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Subspace State Space Approach to Closed Loop System Identification

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Abstract: In this paper, we present an algorithm to identify state space models based on the data obtained from closed loop systems. It works well on both stable and unstable system with serious noises and large delays and also is proved mathematically.

Key words: closed loop identification; coprime factorization; state space models; subspace method

状态空间子空间方法处理闭环系统辨识

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摘要: 本文给出一个由闭环数据辨识状态空间模型的算法.此算法可辨识带有严重噪声干扰和延迟的稳定及不稳定系统,并已为数学理论及实验结果所验证.

关键词: 闭环系统辨识; 互质分解; 状态空间模型; 子空间方法

1 Introduction

The numerical algorithm for subspace state space system identification, N4SID[1], is always convergent and numerically stable since they only make use of QR and Singular Value Decompositions. It derives the state space model directly from the input and output data in a very simple way[2]. One of the main assumptions of N4SID is that the process and measurement noises w are independent of the plant input u. This assumption is repudiated when the system is working in closed loop written into the general feedback system $T(P, C)$ in Fig.1. Using the equivalent open loop identification framework[3] to cope with this problem, we make N4SID work into closed-loop identification.

2 Equivalent open loop identification framework

In the algebraic theory of linear finite dimensional time in variant systems a plant P can be factorized as ND^{-1}. In this we will use the following lemma, where $\mathbb{R}\mathrm{H}_\infty$ denotes the set of all rational stable transfer functions.

Lemma 2.1[3] Let P_0 be an auxiliary model and C a controller such that $T(P_0, C) \in \mathbb{R}\mathrm{H}_\infty$ and let (N_0, D_0) and (N_c, D_c) be a rcf (right coprime factorization) of respectively P_0 and C. Then $P = ND^{-1}$ satisfies $T(P, C) \in \mathbb{R}\mathrm{H}_\infty$ if and only if $\exists\, R \in \mathbb{R}\mathrm{H}_\infty$ with

$$N = N_0 + D_C R, \tag{1}$$

$$D = D_0 - N_c R. \tag{2}$$

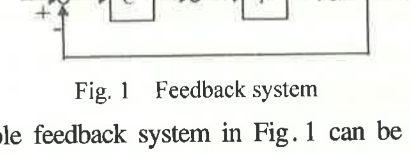

Fig. 1 Feedback system

The stable feedback system in Fig.1 can be recast into Fig.2 with the equivalent open loop identification framework[3]. where x is intermediate.

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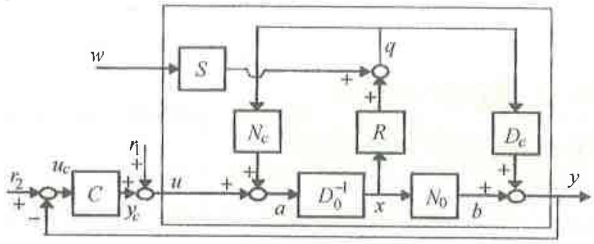

Fig. 2 The open loop identification framework

Theorem 2.1[3] Let a plant P and a known compensator C make a stable $T(P, C)$ as in Fig. 1. Then the intermediate x appearing in between D_0^{-1} and N_0 in Fig. 2 can be reconstructed via

$$x = (D_0 + CN_0)^{-1}(u + Cy), \tag{3}$$

which is independent of w, the closed-loop identification of P can then be conducted through the open loop identification of N and D

$$u = Dx - N_c Sw, \tag{4}$$

$$y = Nx + D_c Sw, \tag{5}$$

provided that r_1 and r_2 are statistically independent of w.

3 Identification algorithm and implementation

In Equations (4) and (5), we ignore the disturbance, D and N can be approximately rewritten as

$$z_D(k+1) = A_D z_D(k) + B_D x(k), \tag{6}$$

$$u(k) = C_D z_D(k) + D_D x(k), \tag{7}$$

and

$$z_N(k+1) = A_N z_N(k) + B_N x(k), \tag{8}$$

$$y(k) = C_N z_N(k) + D_N x(k), \tag{9}$$

where $[A_D, B_D, C_D, D_D]$ and $[A_N, B_N, C_N, D_N]$ are the state space representations of D and N . Then we put u and y together and make them into one system and give the definitions of Y and the new state space equation as

$$Y \doteq \begin{pmatrix} u \\ y \end{pmatrix}, \tag{10}$$

$$z(k+1) \doteq \bar{A}z(k) + \bar{B}x(k), \tag{11}$$

$$Y(k) \doteq \bar{C}z(k) + \bar{D}x(k). \tag{12}$$

Because the contributions of q to x on the two paths N_c, D_0^{-1} and D_c, C, D_0^{-1} are equal and the directions are opposite in Fig. 2, the intermediate x and the disturbance w are uncorrelated

$$E(x, w^t) = 0. \tag{13}$$

Since Equation (13) satisfies the N4SID condition, we can identify $[\bar{A}, \bar{B}, \bar{C}, \bar{D}]$ with N4SID. Then partition Equations (11) and (12) into $[A_D, B_D, C_D, D_D]$ and $[A_N, B_N, C_N, D_N]$ as follows

$$\left[\begin{array}{c|c} \bar{A} & \bar{B} \\ \hline \bar{C} & \bar{D} \end{array}\right] = \left[\begin{array}{c|c} A & B \\ \hline C_D & D_D \\ C_N & D_N \end{array}\right], \tag{14}$$

where

$$A = A_D = A_N, \tag{15}$$

$$B = B_D = B_N, \tag{16}$$

then

$$[A_D, B_D, C_D, D_D] = [A, B, C_D, D_D], \tag{17}$$

$$[A_N, B_N, C_N, D_N] = [A, B, C_N, D_N]. \tag{18}$$

After eliminating the stable uncontrollable and/or unobservable states, we get a lower order controllable and observable state space representation of the plant $P = ND^{-1}$ as follows

$$A_p = A - BD_D^{-1}C_D, \tag{19}$$

$$B_p = BD_D^{-1}, \tag{20}$$

$$C_p = C_N - D_N D_D^{-1} C_D, \tag{21}$$

$$D_p = D_N D_D^{-1}. \tag{22}$$

The proofs are ignored because of limited space.

All the plants we take into account are minimal systems, so we can always find a stable controller C. This is allowed to choose the auxiliary plant P_0 to be 0 and the rcf as $(N_0, D_0) = (0, 1)$ first. Then it is possible to let $r_2 = 0$ and $x = r_1$. For the identification of P we only need controller C and measurements of u and y, or reference r_1, input u and output y.

4 Simulation

To a given unstable system in Equation (23), we can design a stable controller stabilized closed-loop system. Since the disturbance noise is a random signal and has undetermined properties, we identify the plant state space model 3 times under 3 different random disturbance noises conditions. On the other hand, the signal noise ratio $SNR \approx 10$ dB or 20 dB is used in simulation.

$$\left[\begin{array}{c|c} A_{p\text{-unstable}} & B_{p\text{-unstable}} \\ \hline C_{p\text{-unstable}} & D_{p\text{-unstable}} \end{array}\right] = \left[\begin{array}{cc|c} 4.0000 & -0.5000 & 5.0000 \\ 5.0000 & 0.5000 & 5.0000 \\ \hline 5.5000 & -0.3500 & 6.5000 \end{array}\right]. \tag{23}$$

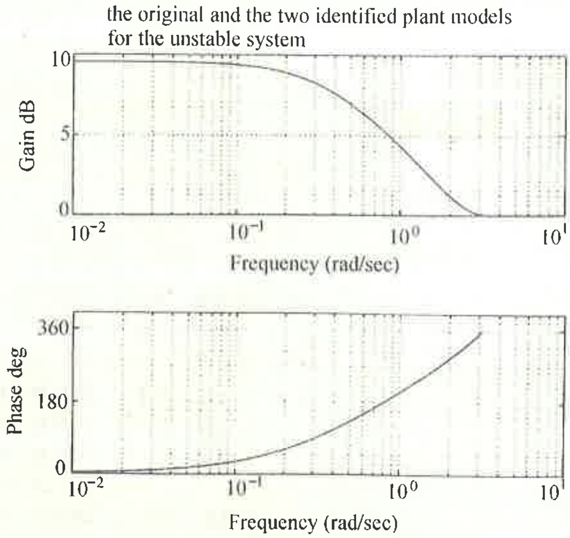

Fig. 3 The original and the 3 identified plant models for the unstable system

Fig. 3 shows three 2nd order models identified by the new algorithm with different disturbances, together with the real unstable plant model. Under very serious noise disturbance conditions, all the identified models are almost identical to the real one. Due to limited space, the too complicated application of a practical glass tube manufacturing process is ignored.

5 Conclusion

The algorithm developed in this paper is able to derive the fairly accurate state space representation of a plant model from closed-loop data in a fast, efficient, reliable way. And it works well on systems with significant noise disturbances and serious delays, as well as on unstable plant models.

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王广雄 见本刊 1999 年第 2 期第 240 页.

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