A Hierarchical Approach for H_∞ Control and Its Application to Airplane Landing under Wind Disturbance *

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Abstract: This paper is concerned with the problem of airplane landing under the influence of wind disturbance. By combining the H_∞ optimization method with the structural perturbation approach of large-scale systems, a hierarchical H_∞ controller is developed to guarantee the robust stability of the system. Since the system matrix is not invertible, a new method is suggested to get the perturbation matrix. The application to flight control shows that the influence of wind disturbance can be restrained by using the new controller.

Key words: robust control; hierarchical H_∞ optimization; wind disturbance; structural perturbation approach; flight control

递阶 H_∞ 方法及其在阵风干扰下飞机着陆控制中的应用

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摘要: 本文对阵风干扰下的飞机着陆控制问题进行了研究,将 H_∞ 方法与大系统结构摄动递阶控制思想相结合,给出了具有鲁棒稳定性的递阶 H_∞ 控制器.在求解摄动矩阵时,针对系统矩阵不可逆的情况,提出了一种摄动矩阵的计算方法.对某型飞机的应用仿真结果表明,采用本文所给的递阶 H_∞ 控制器,阵风干扰能够被抑制在工程允许的范围之内.

关键词: 鲁棒控制; 递阶 H_∞ 控制; 阵风干扰; 结构摄动法; 飞行控制

1 Introduction

The study on airplane landing control is of great importance to promote the efficiency of air transportation and ensure the flight safety. At present, most problems of control, navigation, communication and management in the cruise period of airplane have been solved and also a lot of flight tests have been done to approve the results. However, there are still many unsolved problems in the period of airplane landing, such as control and communications under wind disturbance. Even few theoretical results have been obtained.

Disturbed by wind, the flying airplane may sometimes deviate from its flight path, which may lead to a flight accident or a delay in the landing process. Presently, the research work concerning this problem is mostly focused on two aspects, i.e. Linear Quadratic Gaussian (LQG) or H_2 theory and wind estimation. For LQG, disturbance inputs are modeled as white noise, however, in most cases the effect of disturbance is biased and different from what has been assumed in LQG. Since wind disturbance is stochastic and uncertain, it does not fit very well into the framework of LQG. Moreover, in many circumstances especially for robust control, model uncertainty must be taken into account when we design the controller. The effect of model uncertainty can not be suitably modeled as white noise. For the reasons mentioned above, in this paper, the hierarchical H_∞ optimization method is applied to design a robust controller for the problem of airplane landing control under wind disturbance.

2 Problem description

For airplane landing control, flight state vector can be chosen as

$$x = [h, u, w, q, \theta]^{\mathrm{T}},$$

where h is flight altitude; u and w the velocity components along x axis and z axis respectively; q pitch rate, and θ pitch angle. The input vector is

$$u = [\delta_e, \delta_c]^{\mathrm{T}},$$

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where δ_e and δ_c are the deflections of elevator and canards respectively.

Thus, the model of an airplane can be described by the following linear, time-invariant system:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + \Gamma\xi(t), \\ z(t) = \begin{bmatrix} Dx(t) \\ Cu(t) \end{bmatrix}, \\ y(t) = x(t), \end{cases}$$

where the signal $\xi(t)$ contains all external inputs, including disturbances, sensor noises, and commands; the output $z(t)$ is an error signal; $y(t)$ is the observation vector and $u(t)$ is the control input vector; A, B, Γ are the system matrix, input matrix and disturbance matrix, respectively; C and D are weighting matrices. For the problem (A, B) is controllable, and (A, C) is observable.

Our goal is to design a linear state feedback controller

$$u(t) = Kx(t),$$

to make the system satisfy the following performance specifications:

1) The closed-loop system is internally stable, i.e. $A + BK$ is stable.

2) The closed-loop system satisfies

$$\| T_{\xi z}(j\omega) \|_{\infty} < \gamma,$$

where $T_{\xi z}(j\omega)$ is the transfer function from ξ to z; γ is a positive constant which is selected to be less than 1 to guarantee the robust stability of the system.

If the upper bound of wind disturbance is known, a robust feedback controller can be designed by using the H_∞ method. However, since airplane is a high-order and complicated dynamic system, we have to solve a high-order algebraic Riccati equation if the concentrated H_∞ method is used directly, which may bring us a large quantity of computation work and make the selection of weighting matrices difficult. To avoid this problem, in this paper, we combine the H_∞ optimization method with the theory of large-scale systems and propose a hierarchical H_∞ optimization method based on structural perturbation approach.

3 Design of hierarchical H_∞ controller

The main idea of the hierarchical H_∞ optimization method is to divide the large-scale system into several lower-order subsystems. For each subsystem, H_∞ optimization method is used directly to optimize the subsystem and a coordinator is designed to get the near optimality of the whole system. Therefore, the optimal control of the ith subsystem can be described as

$$u_i(t) = u_i^*(t) + u_i^g(t),$$

where $u_i^*(t)$ is called local control, which is solved by the ith subsystem without considering the interconnections of each subsystem; $u_i^g(t)$ obtained from the coordinator is called global control or rectification control which can compensate the influence of subsystem interconnections.

When the system is working, structural perturbation may occur because of disconnections, reconnections and interconnection variances of these subsystems. The structural perturbation approach emphasizes the reliability of system with structural perturbation. We can obtain the rectification control by solving the perturbation matrix B^P of the block input matrix $\bar{B}$. Under this condition, the system can be reconstructed as

$$\dot{x}(t) = (\bar{A} + G)x(t) - (\bar{B} + B^P)Kx(t), \quad (1)$$

where $\bar{A}, \bar{B}$ are block matrices constructed by the corresponding matrix of each subsystem, i.e.

$$\bar{A} = \text{block-diag}\{A_1, A_2, \cdots, A_N\},$$

$$\bar{B} = [B_1 \vdots B_2 \vdots \cdots \vdots B_N]^{\mathrm{T}};$$

G is a interconnection matrix of the system.

3.1 Optimization of subsystems

It is assumed that the original large-scale system has been divided into N connected subsystems. The ith subsystem can be described as

$$\dot{x} = A_i x_i + B_i u_i + L_i \eta_i + \Gamma_i \xi_i, \quad i = 1, 2, \cdots, N,$$

where A_i and B_i are the system matrix and control matrix of the ith subsystem respectively. η_i is the interconnection input vector of the ith subsystem and L_i is the coefficient matrix of η_i. The performance specification of the subsystem is

$$\| H \|_{\infty, i} < \gamma.$$

Solve the following N independent algebraic Riccati equations

$$P_i A_i + A_i^{\mathrm{T}} P_i + P_i(\gamma_i^{-2} \Gamma_i \Gamma_i^{\mathrm{T}} - B_i R_i^{-1} B_i^{\mathrm{T}}) P_i + Q_i = 0,$$
$$i = 1, 2, \cdots, N.$$

If (A_i, B_i) is controllable, there must exist a positive constant γ_{i0}, and when $\gamma_i > \gamma_{i0}$, the ith subsystem has only one positive definite solution P_i. Thus we can obtain a group of positive definite solutions $P_i, i = 1,2,\cdots,N$.

Let $$P = \text{block-diag}\{P_1, P_2, \cdots, P_N\},$$
$$R = R_i, \quad i = 1,2,\cdots,N$$
and the local optimal feedback matrix K^* can be obtained as the following
$$K^* = -R^{-1}\bar{B}^{\mathrm{T}}P.$$
Next, the hierarchical method will be used to get the near optimal solution of the whole system.

3.2 Solution of the perturbation matrix

In this subsection we would like to solve the perturbation matrix B^P to achieve completely closed control. The following lemma shows how to reach the goal:

Lemma 1 For nonsingular matrix $(\bar{A} + G)$, if there exists a diagonally-symmetric matrix S satisfying the following Lyapunov equation
$$S(\bar{A} + G) + (\bar{A} + G)^{\mathrm{T}}S + G^{\mathrm{T}}P\bar{A} - \bar{A}^{\mathrm{T}}PG = 0 \tag{2}$$
and a matrix $\hat{P} = (S - PG)(\bar{A} + G)^{-1}$ can be given to make the matrix $(P + \hat{P})$ positive definite, the perturbation matrix can be solved by the following equation
$$B^P = -(P + \hat{P})^{-1}\hat{P}\bar{B}. \tag{3}$$
The performance specification of the whole system can be shown as
$$\hat{J}(t_0, x(t_0)) = \frac{1}{2}x^{\mathrm{T}}(t_0)(P + \hat{P})x(t_0).$$

Proof If $\{(\bar{A} + G), (\bar{B} + B^P)\}$ is controllable, let V be the positive definite solution of the following algebraic Riccati equation
$$V(\bar{A} + G) + (\bar{A} + G)^{\mathrm{T}}V - V(\bar{B} + B^P)R^{-1}(\bar{B} + B^P)^{\mathrm{T}}V + Q = 0. \tag{4}$$
Then the closed-loop control
$$\hat{u}(t) = -R^{-1}(\bar{B} + B^P)^{\mathrm{T}}Vx(t), \tag{5}$$
will make the system have the minimal performance specification $\frac{1}{2}x^{\mathrm{T}}(t_0)Vx(t_0)$, and the feedback system can be denoted by
$$\dot{x}(t) = (\bar{A} + G)x(t) - (\bar{B} + B^P)R^{-1}\cdot(\bar{B} + B^P)^{\mathrm{T}}Vx(t). \tag{6}$$
Compare Eq.(1) with Eq.(6) and it can be seen that
$$R^{-1}(\bar{B} + B^P)^{\mathrm{T}}V = R^{-1}\bar{B}^{\mathrm{T}}P. \tag{7}$$
Substitute Eq.(7) into Eq.(4). Considering that matrix P is positive definite, we have
$$VG + G^{\mathrm{T}}V + \bar{A}^{\mathrm{T}}(V - P) + (V - P)\bar{A} = 0. \tag{8}$$
Divide Eq.(8) into two diagonally-symmetric parts and it can be seen that
$$PG + (V - P)(\bar{A} + G) = S, \tag{9}$$
where S is a diagonally-symmetric matrix. Substitute Eq.(9) into Eq.(2). Considering that $(V - P)$ is a symmetric matrix, we can test that S satisfies Eq.(2).

Let
$$\hat{P} = (V - P)$$
and substitute it into Eq.(3). Then Eq.(8) can be achieved.

For the actual plant we use in this paper, the system matrix $(\bar{A} + G)$ is not invertible. So Lemma 1 can not be used directly. To solve the problem, the following result is given.

Let
$$\tilde{A} = (\bar{A} + G)^{\mathrm{T}}, \quad \tilde{B} = (S - PG)^{\mathrm{T}}$$
and $$X = (V - P).$$
Transpose both sides of Eq.(9). Since matrix $(V - P)$ is symmetric, it can be seen that
$$\tilde{A}X = \tilde{B}, \tag{10}$$
Eq.(10) can be written in block form:
$$\begin{bmatrix} 0 & 0 \\ \tilde{A}_{1(4\times1)} & \tilde{A}_{2(4\times4)} \end{bmatrix}\begin{bmatrix} X_{11(1\times1)} & X_{12(1\times4)} \\ X_{21(4\times1)} & X_{22(4\times4)} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \tilde{B}_{1(4\times1)} & \tilde{B}_{2(4\times4)} \end{bmatrix}.$$
Then we have
$$\tilde{A}_1X_{11} + \tilde{A}_2X_{21} = \tilde{B}_1,$$
$$\tilde{A}_1X_{12} + \tilde{A}_2X_{22} = \tilde{B}_2.$$
Since X is a symmetric matrix, if X_{11} is given definitely, X i.e. $(V - P)$ can be determined. Thus, the perturbation matrix B^P and the rectification feedback matrix can be achieved from Eq.(3). The rectification feedback matrix is
$$K^g = -R^{-1}B^{P\mathrm{T}}P.$$
Finally the suboptimal feedback matrix of the whole system can be obtained as
$$K = K^* + K^g = -R^{-1}(\bar{B} + B^P)^{\mathrm{T}}P.$$

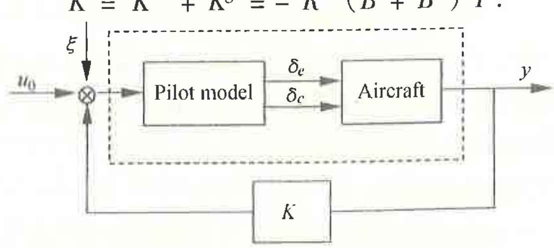

Fig. 1 Control block diagram

The control block diagram is shown in Fig. 1.

4 Simulations

In this section, some simulation results are given to test the performance of the hierarchical H_∞ controller presented in Section 3. The original 5th-order system is divided into two subsystems: $x_1 = [h, u, w]^T$, $x_2 = [q, \theta]^T$, one is 3rd-order and the other is 2nd-order. The two subsystems are

· Subsystem 1

$$A_1 = \begin{bmatrix} 0 & 0.087 & -0.996 \\ 0 & -0.17 & -0.003 \\ 0 & -0.255 & 0.008 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 10.334 & -0.556 \\ 0.692 & 1.09 \end{bmatrix},$$

$$\Gamma_1 = \begin{bmatrix} 0.087 & -0.996 \\ -0.17 & -0.003 \\ -0.255 & 0.008 \end{bmatrix},$$

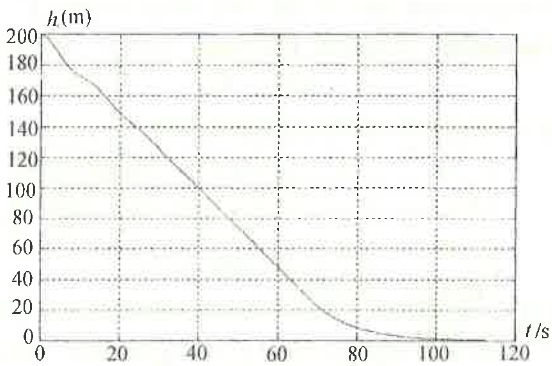

Fig. 2 The response plot of h

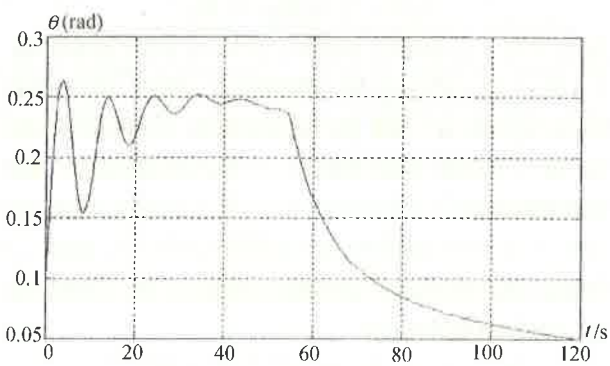

Fig. 4 The response plot of θ

Fig. 2 to Fig. 4 are the response plot of h, $\dot{h}$ and θ, respectively. From them we can see that the speed and steadiness for airplane landing is fairly good.

5 Conclusion

A hierarchical H_∞ method is presented in this paper. The numerical accuracy and computational efficiency

$$L_1 = \begin{bmatrix} 0 & 0.001 & -0.125 \\ 0 & 0 & 0 \end{bmatrix}.$$

· Subsystem 2

$$A_2 = \begin{bmatrix} -1.47 & 0 \\ 1 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -19.8 & 2.47 \\ 0 & 0 \end{bmatrix},$$

$$\Gamma_2 = \begin{bmatrix} 0.001 & -0.125 \\ 0 & 0 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0 & 0.608 \\ -7.014 & -9.773 \\ 200.01 & -0.855 \end{bmatrix}.$$

For the actual plant used in this paper, it can be shown that

$$\bar{A} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad G = \begin{bmatrix} 0 & L_2 \\ L_1 & 0 \end{bmatrix}.$$

Apply the hierarchical H_∞ controller to the actual flight control system, the following optimal feedback matrix can be obtained as

$$K = \begin{bmatrix} 0.017 & -1.301 & -2.615 & 1.256 & -0.244 \\ -1.036 & 0.762 & 1.270 & -1.347 & -6.378 \end{bmatrix}.$$

The simulation results can be shown by Fig. 2 to Fig. 4:

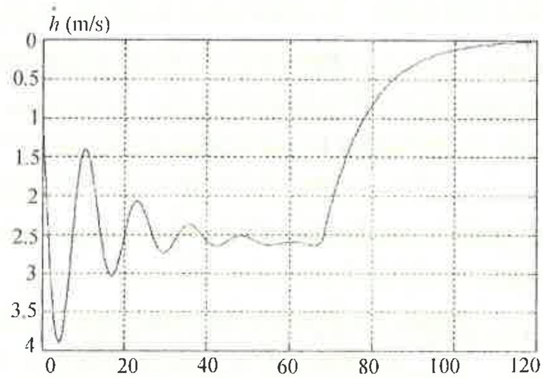

Fig. 3 The response plot of $\dot{h}$

are increased since the solution of high-order Riccati equation is avoided. The new approach is applied to airplane landing control and the satisfactory simulation results are obtained. It can also be applied to solve other control problems related to model uncertainty.

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cal model which is built according to this method has successfully applied in the broad components continuous reforming real-time dynamic simulation system of a certain petrochemical corporation. It has contributed greatly to the simulative running of the equipment's practical running and the training of workers. It has received its user's high praise and had passed the appraise of the ministry level.

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