Minimized Active Control for Flexible Space Structure

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Abstract: Many chaotic phenomena arise from the motion of the large flexible space structure, and the selection of chaotic control methods plays an important role in the design of spacecraft structures[1~3]. In this paper we present how to find the minimized active control for large flexible space sructure, i.e., the structure redesign criterion is to minimize the control power needed to satisfy control law for any given controller, such that chaos would not develop.

Key words: flexible structure; minimized active; passive control; chaotic motion

大型柔性空间结构的极小化主动控制

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摘要:大型柔性空间结构的运动会引起混沌现象的发生,混沌控制方法的选择对航天飞行器的设计起了很大的作用.本注记利用 Smith M J 等人(1992)提出的方法,对大型柔性空间结构进行极小化主动控制,以避免混沌现象的发生.

关键词:极小化主动控制;被动控制;混沌运动

1 Introduction

The model equation of spacecraft flexible structure is still highly nonlinear[4], so if it is not under control, the chaotic motion will develop. In this paper we adopt the idea given by Smith M J, Grigoriadis K M and Skelton R E (SGS)(1992)[5] to find minimized active control for large flexible spacecraft, such that chaos would not develop. SGS's idea is to separate the active control law, Eq.(2), into a passive part (pp) which is implemented into the physical system by the redesign and an active part (AP) which constitutes the remaining active control law required after structure redesign.

Their idea is appreciably different from the idea of the traditional control. In the traditional control of the large flexible space structure, a structure is traditionally designed first, then a controller is designed for the given structure. In this paper, the structure is designed after a controller is given.

2 Main problem and results

Consider the following controlling design problem[4]:

$$M\ddot{q} + D\dot{q} + Kq + \gamma q^3 = Au, \tag{1}$$

$$u = Gx, \tag{2}$$

$$x = B[q^3, q, \dot{q}]^{\mathrm{T}}. \tag{3}$$

This system arises from the Transatmospheric vehicle (National Aerospace Plane USA (1993)). Equations (1) ~ (3) are the generalized SGS structure dynamics system with output feedback control[5]. If $\gamma = 0$ (matrix), Eq.(1) is the well known Duffing equation; if $A = 0$ (matrix), Eq.(1) is the Lienard equation, where

$q(t)$ = state vector $\in \mathbb{R}_q^n$,

$u(t)$ = control force $\in \mathbb{R}_u^n$,

$x(t) \in \mathbb{R}_x^n$,

M = mass matrix = $M^{\mathrm{T}} > 0$,

K = stiffness matrix = $K^{\mathrm{T}} \geqslant 0$,

γ = constant matrix = $\gamma^{\mathrm{T}} \geqslant 0$,

D = damping matrix = $D^{\mathrm{T}} \geqslant 0$,

A = input matrix has full column rank,

G = controller(assume that is given),

B = output matrix has full row rank,

$AA^* = I$ and $B^*B = I$.

Suppose that G is given; i.e., the control law, Eq.(2), has already been designed such that the closed-loop system has acceptable performance.

Using SGS's idea, the structure redesign concept is

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to change the mass matrix by ΔM, the damping matrix by ΔD, the stiffness matrix by ΔK and the constant matrix by $\Delta\gamma$ to produce the closed-loop system after redesign:

$$(M+\Delta M)\ddot{q}+(D+\Delta D)\dot{q}+(K+\Delta K)q+(\gamma+\Delta\gamma)q^3 = AG_aB[q^3,q,\dot{q}]^{\mathrm{T}}, \tag{4}$$

where

$$u_a = G_aB[q^3,q,\dot{q}]^{\mathrm{T}}. \tag{5}$$

G_a = the active part of the controller after redesign, and

$$\Delta M\ddot{q}+\Delta D\dot{q}+\Delta Kq+\Delta\gamma q^3 = \text{the passive part}.$$

The structure redesign criterion is to minimize the control power (that is $\| u_a \|$) needed to satisfy the control law:

$$Au = AGB[q^3,q,\dot{q}]^{\mathrm{T}} = AG_aB[q^3,q,\dot{q}]^{\mathrm{T}} - [\Delta\gamma \quad \Delta K \quad \Delta D][q^3,q,\dot{q}]^{\mathrm{T}} - \Delta M\ddot{q} \tag{6}$$

for any given G.

Since the closed-loop system response remains unchanged before and after redesign, all of the designed closed loop system properties remain unchanged. Our objective is to find the passive control ($\Delta M,\Delta D,\Delta K,\Delta\gamma$) to minimize the active control:

$$\| u_a \| = \text{norm of } G_aB[q^3,q,\dot{q}]^{\mathrm{T}}$$

for the given controller G.

Let A_γ, A_K, A_d, A_m be the spring, stiffness, damping, and mass connectivety matrices of the structural system. According to the representation formulas propounded by Skelton R E, Hanks B R and Smith M J (1992)[6], any changes in the structural parameters γ_j to $\gamma_j+\Delta\gamma_j$, k_j to $k_j+\Delta k_j$, d_j to $d_j+\Delta d_j$, and m_j to $m_j+\Delta m_j$ can be represented in the following form:

$$\gamma+\Delta\gamma = \gamma + A_\gamma G_\gamma A_\gamma^{\mathrm{T}}, \tag{7}$$

$$K+\Delta K = K + A_K G_K A_K^{\mathrm{T}}, \tag{8}$$

$$D+\Delta D = D + A_d G_d A_d^{\mathrm{T}}, \tag{9}$$

$$M+\Delta M = M + A_m G_m A_m^{\mathrm{T}}, \tag{10}$$

where

$$G_\gamma = \mathrm{diag}(\cdots \quad \Delta\gamma_j \quad \cdots), \tag{11}$$

$$G_k = \mathrm{diag}(\cdots \quad \Delta k_j \quad \cdots), \tag{12}$$

$$G_d = \mathrm{diag}(\cdots \quad \Delta d_j \quad \cdots), \tag{13}$$

$$G_m = \mathrm{diag}(\cdots \quad \Delta m_j \quad \cdots). \tag{14}$$

These produce the following expression for the desired control law, Eq.(6); substituting the solution of $\ddot{q}$ from the system (1), (2) and (3), we get

$$AGBW = (\tilde{G}_a + \tilde{G}_p)W, \tag{15}$$

where

$$\tilde{G}_a = G_{\text{active}}, \quad \tilde{G}_p = G_{\text{passive}}; \quad W = [q^3,q,\dot{q}]^{\mathrm{T}}. \tag{16}$$

Make use of Eqs. (7 ~ 15), we can obtain the control law in the following expression:

$$\tilde{G}_a = A\tilde{G}_aB, \quad G_p = -I_0A_pG_pA_p^{\mathrm{T}}R, \tag{17}$$

where

$$I_0 = [I \quad I \quad I \quad I], \tag{18}$$

$$A_p = \begin{bmatrix} A_\gamma & 0 & 0 & 0 \\ 0 & A_k & 0 & 0 \\ 0 & 0 & A_d & 0 \\ 0 & 0 & 0 & A_m \end{bmatrix}, \tag{19}$$

$$G_p = \begin{bmatrix} G_\gamma & 0 & 0 & 0 \\ 0 & G_k & 0 & 0 \\ 0 & 0 & G_d & \\ 0 & 0 & 0 & \end{bmatrix} \tag{20}$$

and

$$R = \begin{bmatrix} I \\ I \\ I \\ M^{-1}(AGB - [\gamma \quad K \quad D]) \end{bmatrix}. \tag{21}$$

On the basis of the hypothesis of this paper that the input matrix A has full column rank and that the output matrix B has full row rank (which does not dependent upon control and sensors), we can get:

3 Conclusion

If and only if that the equation

$$AA^* I_0A_pG_pA_p^{\mathrm{T}}RB^* B = I_0A_pG_pA_p^{\mathrm{T}}R \tag{22}$$

holds, then there exists an active controller G_a to satisfy

$$AGB = AG_aB - I_0A_pG_pA_p^{\mathrm{T}}R \tag{23}$$

and G_a is given by

$$G_a = G + A^* I_0A_pG_pA_p^{\mathrm{T}}RB^*, \tag{24}$$

where $*$ denotes the Moore-Penrose inverse of a matrix.

Solving Eq.(22), we can obtain a G_p (which always exists, e.g. $G_p = 0$).

By the use of this method, if given an output feed-

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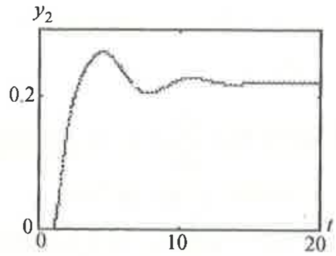

(b)

Fig. 5 Time response diagram

5 Conclusion

The locally stability criteria for the polynomial class of MIMO nonlinear closed-loop control systems based GFRFM's are similar to those of linear closed-loop control system. Due to not considering the problem of GFRFM's power series convergence of nonlinear closed-loop, the criteria is very simple and practical.

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back (initial) controller C_0 for an initial structure S_0 is given, then we can, redesign the new structure (changing physical parameters in S_0) and the new controller C_1 (so as to minimize the amount of active control power that will be needed after the structure S_1 redesign). Consequently S_1 and C_1 yield a closed-loop response which matches that of S_0 in closed-loop with C_0.

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