller for closed loop system to satisfy robust H$_\infty$ performance indices.

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