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New Multivariable Generalized Predictive Self-Tuning Controller

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Abstract: This paper presents a generalized predictive self-tuning controller for multivariable linear systems (MGPC). As this algorithm is to solve the controller using the identification results and not to compute the Diophantine equation on-line, which reduces the calculations greatly and simplifies designing of control systems. Simulating results showthat the proposed method can be applied to the multivariable stochastic systems which can be unstable and/or nonminimum-phase plants with color-noise disturbances.

Key words: multivariable control; predictive control; self-tuning control

1 Introduction

A novel generalized predictive control (GPC) was developed by Clarke, D. W., Mohtadi, C. and Tuffs, P. S. (1987), YUAN Zhuzhi and LIU Ruihua (1988), Albertos, P. and Ortega, R. (1989) et al. to improve the robustness of the commonly used self-tuning controllers. It was shown that the algorithm of GPC is effective in control of single variable systems with variable parameters, variable dead-times, and variable orders of models etc. The extension of the algorithm of GPC to multivariable systems was carried out by Kinnaert, M. (1987), Shah, S. L. (1987), HU Weili and LI Qingyuan (1988). However, their methodsare applicable to the systems without disturbance or wtih white-noise disturbances only. The purpose of this paper is to generalize the algorithm of GPC to multivariable systems with color-noise disturbances. First, using discrete-time input-output CARIMA model, we present controller of the plant. Second, it is shown, via simple algebraic manipulations, that the key prediction equation used for MGPC can be easily derived from the identification result coefficients efficiently replacing the Diophantine equation recursions.

2 System Model and Cost-Function

Consider a multivariable linear discrete-time system described by thefollowing CARIMA model

$$A(z^{-1})y(k) = B(z^{-1})u(k-1) + C(z^{-1})\xi(k)/\Delta, \tag{1}$$

Where $y(k)$ is the output vector, $u(k)$ is the input vector, and $\{\xi(k)\}$ is a sequence of independent, equally distributed, randomvectors with a zero-mean value and variance matrix

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$\mathscr{E}[\xi(k)\xi^{T}(k)]=R$, and the Δ represents the differenceoperator $1-z^{-1}$. The vectors $y(k)$, $u(k)$ and $\xi(k)$ areall of dimension p, respectively. The polynomial matrices (in the backward shift operator z^{-1}) A, B and C are all of dimension $p\times p$ and can be expressed as

$$A(z^{-1}) = I + A_1z^{-1} + \cdots + A_{Na}z^{-Na},$$
$$B(z^{-1}) = B_0 + B_1z^{-1} + \cdots + B_{Nb}z^{-Nb},$$
$$C(z^{-1}) = I + C_1z^{-1} + \cdots + C_{Nc}z^{-Nc}.$$

Det (B_0) is assumed to be nonsingular, and the zeros of det $(C(z^{-1}))$ are to lie inside the unit circle. The cost function of interest is presented as

$$J(N) = \mathscr{E}\left\{\sum_{j=1}^{N} \| y(k+j) - w(k+j) \|_I^2 + \sum_{j=1}^{N} \| Q(z^{-1})\Delta u(k+j-1) \|_\lambda^2\right\}. \quad (2)$$

Where N is the maximum costing horizon, $\lambda=\mathrm{diag}[\lambda_1, \cdots, \lambda_p]$ is a positive definite matrix and $Q(z^{-1})=I_p+Q_1z^{-1}+\cdots+Q_{Nq}z^{-Nq}$ is a control-weighting function matrix. The $w(k)$ is the reference signal which can be obtained from the simple first-order model as follows

$$w(k) = y(k),$$
$$w(k+j) = \alpha w(k+j-1) + (I-\alpha)y_r(k).$$

Where $y_r(k)$ is the set-vector at the sample-instant k, and $\alpha=\mathrm{diag}[\alpha_1,\cdots,\alpha_p]$ is a positive semi-definite matrixwith α_i being between 0 and 1 for each i.

The multivariable predictive model can be described as follows:

We consider the following identity

$$C(z^{-1}) = E_j(z^{-1})\overline{A}(z^{-1}) + z^{-j}R_j(z^{-1}). \quad (3)$$

Where $\overline{A}(z^{-1})=A(z^{-1})\Delta\triangleq I+\overline{A}_1z^{-1}+\cdots+\overline{A}_{Na+1}z^{-(Na+1)}$, the polynomial matrices $E_j(z^{-1})$ and $R_j(z^{-1})$ of dimension $p\times p$ are uniquely defined by the given $A(z^{-1})$, $C(z^{-1})$ and the prediction internal j. The degrees of $E_j(z^{-1})$ and $R_j(z^{-1})$ are $j-1$ and $N_r=\mathrm{Max}(Na,N_c-1)$, respectively. Also, we introduce new polynomial matrices $H_j(z^{-1})$, $S_j(z^{-1})$, $E_{1j}(z^{-1})$, $E_{2j}(z^{-1})$ given by

$$E_j(z^{-1})B(z^{-1}) = H_j(z^{-1}) + z^{-j}S_j(z^{-1}), \quad (4)$$
$$E_j(z^{-1})C(z^{-1}) = E_{1j}(z^{-1}) + z^{-j}E_{2j}(z^{-1}). \quad (5)$$

Where the degrees of $H_j(z^{-1})$ and $E_{1j}(z^{-1})$ are $j-1$, $S_j(z^{-1})$ and $E_{2j}(z^{-1})$ are N_b-1 and N_c-1 respectively.

We rewrite the process model (1) as

$$\overline{A}(z^{-1})y(k+j) = B(z^{-1})\Delta u(k+j-1) + C(z^{-1})\xi(k+j). \quad (6)$$

Premultiply the above equation with $E_j(z^{-1})$ results in

$$E_j\overline{A}y(k+j) = E_jB\Delta u(k+j-1) + E_jC\xi(k+j). \quad (7)$$

Deduction of (3), (4) and (5) yields

$$\begin{aligned} Cy(k+j) &= S_j\Delta u(k-1) + R_jy(k) + H_j\Delta u(k+j-1) + E_{1j}\xi(k+j) + E_{2j}\xi(k) \\ &= \theta_j\varphi(k) + H_j\Delta u(k+j-1) + E_{1j}\xi(k+j) + E_{2j}\xi(k). \end{aligned} \quad (8)$$

Where $\theta_j\in\mathscr{R}^{p\times(p\times(N_r+N_b+1))}$ contains the coefficients of $S_j(z^{-1})$ and $R_j(z^{-1})$, transposition is indicated by T and

$$\varphi(k) \triangleq [\Delta u^{\mathrm{T}}(k-N_b),\cdots,\Delta u^{\mathrm{T}}(k-1);y^{\mathrm{T}}(k-N_r),\cdots,y^{\mathrm{T}}(k)]^{\mathrm{T}} \in \mathscr{R}^{((N_r+N_b+1)\times p)\times 1}. \quad (9)$$

By combining all the outputs into vectors $\bar{y}(kN)$ and $\bar{y}[(k-1)N]$, all the inputs into a vector $\Delta\bar{u}(kN)$, all the noise vectors into $\bar{\xi}(kN)$ and $\bar{\xi}[(k-1)N]$, one obtains the key prediction equation

$$\mathscr{C}_1\bar{y}(kN) + \mathscr{C}_2\bar{y}[(k-1)N] = \theta\varphi(k) + H\Delta\bar{u}(kN) + \mathscr{E}_1\bar{\xi}(kN) + \mathscr{E}_2\bar{\xi}[(k-1)N]. \quad (10)$$

Where $\mathscr{C}_1,\mathscr{C}_2 \in \mathscr{R}^{(N\times p)\times(N\times p)}$ are Toeplitzmatrices defined as follows:

$\mathscr{C}_1 \triangleq$ lower triangular with first column $[I,\ C_1^{\mathrm{T}},\cdots,C_{N_c}^{\mathrm{T}},0,\cdots,0]^{\mathrm{T}}$

$\mathscr{C}_2 \triangleq$ upper triangular with first row $[0,\cdots,0,C_{N_c},\cdots,C_1]$ and

$$H \triangleq \begin{bmatrix} H_0 & 0 & \cdots & 0 \\ H_1 & H_0 & \cdots & 0 \\ & & \cdots & \\ H_{N-1} & H_{N-2} & \cdots & H_0 \end{bmatrix}, \quad \bar{E} \triangleq \begin{bmatrix} E_{1\,0} & 0 & \cdots & 0 \\ E_{1\,1} & E_{1\,0} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ E_{1\,N-1} & E_{1\,N-2} & \cdots & E_{1\,0} \end{bmatrix},$$

$$\mathscr{E}_1 = \bar{E}\mathscr{C}_1, \quad \mathscr{E}_2 = \bar{E}\mathscr{C}_2, \quad (11)$$

$$\theta \triangleq [\theta_1^{\mathrm{T}},\theta_2^{\mathrm{T}},\cdots,\theta_N^{\mathrm{T}}]^{\mathrm{T}} \in \mathscr{R}^{(N\times p)\times((N_r+N_b+1)\times p)}, \quad (12a)$$

$$\bar{y}(kN) \triangleq [y^{\mathrm{T}}(k+1),y^{\mathrm{T}}(k+2),\cdots,y^{\mathrm{T}}(k+N)]^{\mathrm{T}} \in \mathscr{R}^{(N\times p)\times 1}, \quad (12b)$$

$$\Delta\bar{u}(kN) \triangleq [\Delta u^{\mathrm{T}}(k),\Delta u^{\mathrm{T}}(k+1),\cdots,\Delta u^{\mathrm{T}}(k+N-1)]^{\mathrm{T}} \in \mathscr{R}^{(N\times p)\times 1}, \quad (12c)$$

$$\bar{\xi}(kN) \triangleq [\xi^{\mathrm{T}}(k+1),\xi^{\mathrm{T}}(k+2),\cdots,\xi^{\mathrm{T}}(k+N)]^{\mathrm{T}} \in \mathscr{R}^{(N\times p)\times 1}. \quad (12d)$$

Eq. (10) can be written as

$$\bar{y}(kN) = -\mathscr{C}_1^{-1}\mathscr{C}_2\bar{y}[(k-1)N] + \mathscr{C}_1^{-1}H\Delta\bar{u}(kN) + \mathscr{C}_1^{-1}\theta\varphi(k) + \mathscr{C}_1^{-1}\mathscr{E}_1\bar{\xi}(kN) + \mathscr{C}_1^{-1}\mathscr{E}_2\bar{\xi}[(k-1)N]. \quad (13)$$

Also, we define

$$Q(z^{-1}) = Q_{1j}(z^{-1}) + z^{-j}Q_{2j}(z^{-1}),$$

$$\partial Q_{1j}(z^{-1}) = j-1, \quad \partial Q_{2j}(z^{-1}) = N_q - j(\text{when } N_q > j),$$

$\mathscr{Q}_1 \triangleq$ lower triangular with first column $[I,Q_1^{\mathrm{T}},\cdots,Q_{N_q}^{\mathrm{T}},0,\cdots,0]^{\mathrm{T}}$, (14)

$\mathscr{Q}_2 \triangleq$ upper triangular with first row $[0,\cdots,0,Q_{N_q},\cdots,Q_1]$.

Let the reference signal be

$$\bar{w}(kN) \triangleq [w^{\mathrm{T}}(k+1),w^{\mathrm{T}}(k+2),\cdots,w^{\mathrm{T}}(k+N)]^{\mathrm{T}} \in \mathscr{R}^{(N\times p)\times 1} \quad (15)$$

and the input weighting matrix be

$$\bar{\lambda} \triangleq [\lambda,\lambda,\cdots,\lambda] \in \mathscr{R}^{(N\times p)\times(N\times p)}. \quad (16)$$

Then the expectation of the cost-function can be written as

$$J(N) = \mathscr{E}\{\|\bar{y}(kN) - \bar{w}(kN)\|_I^2 + \|\mathscr{Q}_1\Delta\bar{u}(kN) + \mathscr{Q}_2\Delta\bar{u}[(k-1)N]\|_{\bar{\lambda}}^2\},$$

i.e.,

$$J(N) = \mathscr{E}\{\| -\mathscr{C}_1^{-1}\mathscr{C}_2\bar{y}[(k-1)N] + \mathscr{C}_1^{-1}H\Delta\bar{u}(kN) + \mathscr{C}_1^{-1}\theta\varphi(k) + \mathscr{C}_1^{-1}\mathscr{E}_1\bar{\xi}(kN) + \mathscr{C}_1^{-1}\mathscr{E}_2\bar{\xi}[(k-1)N] - \bar{w}(kN)\|_I^2 + \|\mathscr{Q}_1\Delta\bar{u}(kN) + \mathscr{Q}_2\Delta\bar{u}[(k-1)N]\|_{\bar{\lambda}}^2\}. \quad (17)$$

3 The Control Law

Now minimizing (17) with respect to $\Delta\bar{u}(kN)$ yields

$$\Delta\bar{u}(kN) = (H^{\mathrm{T}}\mathscr{C}_1^{-\mathrm{T}}\mathscr{C}_1^{-1}H + \mathscr{Q}_1^{\mathrm{T}}\bar{\lambda}\mathscr{Q}_1)^{-1}\{H^{\mathrm{T}}\mathscr{C}_1^{-\mathrm{T}}[\bar{w}(kN) - \mathscr{C}_1^{-1}\theta\varphi(k)$$

$$+\mathscr{C}_1^{-1}\mathscr{C}_2\bar{y}((k-1)N)]-\mathscr{D}_1^{\mathrm{T}}\bar{\lambda}\mathscr{D}_2\Delta\bar{u}[(k-1)N]\}. \tag{18}$$

Where $(\cdot)^{\mathrm{T}}$ denotes $[(\cdot)^{\mathrm{T}}]^{-1}$.

In the following section we assume $N\geqslant\max(Na+1,\ Nb,\ Nc)+1$ andderive one expression of $\theta\varphi(k)$ in the key predictionequation (18).

Define $\mathscr{A}_1, \mathscr{A}_2, \mathscr{B}_1, \mathscr{B}_2\in\mathscr{R}^{(N\times p)\times(N\times p)}$ (Toeplitz matrices) as follows:

$$\begin{aligned}
&\mathscr{A}_1 \triangleq \text{lower triangular with first column } [I,\ \bar{A}_1^{\mathrm{T}},\cdots,\bar{A}_{Na+1}^{\mathrm{T}},0,\cdots,0]^{\mathrm{T}},\\
&\mathscr{A}_2 \triangleq \text{upper triangular with first row } [0,\cdots,0,\bar{A}_{Na+1},\cdots,\bar{A}_1],\\
&\mathscr{B}_1 \triangleq \text{lower triangular with first column } [B_0^{\mathrm{T}},B_1^{\mathrm{T}},\cdots,B_{Nb}^{\mathrm{T}},0,\cdots,0]^{\mathrm{T}},\\
&\mathscr{B}_2 \triangleq \text{upper triangular with first row } [0,\cdots,0,B_{Nb},\cdots,B_1].
\end{aligned} \tag{19}$$

From (6), (12b), (12c), (12d) and the matrices defined by (11), (19), it is easy to show that

$$\mathscr{A}_1\bar{y}(kN)+\mathscr{A}_2\bar{y}[(k-1)N]=\mathscr{B}_1\Delta\bar{u}(kN)+\mathscr{B}_2\Delta\bar{u}[(k-1)N] + \mathscr{C}_1\bar{\xi}(kN)+\mathscr{C}_2\bar{\xi}[(k-1)N],$$

i. e.

$$\bar{y}(kN)=\mathscr{A}_1^{-1}\{-\mathscr{A}_2\bar{y}[(k-1)N]+\mathscr{B}_1\Delta\bar{u}(kN)+\mathscr{B}_2\Delta\bar{u}[(k-1)N] + \mathscr{C}_1\bar{\xi}(kN)+\mathscr{C}_2\bar{\xi}[(k-1)N]\}. \tag{20}$$

Considering (3), (4), (5) and the matrices defined by (11), (19), we get the identities

$$\mathscr{C}_1=\bar{E}\mathscr{A}_1,\quad \bar{E}\mathscr{B}_1=H,\quad \bar{E}\mathscr{C}_1=\mathscr{E}_1,\quad \bar{E}\mathscr{C}_2=\mathscr{E}_2,$$

i. e.

$$\mathscr{A}_1^{-1}\mathscr{B}_1=\mathscr{C}_1^{-1}H,\quad \mathscr{A}_1^{-1}\mathscr{C}_1=\mathscr{C}_1^{-1}\mathscr{E}_1,\quad \mathscr{A}_1^{-1}\mathscr{C}_2=\mathscr{C}_1^{-1}\mathscr{E}_2. \tag{21}$$

Comparing (20) with the key prediction equation (13), we have

$$\mathscr{C}_1^{-1}\theta\varphi(k)=(\mathscr{C}_1^{-1}\mathscr{C}_2-\mathscr{A}_1^{-1}\mathscr{A}_2))\bar{y}[(k-1)N]+\mathscr{A}_1^{-1}\mathscr{B}_2\Delta\bar{u}[(k-1)N]. \tag{22}$$

Replacing (22) in (18) we have the control law:

$$\begin{aligned}
\Delta\bar{u}(kN)&=(\mathscr{B}_1^{\mathrm{T}}\mathscr{A}_1^{-\mathrm{T}}\mathscr{A}_1^{-1}\mathscr{B}_1+\mathscr{D}_1^{\mathrm{T}}\bar{\lambda}\mathscr{D}_1)^{-1}\{\mathscr{B}_1^{\mathrm{T}}\mathscr{A}_1^{-\mathrm{T}}[\bar{w}(kN)+\mathscr{A}_1^{-1}\mathscr{A}_2\bar{y}[(k-1)N]\\
&\quad-\mathscr{A}_1^{-1}\mathscr{B}_2\Delta\bar{u}((k-1)N)]-\mathscr{D}_1^{\mathrm{T}}\bar{\lambda}\mathscr{D}_2\Delta\bar{u}[(k-1)N]\}\\
&=(\mathscr{B}_1^{\mathrm{T}}\mathscr{A}_0\mathscr{B}_1+\mathscr{D}_1^{\mathrm{T}}\bar{\lambda}\mathscr{D}_1)^{-1}\{\mathscr{B}_1^{\mathrm{T}}\mathscr{A}_0[\mathscr{A}_1\bar{w}(kN)+\mathscr{A}_2\bar{y}[(k-1)N]\\
&\quad-\mathscr{B}_2\Delta\bar{u}((k-1)N)]-\mathscr{D}_1^{\mathrm{T}}\bar{\lambda}\mathscr{D}_2\Delta\bar{u}[(k-1)N]\}.
\end{aligned} \tag{23a}$$

Where

$$\mathscr{A}_0=(\mathscr{A}_1\mathscr{A}_1^{\mathrm{T}})^{-1}. \tag{23b}$$

4 Simulation Results

To illustrate the proposed method, two different models are utilized for simulations, and their respective mathematical models are represented as follows:

Model 1 Stable and nonminimum-phase plant

$$y(k)=A_1y(k-1)+A_2y(k-2)+B_1u(k-1)+B_2u(k-2)+\xi(k)+C_1\xi(k-1).$$

Where $A_1=\begin{bmatrix}1.4 & 0.2\\ 0.1 & 0.9\end{bmatrix}$, $A_2=\begin{bmatrix}-0.48 & -0.1\\ 0 & -0.2\end{bmatrix}$, $B_1=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$,

$$B_2=\begin{bmatrix}1.5 & 1\\ 0 & 1\end{bmatrix},\quad C_1=\begin{bmatrix}-0.5 & 0\\ 0.1 & -0.3\end{bmatrix}.$$

Model 2 Unstable plant

$$y(k) = A_1 y(k-1) + B_1 u(k-2) + \xi(k) + C_1 \xi(k-1).$$

Where $A_1 = \begin{bmatrix} 0.9 & -0.5 \\ -0.5 & 0.2 \end{bmatrix}$, $B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $C_1 = \begin{bmatrix} -0.2 & -0.4 \\ 0.2 & -0.8 \end{bmatrix}$.

The disturbance noise for each of the two models has a zero-mean valueand variance matrix diag[0, 1, 0.08]. The initial value ofthe convariance matrix was chosen to be 100 times the unit matrix for alltwo models. A standard recursive-least-sequares parameter estimator isutilized to estimate the parameter matrices in$\bar{A}(z^{-1})$, $B(z^{-1})$ and $C(z^{-1})$. The forgetting factors β_1 and β_2 are setequal to 0.98 and 0.95 for model 1; 0.98 and 0.96 for model 2,respectively.

For model 1, the parameters of MGPC are chosen as follows

$Na = 2$, $Nb = 3$, $Nc = 1$, $N = 4$, $\alpha = \mathrm{diag}[0.45, 0.4]$, $\lambda = \mathrm{diag}[0.15, 0.2]$,

$Q(z^{-1}) = I + \mathrm{diag}[0.4, 0.3]z^{-1}$.

The simulation results are shown in Fig. 1.a (the first circuit) and Fig. 1.b (the second circuit).

For model 2, the parameters of MGPC are chosen as follows

$Na = 2$, $Nb = 2$, $Nc = 1$, $N = 4$, $\alpha = \mathrm{diag}[0.4, 0.45]$, $\lambda = \mathrm{diag}[1.1, 1.0]$,

$Q(z^{-1}) = I + \mathrm{diag}[0.4, 0.45]z^{-1}$.

The simulation results are shown in Fig. 2.a (the first circuit) and Fig. 2.b (the second circuit).

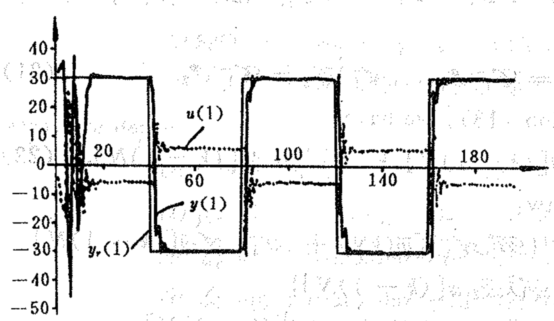

Fig 1a MGPC ST control to model 1 of the first circuit

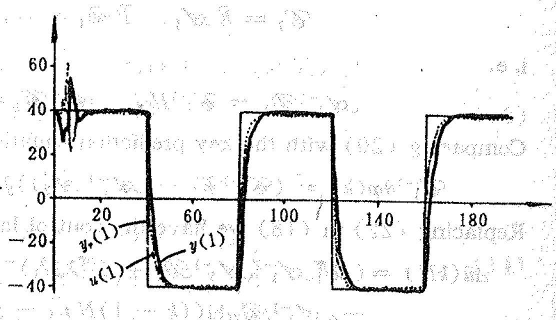

Fig 2a MGPC ST control to model 2 of the first circuit

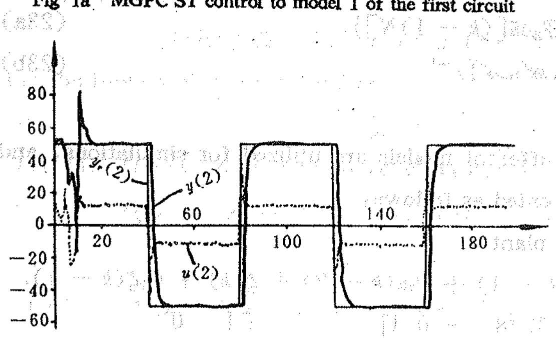

Fig 1b MGPC ST control to model 1 of the second circuit

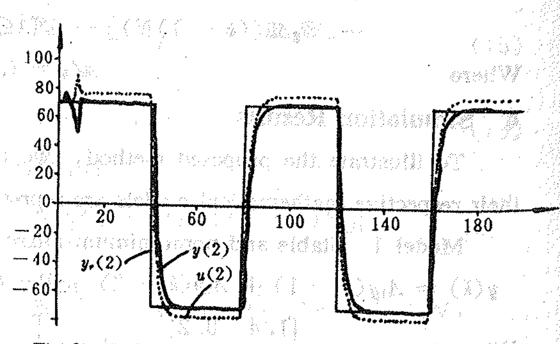

Fig 2b MGPC ST control to model 2 of the second circuit

5 Conclusions

A generalized predictive controller of multivariable linear systems with color-noise disturbances has been developed. This algorithm is directly to solve the controller using the identifiçation results, so that it reduces the calculations greatly. Simulations have shown that the proposed method is good in adaptivity and robustness.

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新型多变量广义预测自校正控制器

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摘要：本文对多变量线性系统提出了一种新的广义预测自校正控制器(MGPC)．由于本算法是利用辨识结果直接求解控制器，不需要在线求解 Diophantine 方程，因而大大减少了计算量，并简化了控制系统的设计过程．仿真结果表明：本文提出的算法能适用于具有有色噪声干扰的不稳定和(或)非最小相位多变量随机系统．

关键词：多变量控制；预测控制；自校正控制

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