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The Passive Stabilization of the Motion of a Class of Lagrange System *

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Abstract: The idea of the passive stabilization of nonasymptotically stable motion of the dynamical systems by introducing supplementary degrees of freedom is advanced for the first time in the paper [1]. The effectiveness of the application to the study of Lagrangian system is shown by the special example in the present paper, which has a unique scientific meaning. In the system a nonlinear friction and inelastic potential energy are adopted. It is shown that this problem can also be solved in the nonlinear case.

Key words: passive stabilization; equilibrium; nonliear; generalized force

一类 Lagrange 系统的 PSM 问题

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摘要: PSM 思想在[1]中第一次提出, 得到了广泛的应用. 本文根据 PSM 思想, 建立一个特殊模型, 在非线性系统中研究拉格朗日系统的 PSM, 解决了模型的 PSM 控制.

关键词: PSM; 平衡稳定性; 非线性; 广义力

1 Statement of the problem

The attitude of a satellite is often controlled by reactive forces which requires some additional energy. But the satellite can also be stabilized by means of the relative motion of some piece of the satellite moving in nonideal fluid as an oscillator with damping. This does not require additional energy and is called "passive stabilization". Here we illustrate the passive stabilization for Lagrangian system with a specific example, which has a unique scientific meaning.

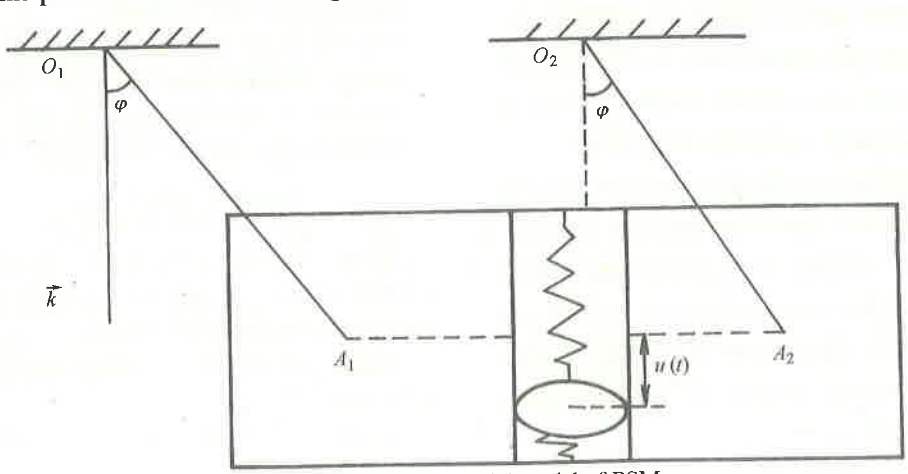

Fig. 1 The special model of PSM

Let an absolutely rigid body S with mass M perform plane-parallel movement under the action of gravity as the parallelogram pendulum (Fig. 1). Vector k is parallel to the vector of gravity, lines O_1O_2 and A_1A_2 are vertical to vector $\vec{k}$ during the whole motion. The position of S in space is determined by angle φ (Fig. 1). Moreover, we suppose a block s with mass m is contained in S. Now we consider two cases. In the first case, s is fixed

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to body S (or is frozen in S). In the second one, s can move with a friction (nonlinear) under the action of some spring fixed to body S and its moving direction is vertical to A_1A_2. The position of the block s to body S is determined by coordinate u (Fig. 1). Thus, we can say, $u = u_0$ = constant in the first situation and $u = u(t)$ in the second. Obviously, in the first situation the equilibrium state defined by $\varphi = 0$ and $u = u_0$ is stable, yet it is nonasymptotically stable. Thus the problem arises: Is the equilibrium state asymptotically stable when s is defrozen? If it is asymptotically stable, we can say the problem of passive stabilization is solved. The following statement will verify that this problem can be solved only in nonlinear case by the method in the paper [2].

2 Movement equations and their reduction to special form

The kinetic energy of the studied mechanical system is

$$T = \frac{1}{2}(M_* l^2\dot{\varphi}^2 + m\dot{u}^2 - 2ml\dot{\varphi}\dot{u}\sin\varphi),$$

where $M_* = M + m$. And the inelastic potential energy of this system is

$$H = -(M_* l\cos\varphi + mu)g + \frac{1}{2}\beta u^2 + \frac{1}{4}\bar{\beta}u^4,$$

where $\beta > 0, \beta$ and $\bar{\beta}$ are a coefficients. Block s moves with an inelastic potential energy.

$$V_u = \frac{1}{2}\beta u^2 + \frac{1}{4}\bar{\beta}u^4.$$

Block s moves with a nonlinear friction $Q_u = -\alpha_1\dot{u} - \alpha_2\dot{u}^3$, namely, the generalized force which is denoted by Q_u where $\alpha_1 > 0$. Thus, the studied mechanical system has Lagrange's equation of second type as follows

$$\begin{aligned} &M_* l^2\ddot{\varphi} - ml\sin\varphi\ddot{u} + M_* gl\sin\varphi = 0, \\ &m\ddot{u} - ml\sin\varphi\ddot{\varphi} - ml\dot{\varphi}^2\cos\varphi - \\ &mg + \beta u + \bar{\beta}u^3 + \alpha_1\dot{u} + \alpha_2\dot{u}^3 = 0. \end{aligned} \tag{1}$$

From Equation (1), we obtain

$$\begin{aligned} \ddot{\varphi} &= \frac{\sin\varphi(ml\dot{\varphi}^2\cos\varphi - Mg - \beta u - \bar{\beta}u^3 - \alpha_1\dot{u} - \alpha_2\dot{u}^3)}{l(M + m\cos^2\varphi)}, \\ \ddot{u} &= \frac{M_*(ml\dot{\varphi}^2\cos\varphi + mg\cos^2\varphi - \beta u - \bar{\beta}u^3 - \alpha_1\dot{u} - \alpha_2\dot{u}^3)}{m(M + m\cos^2\varphi)}, \end{aligned} \tag{2}$$

Equation (2) admits the particular solution

$$\varphi = 0, \quad u = u_0 \quad \text{and} \quad mg - \beta u_0 - \bar{\beta}u_0^2 = 0,$$

$$\text{If} \quad \bar{\beta} = 0, \quad u = u_0 = \frac{mg}{\beta} \tag{3}$$

corresponding to an equilibrium state of (2).

Supposing $\varphi = x$ and $u = u_0 + v$ in perturbed movement, we can decompose the right side of Equation (2) to series about perturbation $x, \dot{x}, v, \dot{v}$ till terms of the third order smallness (including the third order) as follows:

$$\begin{aligned} \ddot{x} &= -\lambda^2 x + b_1 xv + b_2 x\dot{v} + c_1 x^3 + c_2 x\dot{x}^2 + \cdots, \\ \ddot{v} &= g + d_1(u_0 + v) + d_2\dot{v} + a_1 x^2 + e_1\dot{x}^2 + \\ &\quad \frac{1}{2}b_1(u_0 + v)x^2 + \frac{1}{2}b_2 x^2\dot{v} + e_2\dot{v}^3 + \\ &\quad f(u_0 + v)^3 + \cdots, \end{aligned} \tag{4}$$

where

$$a_1 = \frac{-Mg}{lM_*}, \quad b_1 = -\frac{\beta}{lM_*}, \quad b_2 = -\frac{\alpha_1}{lM_*},$$

$$c_1 = \frac{Mg(6M + 5m)}{6lM_*}, \quad c_2 = \frac{m}{M_*},$$

$$d_1 = -\frac{\beta}{m}, \quad d_2 = -\frac{\alpha_1}{m}, \quad e_1 = l,$$

$$e_2 = -\frac{\alpha_2}{m}, \quad f = -\frac{\bar{\beta}}{m}, \quad \lambda^2 = \frac{g}{l}.$$

By introducing variables

$$\dot{x} = \lambda y, \quad \dot{v} = w, \tag{5}$$

we have the following equation, which consists of four differential equation of the first order

$$\begin{aligned} \dot{x} &= \lambda y, \\ \dot{y} &= -\lambda x + p_1 xw + p_2 xw + q_1 xy^2 + q_2 x^3 + \cdots, \\ \dot{v} &= w, \\ \dot{w} &= \bar{d}_1 v + d_2 w - l_1 x^2 + l_2 y^2 + l_3 v^2 + \\ &\quad f_1 x^2 v + f_2 x^2 w + f_3 w^3 + fv^3 + \cdots, \end{aligned} \tag{6}$$

where

$$p_1 = -\frac{\beta}{M_*\sqrt{lg}}, \quad p_2 = -\frac{\alpha_1}{M_*\sqrt{lg}},$$

$$q_1 = \frac{1}{M_*}\sqrt{\frac{l}{g}}, \quad q_2 = \frac{M(M + 5m)}{6M_*^2}\sqrt{\frac{g}{l}},$$

$$\bar{d}_1 = -\beta - 3\bar{\beta}u_0^2, \quad l_1 = l_2 = -g, \quad l_3 = -3\bar{\beta}u_0,$$

$$f_1 = -\frac{\alpha_1}{M_*}, \quad f_2 = -\frac{\beta}{m}, \quad f_3 = -\frac{\beta}{M_*}.$$

It is possible to point out that the linearized system of Equation (6) falls into two independent linear systems, the first

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -\lambda x, \end{aligned} \tag{7}$$

corresponds to a pair of purely imaginary roots of its characteristic equation, and the second

$$\begin{aligned} \dot{v} &= w, \\ \dot{w} &= \bar{d}_1 v + d_2 w, \end{aligned} \tag{8}$$

corresponds to a pair of complex roots with negative real parts. According to the conclusion of the known Lyapunov's theorem (see[3]), we have a critical case, from which it is impossible to obtain the conclusion about the stability or instability of solution (3) in system (4) without using nonlinear terms.

3 Constructing auxiliary function of Lyapunov's type

Since linear system (8) has eigenvalues of negative real part (since $\bar{d}_1 < 0, d_2 > 0$), there exists the definitely positive Lyapunov's function $V^{(2)}(v,w)$ such that its time derivative $\dot{V}^{(2)}(v,w)$ along Equation (8) is negatively definite. For example, function

$$V^{(2)}(v,w) = \frac{1}{2}(m_{11}v^2 + 2m_{12}vw + m_{22}w^2) \tag{9}$$

satisfies those conditions, where

$$m_{11} = \frac{\beta' + 3\bar{\beta}' u_0^2 + 1}{\alpha'_1} + \frac{\alpha'_1}{\beta' + 3\bar{\beta}' u_0^2},$$

$$m_{12} = \frac{1}{\beta' + 3\bar{\beta}' u_0^2},$$

$$m_{22} = \frac{\beta' + 3\bar{\beta}' u_0^2 + 1}{\alpha'_1(\beta' + 3\bar{\beta}' u_0^2)}).$$

where

$$\beta' = \frac{\beta l}{mg}, \quad \bar{\beta}' = \frac{\bar{\beta} l^3}{mg}, \quad \alpha'_1 = \frac{\alpha_1}{m}\sqrt{\frac{l}{g}},$$

since $m_{11} > 0, m_{11}m_{22} - m_{12}^2 = \dfrac{(\beta' + 3\bar{\beta}' u_0^2 + 1)^2}{\alpha'_1(\beta' + 3\bar{\beta}' u_0^2)} + \dfrac{1}{\beta' + 3\bar{\beta}' u_0^2} > 0$ and its time derivative $\dot{V}^{(2)}(v,w)$ along Equation (8) is

$$\dot{V}^{(2)}(v,w) = -(\lambda^2 v^2 + w^2) < 0. \tag{10}$$

Moreover, according to the methodology of critical case of n pair purely imaginary roots [2], we construct auxiliary function $V(x,y,v,w)$ as follows:

$$\begin{aligned} V(x,y,v,w) = \\ \frac{p}{2}(x^2 + y^2) + qV^{(2)}(v,w) + \\ V^{(3)}(x,y,v,w) + \\ V^{(4)}(x,y,v,w), \end{aligned} \tag{11}$$

where $V^{(2)}(v,w)$ is defined by formula (9), and

$$V^{(r)}(x,y,v,w) = \sum_{i+j+k+n=r} a_{ijkn}x^i y^j v^k w^n, \quad r = 3,4 \tag{12}$$

are the forms of the third and the fourth orders about x, y, v, w, here a_{ijkn} are constant coefficients and its algorithm will be given later, p and q are arbitrary constants.

The derivative $\dot{V}(x,y,v,w)$ along Equation (6) by virtue of Equality (10) has the following form:

$$\dot{V}(x,y,v,w) = -q(\lambda^2 v^2 + w^2) + \dot{V}^{(3)}(x,y,v,w) + \dot{V}^{(4)}(x,y,v,w) + \cdots,$$

where $\dot{V}^{(3)}$ and $\dot{V}^{(4)}$ are the forms of the third and the fourth orders respectively defined by:

$$\begin{aligned} \dot{V}^{(r)}(x,y,v,w) = \lambda y \frac{\partial V^{(r)}}{\partial x} - \lambda x \frac{\partial V^{(r)}}{\partial y} + \\ w\frac{\partial V^{(r)}}{\partial v} + (\bar{d}_1 v + dw)\frac{\partial V^{(r)}}{\partial w} + \\ W^{(r)}(x,y,v,w), \quad r = 3,4, \end{aligned} \tag{13}$$

where

$$\begin{aligned} W^{(3)}(x,y,v,w) = \\ py(p_1xv + p_2xw) + \\ q(m_{12}v + m_{22}w)(-gx^2 + gy^2 + l_3v^2), \end{aligned} \tag{14}$$

$$\begin{aligned} W^{(4)}(x,y,v,w) = \\ py(q_2x^3 + q_1xy^2) + \\ q(m_{12}v + m_{22}w)(f_1x^2v + f_2x^2w + \\ f_3w^3 + fv^3) + (p_1xv + p_2xw)\frac{\partial V^{(3)}}{\partial y} + \\ (-gx^2 + gy^2 + l_3v^2)\frac{\partial V^{(3)}}{\partial w}. \end{aligned} \tag{15}$$

We can seek the coefficients of form $V^{(3)}(x,y,v,w)$ from the condition

$$\begin{aligned} \dot{W}^{(3)}(x,y,v,w) = \lambda y\frac{\partial V^{(3)}}{\partial x} - \lambda x\frac{\partial V^{(3)}}{\partial y} + w\frac{\partial V^{(3)}}{\partial v} + \\ (\bar{d}_1 v + d_2 w)\frac{\partial V^{(3)}}{\partial w} + W^{(3)} \equiv 0, \end{aligned} \tag{16}$$

where $W^{(3)}$ is defined by formula (14).

According to [3], this equation by the form $V^{(3)}(x,y,v,w)$ has a unique solution.

Substituting $V^{(3)}(x,y,v,w)$ (defined by Equation (12)) into Equation (16), we obtain coefficients of $V^{(3)}(x,y,v,w)$.

For the sake of convenience, we study firstly the situation of $\beta = 0$ (In the next paper we will study further the situation of $\beta \neq 0$), thus we obtain

$$a_{2001} = - a_{0201} = a_p p + a_q q, \tag{17}$$

where

$$a_p = \frac{4\alpha_1\lambda^2}{M_* l\Delta}, \quad a_q = \frac{-5\lambda g^2}{\Delta}, \quad \Delta = \frac{4\lambda^2}{m^2}\alpha_1^2 + (4\lambda^2 - \frac{\beta}{m})^2 \tag{18}$$

and we can see that all nonzero coefficients of $V^{(3)}$ are the function of arbitrary parameters p and q. In

$$W^{(4)}(x,y,v,w) = \sum_{i+j+k+n=4} w_{ijkn} x^i y^j v^k w^n, \tag{19}$$

we only need to find out the coefficients w_{4000}, w_{0400}, w_{2200}, which are necessry to the solution of the problem of passive stabilization.

According to (12), (19) and (15), we find

$$w_{4000} = - a_{2001}, \quad w_{0400} = a_{0201}, \quad w_{2200} = a_{2001} - a_{0201}, \tag{20}$$

and a_{2001}, a_{0201} are determined by the relation between (17) and (18).

Now we can seek the coefficients of form $V^{(4)}(x,y,v,w)$ from the condition

$$\dot{V}^{(4)}(x,y,v,w) = \lambda y \frac{\partial V^{(4)}}{\partial x} - \lambda x \frac{\partial V^{(4)}}{\partial y} + w \frac{\partial V^{(4)}}{\partial v} + (\bar{d}_1 v + d_2 w)\frac{\partial V^{(4)}}{\partial w} + W^{(4)}(x,y,v,w) = C(x^2+y^2)^2, \tag{21}$$

where $V^{(4)}$ and $W^{(4)}$ are defined by Equations (12) and (19) respectively.

From the lemma of [2], there exists a unique C, in which Equation (21) has the solution by the form $V^{(4)}$, and also

$$C = \frac{1}{8}(3w_{4000} + w_{2200} + 3w_{0400}), \tag{22}$$

or by virtue of Equation (20)

$$C = \frac{1}{4}(a_{0201} - a_{2001}) = -\frac{1}{2}(a_p p + a_q q) = -pC_p + qC_q.$$

According to Equation (18)

$$C_p = \frac{1}{2}a_p > 0, \quad C_q = -\frac{1}{2}a_q > 0. \tag{23}$$

Thus we obtain

$$V = \frac{p}{2}(x^2+y^2) + qV^{(2)}(v,w) + V^{(3)}(x,y,v,w) + V^{(4)}(x,y,v,w), \tag{24}$$

where the coefficients of $V^{(4)}$ are linear functions of arbi-

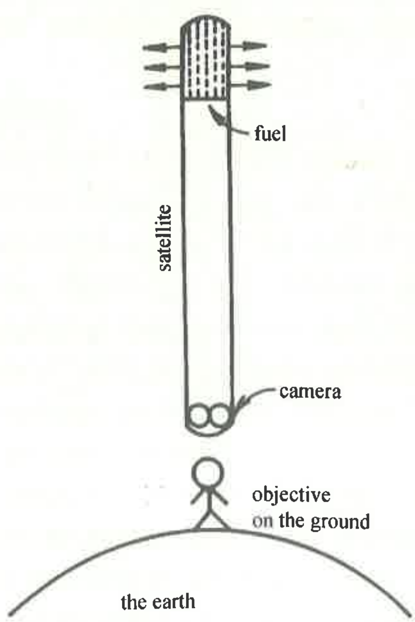

Fig. 2 The satellite can be stabilized by means of additional energy

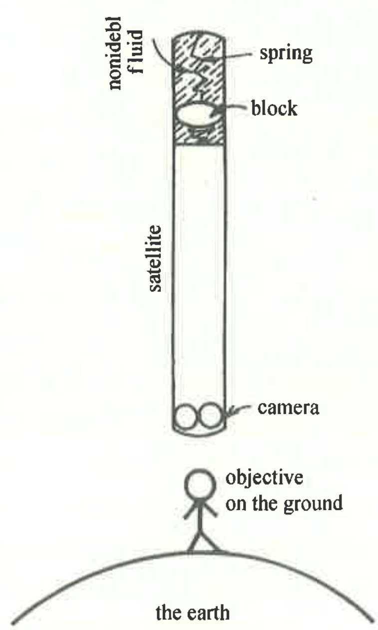

Fig. 3 The satellite can be stabilized by means of the relative motion

trary parameters p and q as those of $V^{(3)}$, and its derivative along Equation (6) is

$$\dot{V} = -q(\lambda^2 v^2 + w^2) + (-pC_p + qC_q)(x^2 + y^2)^2 + \cdots, \quad (25)$$

where the ellipses denote infinitely small terms whose orders are higher than the fourth.

So the particular solution in Equation (3) is asymptotically stable. This shows that the problem of passive stabilization about oscillating of body S is sloved by introducing supplementary degree of freedom which is determined by coordinate u (namely, block s is defrozen).

We mark that the oscillating stabilization of the studied system is realized without supplementary sources of energy.

Remark We point out that if we use a geostationary satellite to take a picture of a certain object on the ground, it is very important for the camera to take aim, namely, the motion of the satellite must be asymptotically stable. Usually, the effect brought by a small perturbation is removed by the reaction of a jet stream, which is produced by burning fuel, but the astronautic fuel is very expensive (Fig.2, Fig.3). The obtained result by us shows that the satellite can also be stabilized by means of the relative motion of some piece of the satellite moving in a nonideal fluid as an oscillator with damping and this does not require additional energy.

References

1 Gorr G V at al. Nonlinear analysis of the behaviour of mechanical systems. Naukova Dumka, Kiev, 1984(in Russian)

2 Savchenko A Ya and Ignatyev A O. Some problem of stability of nonautonomous dynamical systems. Naukova Dumka, Kiev, 1989 (in Russian)

3 Lyapunov A M. Collected work, Vol.2, Press of the Academy of Science SSSR, Moscow, 1956 (in Russian)

4 Savchenko A Ya. On passive stabilization. Louvain-la-Neuve, Catholique University, 1994,(1) (in Russian)

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