第 16 卷第 6 期
1999 年 12 月

控制理论与应用
CONTROL THEORY AND APPLICATIONS

Vol.16, No.6
Dec., 1999

Study on Stability of Nonlinear Closed-Loop Control Systems Based on Generalized Frequency Response Function Matrices*

Fang Yangwang and Han Chongzhao

(Institute of System Engineering, Xi'an Jiaotong University · Xi'an, 710049, P.R. China)

Abstract: Based on the representation of generalized frequency response function matrices (GFRFM), this paper proposes a stability criterion for a class of nonlinear multi-input multi-output closed-loop control systems by the use of open-loop stability of its subsystem, and this criterion is demonstrated by numerous simulation examples.

Key words: generalized frequency response function matrix; nonlinear MIMO control system; local stability

基于广义频率响应函数矩阵的非线性闭环控制系统的稳定性研究

方洋旺　韩崇昭

(西安交通大学系统工程研究所·西安, 710049)

摘要：本文基于广义频率响应函数矩阵表示，针对一类多输入多输出非线性系统，提出了直接利用开环稳定性来判断系统闭环稳定性的新方法，并用实例仿真来验证了此判据的有效性.

关键词：广义频率响应函数矩阵；非线性控制系统；局部稳定性

1 Introduction

The nonlinear frequency analysis is an extension of the classical linear frequency analysis method and its most importance lies in the experimental verifiability. There are many important results in the nonlinear system simulation, nonlinear system identification, GFRF's computational method and its applications in industrial control systems[1~4]. Recently, the stability of open-loop and closed-loop for a class of nonlinear SISO control system based on GFRF method have been discussed respectively in paper [4] and [5], but it is very difficult to deduce the stability of the closed-loop system from the open-loop subsystem for its complex series computation.

In this paper, based on GFRFM, the closed-loop stability criterion for a class of nonlinear MIMO system according to open-loop subsystem stability is presented. In the second section, the nonlinear MIMO control system is described; the stability criterion is given in the third section; finally, a simulation example is offered to illustrate the efficiency of the stability criterion.

2 Description of the nonlinear control system

The polynomial class of nonlinear multi-input multi-output control system is considered as

$$\sum_{n=1}^{N}\Big\{\sum_{p_1=0}^{N}\cdots\sum_{p_n=0}^{N}\Big[a_{n,p_1,\cdots,p_n}\prod_{i=1}^{n}D^{p_i}y(t)+c_{n,p_1,\cdots,p_n}\prod_{i=1}^{n}D^{p_i}u(t)\Big]\Big\}+\sum_{n=1}^{N-1}\sum_{q=1}^{N-n}\Big[\sum_{p_1=0}^{M}\cdots\sum_{p_{n+q}=0}^{M}b_{n,q,p_1,\cdots,p_{n+q}}\prod_{i=1}^{n}\prod_{k=n+1}^{N}D^{p_i}y(t)D^{p_k}u(t)\Big]=0, \quad (1)$$

where D is the differential operator, M is the maximum of differential order, and N is the maximum of multiple degree, $a's$, $b's$, $c's$ are coefficient matrices, $u\in\mathbb{R}^r$, $y\in\mathbb{R}^m$ are the input and output of the system respectively.

Assume "$\otimes$" represents Kroneker product, and the nonlinear control system (1) possesses the Volterra series solution, i.e.

$$y(t)=\sum_{n=1}^{\infty}y_n(t), \quad (2)$$

$$y_1(t)=\int_{-\infty}^{\infty}H_1(\tau)u(t-\tau)\mathrm{d}\tau,$$

* This work is supported by National Natural Science Foundation grant(69674009).
Manuscript received Oct.28, 1997, revised Dec.9, 1998.

$$y_2(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H_2(\tau_1,\tau_2)(u(t-\tau_1)\otimes u(t-\tau_2))\mathrm{d}\tau_1\mathrm{d}\tau_2\cdots,$$

$$y_n(t) = \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} H_n(\tau_1,\cdots,\tau_n)\cdot(\bigotimes_{i=1}^{n} u(t-\tau_i))\mathrm{d}\tau_1\cdots\mathrm{d}\tau_n,$$

where

$$H_n(\tau_1,\tau_2,\cdots,\tau_n)\in\mathbb{R}^{m\times r^n},\quad n=1,2,\cdots$$

are the nth degree Volterra kernels or the generalized pulse response function matrices. Furthermore, by using the Fourier transform, the Volterra series can be represented as:

$$\hat{y}(w) = \sum_{n=1}^{\infty}\hat{y}_n(w), \tag{3}$$

$$\hat{y}_n(w) = \int_{-\infty}^{\infty} y_n(t)\mathrm{e}^{-\mathrm{j}\omega t}\mathrm{d}t = (2\pi)^{-(n-1)}\cdot \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\hat{H}_n(w-w_2-\cdots-w_n, w_2,\cdots,w_n)\hat{u}(w-w_2-\cdots-w_n)\otimes(\bigotimes_{i=2}^{n}\hat{u}(w_i))\mathrm{d}w_2\cdots\mathrm{d}w_n,$$

where $\hat{y}, \hat{y}_n, \hat{u}$ are Fourier transforms of y, y_n and u, $\hat{H}_n$ is the multi-dimensional Fourier transform of H_n, i.e.

$$\hat{H}_n(w_1,\cdots,w_n) = \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} H_n(\tau_1,\cdots,\tau_n)\prod_{i=1}^{n}\mathrm{e}^{-\mathrm{j}\omega_i\tau_i}\mathrm{d}\tau_i,$$

$\hat{H}_n$ is said to be degree-n generalized frequency response function matrix, i.e. GFRFM.

Some notation: consider that $x(t):\mathbb{R}\to\mathbb{R}^n$ is time-domain signal, $\hat{x}$ is the Fourier transform of x. Let

$$L_p^n(-\infty,\infty): = \{x: \|x\|_p: = \Big[\int_{-\infty}^{\infty}\sum_{i=1}^{n}|x(t)|^p\mathrm{d}t\Big]^{1/p} < \infty\},$$

$$L_\infty^n(-\infty,\infty): = \{x: \|x\|_\infty: = \operatorname*{ess\,sup}_{t\in\mathbb{R}}\|x(t)\|_2 < \infty\},$$

$$H_p^n(-\infty,\infty): = \{\hat{x}: \|\hat{x}\|_p: = \Big[(2\pi)^{-1}\int_{-\infty}^{\infty}\sum_{i=1}^{n}|\hat{x}_i(w)|^p\mathrm{d}w\Big]^{1/p} < \infty\},$$

$$H_\infty^n(-\infty,\infty): = \{x: \|\hat{x}\|_\infty: = \sup_{w\in\mathbb{R}}\|\hat{x}(w)\|_2 < \infty\},$$

$$H_\infty^{m\times n}: = \{F(w_1,\cdots,w_m)\in\mathbb{C}^{m\times n}: \|F\|_\infty = \sup_{w_1,\cdots,w_n}(\|F(w_1,\cdots,w_n)\|_2) < \infty\},$$

where $1\leqslant p<\infty$.

Remark It is assumed that the following nonlinear systems are the polynomial classes of MIMO nonlinear systems without any special explanation.

3 Main result

Definition 3.1 The nonlinear control system (1) is said to be locally L_2 stable, if $u\in L_2^r(-\infty,\infty)$, there exists $L>0$ such that $\|\hat{u}\|_1 < L$, then $y\in L_2^m(-\infty,\infty)$.

Lemma 3.1 Assume

$$\hat{H}_n\in H_\infty^{m\times r^n}(-\infty,\infty),\quad \text{i.e.}\quad \sup_{w_1,\cdots,w_n}\|\hat{H}_n(w_n,\cdots,w_n)\|_2 < \infty,$$

and

$$\hat{u}\in H_1^r\cap H_2^r,$$

$$\hat{y}_n(w) = (2\pi)^{-(n-1)}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\hat{H}_n(w-w_2-\cdots-w_n, w_2,\cdots,w_n)\times\hat{u}(w-w_2-\cdots-w_n)\otimes(\bigotimes_{i=2}^{n}\hat{u}(w_i))\mathrm{d}w_2\cdots\mathrm{d}w_n,$$

then

$$\|\hat{y}_n\|_2\leqslant\|\hat{H}_n\|_\infty\|\hat{u}\|_1^{n-1}\|\hat{u}\|_2.$$

Proof Since

$$\|\hat{y}_n\|_2^2 = (2\pi)^{-1}\int_{-\infty}^{\infty}\|\hat{y}_n(w)\|_2^2\mathrm{d}w = (2\pi)^{-1}\int_{-\infty}^{\infty}\langle\hat{y}_n(w),\hat{y}_n(w)\rangle\mathrm{d}w,$$

hence

$$\langle\hat{y}_n(w),\hat{y}_n(w)\rangle = (2\pi)^{-2(n-1)}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\langle\hat{H}_n(w-w_2-\cdots w_n, w_2,\cdots,w_n)(\hat{u}(w-w_2-\cdots w_n)\otimes\hat{u}(w_2)\otimes\cdots\otimes\hat{u}(w_n)), \hat{H}(w-\tilde{w}_2-\cdots\tilde{w}_n,\tilde{w}_2,\cdots,\tilde{w}_n)\cdot(\hat{u}(w-\tilde{w}_2-\cdots\tilde{w}_n)\otimes\hat{u}(\tilde{w}_2)\otimes\cdots\otimes\hat{u}(\tilde{w}_n))\rangle\mathrm{d}w_2\cdots\mathrm{d}w_n\mathrm{d}\tilde{w}_2\cdots\mathrm{d}\tilde{w}_n,$$

Since

$$|\langle\hat{H}_n(w-w_2-\cdots w_n,w_2,\cdots,w_n)\cdot(\hat{u}(w-w_2-\cdots w_n)\otimes\hat{u}(w_2)\otimes\cdots\otimes\hat{u}(w_n)), \hat{H}_n(w-\tilde{w}_2-\cdots\tilde{w}_n,\tilde{w}_2,\cdots,\tilde{w}_n)\cdot(\hat{u}(w-\tilde{w}_2-\cdots\tilde{w}_n)\otimes\hat{u}(\tilde{w}_2)\otimes\cdots\otimes\hat{u}(\tilde{w}_n))\rangle|\leqslant$$

$$\|\hat{H}_n(w-w_2-\cdots-w_n,w_2,\cdots,w_n)\|_2\cdot\|\hat{u}(w-w_2-\cdots-w_n)\|_2\|\hat{u}(w_2)\|_2\cdots\|\hat{u}(w_n)\|_2\times$$

$\|\hat{H}_n(w-\tilde{w}_2-\cdots-\tilde{w}_n)\|_2\|\hat{u}(w-\tilde{w}_2-\cdots-\tilde{w}_n)\|_2\cdot$
$\|\hat{u}(\tilde{w}_2)\|_2\cdots\|\hat{u}(\tilde{w}_n)\|_2.$

Then

$$\langle\hat{y}_n(w),\hat{y}_n(w)\rangle\leqslant$$
$$(2\pi)^{-2(n-1)}\Big(\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\|\hat{H}_n(w-w_2-\cdots-w_n,$$
$$w_2,\cdots,w_n)\|_2\|\hat{u}(w_2)\|_2\cdots\|\hat{u}(w_n)\|_2\cdot$$
$$\|(\hat{u}(w-w_2-\cdots-w_n)\|_2^2\mathrm{d}w_2\cdots\mathrm{d}w_n)\times$$
$$(\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\|\hat{H}_n(w-w_2-\cdots-w_n,w_2,\cdots,w_n\|_2\cdot$$
$$\|\hat{u}(w_2)\|_2\cdots\|\hat{u}(w_n)\|_2\mathrm{d}w_2\cdots\mathrm{d}w_n\leqslant$$
$$\|\hat{H}_n\|_2^2\|\hat{u}\|_1^{n-1}(2\pi)^{-(n-1)}\cdot$$
$$\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\|\hat{u}(w_2)\|_2\cdots\|\hat{u}(w_n)\|_2\cdot$$
$$\|\hat{u}(w-w_2-\cdots-w_n)\|_2^2\mathrm{d}w_2\cdots\mathrm{d}w_n.$$

Furthermore

$$\|\hat{y}_n\|_2^2=(2\pi)^{-1}\int_{-\infty}^{\infty}\|\hat{y}_n(w)\|_2^2\mathrm{d}w\leqslant$$
$$\|\hat{H}\|_2^2\|\hat{u}\|_1^{n-1}(2\pi)^{-(n-1)}\cdot$$
$$\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\|\hat{u}(w_2)\|_2\cdots\|\hat{u}(w_n)\|_2\cdot$$
$$\mathrm{d}w_2\cdots\mathrm{d}w_n\times(2\pi)^{-1}\cdot$$
$$\int_{-\infty}^{\infty}\|\hat{u}(w-w_2-\cdots-w_n)\|_2^2\mathrm{d}w=$$
$$\|\hat{H}_n\|_2^2\|\hat{u}\|_1^{2(n-1)}\|\hat{u}\|_2^2.$$

Hence

$$\|\hat{y}_n\|_2\leqslant\|\hat{H}_n\|_2\|\hat{u}\|_1^{n-1}\|\hat{u}\|_2.$$

Theorem 3.1 Suppose that nonlinear control system (1) has Volterra series solutions (2) and (3), and if its 1th order subsystem i.e. linear control subsystem is asymptotically stable, and power series $\sum_{n=1}^{\infty}\|\hat{H}_n\|_\infty x^n$ is convergent. Then system (1) is locally L_2 stable.

Proof From Lemma 3.1 and Parseval Theorem, we have

$$\|y\|_2=(2\pi)^{-1/2}\|\hat{y}\|_2\leqslant(2\pi)^{-1/2}\sum_{n=1}^{\infty}\|\hat{y}_n\|_2\leqslant$$

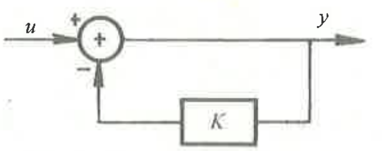


Fig. 1 Single-loop feedback system

$$(2\pi)^{-1/2}\sum_{n=1}^{\infty}\|\hat{H}_n\|_\infty\|\hat{u}\|_2^{n-1}\|\hat{u}\|_2=$$
$$\|\hat{u}\|_1^{-1}\|\hat{u}\|_2\sum_{n=1}^{\infty}\|\hat{H}_n\|_\infty\|\hat{u}\|_1^n,$$

and when $\|\hat{u}\|_1<r$, we have $\sum_{n=1}^{\infty}\|\hat{H}_n\|_\infty\|\hat{u}\|_1^n<+\infty$, hence $\|y\|_2<+\infty$, then the system (1) is locally L_2 stable.

For the convenience of discussing the local L_2 stability of closed-loop nonlinear control system, we will modify Definition 3.1 as following:

Definition 3.2 Nonlinear system (1) is said to be locally stable, if it satisfies:

1) Its linear subsystem is asymptotically stable.

2) Power series $\sum_{n=1}^{\infty}\|\hat{H}_n\|_\infty x^n$ is convergent.

Lemma 3.2 Assume nonlinear plant H, P are local stable, then cascade system $G=H\circ P$ is also locally stable.

Proof This proof is similar to that in [9].

Lemma 3.3 If $F:=(I+L)^{-1}$ exists, and its GFRFM is $\{\hat{F}_n\}$, where L is a nonlinear plant, I is identity operator, then $\|\hat{L}_n\|_\infty<+\infty$, $n=1,2,\cdots$, $I+\hat{L}_1(\infty)$ is non-singular matrix, and $(I+\hat{L}_1(s))^{-1}$ is stable matrix, then $\|\hat{F}_n\|_\infty<+\infty$, $n=2,3,\cdots$.

Theorem 3.2 Consider nonlinear system shown is Fig. 1, it is assumed that its 1th degree Volterra subsystem $F_1:=(I+K_1)^{-1}$ of closed-loop system F is asymptotically stable, and $I+\hat{K}_1(\infty)$ in non-singular matrix, where K_1 is 1th degree Volterra subsystem of system K. If K is locally stable, then the closed-loop system F is also locally stable.

Proof since $I+\hat{K}_1(\infty)$ is non-singular matrix, hence $\hat{F}(s)=(I+\hat{K}(s))^{-1}$ is regular, and $F_1=(I+k_1)^{-1}$ is asymptotically stable linear system. The feedback system shown in Fig. 1 can be redrawn as in Fig. 2, which is described by the operator equation

$$F=F_1\circ(I-(K-K_1))\circ F,$$

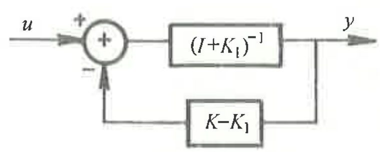


Fig. 2 Single-loop feedback system

where $F_1 = (I + K_1)^{-1}$.

Writing out the hemegeneous terms for F and $(K - K_1)$ gives

$F_1 + F_2 + F_3 + \cdots =$

$F_1 - F_1 \circ (K_1 + K_3 + \cdots) \circ (F_1 + F_2 + F_3 + \cdots) =$

$F_1 + Q \circ (F_1 + F_2 + F_3 + \cdots)$,

where

$$Q = - F_1 \circ (K_2 + K_3 + \cdots)$$

is a cascade Volterra system that contains no degree-0 and degree-1 terms. Since $\| \hat{F}_1 \|_{\infty} < +\infty$, and $(K - K_1)$ is locally stable, by Lemma 3.2, Q is locally stable, i.e. $\sum_{n=1}^{\infty} q_n x^n, x > 0$ is convergent, r_Q is its radius of convergence, where $q_n := \| \hat{Q}_n \|_{\infty}$. According to Lemma 3.3, $f_n := \| \hat{F}_n \|_{\infty}$ exists.

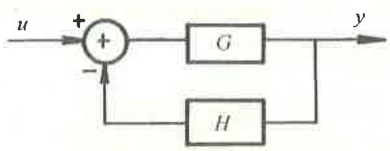


Fig. 3 Closed-loop feedback system

We will prove that $\sum_{n=1}^{\infty} f_n x^n$ is convergent. The following proof is similar to that in [6].

Theorem 3.3 Suppose that the nonlinear system shown in Fig.3 satisfies:

1) G, H are locally stable nonlinear subsystem respectively.

2) $I + \hat{G}_1(\infty)\hat{H}_1(\infty)$ is non-singular matrix, and $F_1 := (1 + G_1 H_1)^{-1}$ is asymptotically stable. Then the feedback nonlinear system is locally stable.

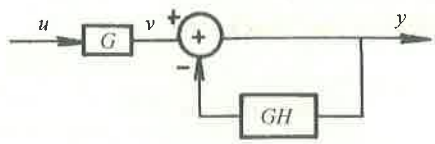


Fig. 4 Closed-loop feedback system

Proof The nonlinear system shown in Fig.3 can be redrawn as in Fig.4. Let L be the feedback nonlinear subsystem from input v to output y. Applying Theorem 3.2, we know that L is locally stable. Since the closed-loop nonlinear system from input u to output y is cascade system of G and L, by Lemma 3.2, the nonlinear system shown in Fig.3 is, thus, locally stable.

Corollary 3.1 Suppose G is locally stable nonlinear system, $H: y = f(u)$ is a nonlinear memoryless system which satisfies $f(0) = 0$, and $y = f(u)$ is analytic, where $u \in \mathbb{R}^m, y \in \mathbb{R}^r$ are input and output vectors of the system respectively. If $I + \hat{G}_1(\infty)f'(0)$ is non-singular matrix, and $(I + G(s)f'(0))^{-1}$ is asymptotically stable, then the feedback system shown in Fig.3 is locally stable.

4 Simulation examples

Example Suppose the pure input nonlinear system

$$G: \begin{cases} y_1'' + 2y_1' + 2y_1 - u_1 - u_1^3 = 0, \\ y_2'' + 3y_2' + 2y_2 - u_2 - u_2^2 = 0, \end{cases}$$

the pure output nonlinear system

$$H: \begin{cases} y_1' + 2y_1 + 0.1 y_1 y_2 - u_1 = 0, \\ y_2' + y_2 + 0.2 y_1^2 - u_2 = 0, \end{cases}$$

by using the criterion in [2], G, H are both locally stable. Since the GFRFM of G, H are

$$\hat{G}_1(w) = \begin{bmatrix} \dfrac{1}{2 - w^2 + 2jw} & 0 \\ 0 & \dfrac{1}{2 - w^2 + 3jw} \end{bmatrix},$$

$$\hat{H}_1(w) = \begin{bmatrix} \dfrac{1}{jw + 2} & 0 \\ 0 & \dfrac{1}{jw + 1} \end{bmatrix},$$

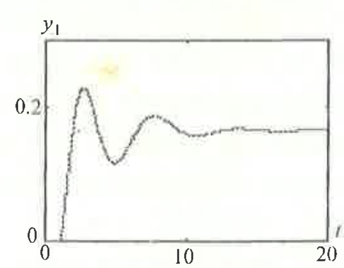
$$I + \hat{G}_1(\infty)\hat{H}_1(\infty) = I,$$

and

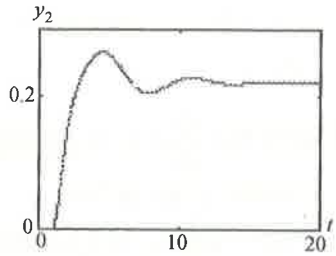
$$(I + \hat{G}_1(s)\hat{H}_1(s))^{-1} =$$

$$\begin{bmatrix} \dfrac{s^3 + 4s^2 + 6s + 4}{S^3 + 4s^2 + 6s + 5} & 0 \\ 0 & \dfrac{s^3 + 4s^2 + 5s + 2}{s^3 + 4s^2 + 5s + 3} \end{bmatrix}$$

is asymptotically stable; by Theorem 3.3, the closed-loop system shown in Fig.3 is locally stable. The response diagrams of the system are shown in Fig.5.



(a)



(b)

Fig. 5 Time response diagram

5 Conclusion

The locally stability criteria for the polynomial class of MIMO nonlinear closed-loop control systems based GFRFM's are similar to those of linear closed-loop control system. Due to not considering the problem of GFRFM's power series convergence of nonlinear closed-loop, the criteria is very simple and practical.

References

1 Billings S A and Tsang K M. Spectral analysis for nonlinear systems, Part Ⅱ: interpretation of nonlinear fequency response functions. Mechanical Systems and Signal Processing, 1989, 3(4): 341 – 369

2 Billings S A and Tsang K M. Spectral analysis for nonlinear systems, Part Ⅲ: case study examples. Mechanical Systems and Signal Processing, 1989, 4(1): 3 – 21

3 Zhang H and Billings S A. Analysing the transfer function of nonlinear systems in the frequency domain. J. Mech. Systems and Signal Processing, 1993, 11(7): 531 – 550

4 Cao Jianfu. Study on frequency analysis and stability of nonlinear control system based on nonlinear transfer functions: [Ph. D. dissertation]. Xi'an: Xi'an Jiaotong University, 1996

5 Han Chongzhao and Cao Jianfu. Study on stability of nonlinear control system based on generalized fequency response functions. Control Theory and Applications, 1996, 13(5): 573 – 582

6 Rugh W J. Nonlinear System Theory: The Volterra/Wiener Approach. London: The Johns Hopkins University Press, 1981

7 Schetzen M. The Volterra and Wiener Theories of Nonlinear System. New York: Wiley, 1980

本文作者简介

方洋旺 见本刊 1999 年第 4 期第 586 页.

韩崇昭 见本刊 1999 年第 3 期第 451 页.

(Continued from page 915)

back (initial) controller C_0 for an initial structure S_0 is given, then we can, redesign the new structure (changing physical parameters in S_0) and the new controller C_1 (so as to minimize the amount of active control power that will be needed after the structure S_1 redesign). Consequently S_1 and C_1 yield a closed-loop response which matches that of S_0 in closed-loop with C_0.

References

1 Huang Lie-de. The necessary condition for optimal control of nonlinear evolution system. In General Systems Studies and Applications IIGS 5-CB, Ed. by Chen Mainyun et al. HUST Press, 1997, 422 – 425

2 Huang Lie-de. Statistical properties of an infinite dimensional dynamical system which exhibit strange attractors and their control. In Modern Mech. & Sci-Tech Progress Ed. by Zhang Fenggan et al. Beijing: Tsinghua University Press, 1997, 2: 790 – 793 (in Chinese)

3 Hall II E K and Hanagud S V. Control of nonlinear structural dynamic systems. J. Guid. Cont. Dyna., 1993, 16(3): 470 – 476

4 Jon lee. Large-amplitued plate vibration in an elevated thermal environment. Appl. Mech. Rev. 1993, 46(11): 5242 – 5254

5 Smith M J, Grigoriadis K M and Skelton R E. Optimal mix of passive and active control in structures. J. Guid. Cont. Dyna., 1992, 15(4): 912 – 919

6 Skelton R E, Hanks B R and Smith M J. Structure redesign for improve dynamic response, J. Guid. Cont. Dyna., 1992, 15(5): 1272 – 1278

本文作者简介

黄烈德 1922 年生. 教授. 研究航天数学, 积分方程与偏微分方程.