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A CAD Algorithm for Calculation of Degrees of Importance of Components in Fault-Tolerant Control Systems*

GE Jianghua and SUN Youxian

(Institute of Industrial Process Control, Zhejiang University · Hangzhou, 310027, PRC)

Abstract: Failure of a component, such as a sensor or an actuator, may results in deterioration of properties or even complete failure of a control system. Therefore, it is very important to determine the degree of importance of a component in the design process of a fault-tolerant control system. In this paper, both degrees of static and dynamic importance of a component are presented, which denote the changed amplitude of static and dynamic properties respectively when a step change occurs in the output of a component. After simulation of control systems and computation of transfer matrix, a CAD algorithm is presented, which can simultaneously compute degrees of static and dynamic importance of r components. An example showing computation of degrees of importance in a process control system is studied, and the theoretical results are vertified by experiment.

Key words: degree of importance; fault-tolerant control; sensors; actuators; failure

1 Introduction

With the development in control theory, the improvement of control system reliability becomes more and more interested. The study mainly includes two categories[1~7]: 1) analysis of the reliability of a control system; 2) design of a control system with high reliability. In a control system, its reliability is highly related to property index. Failure of components will affect the system performance, and may change the working state of the control system. From the view of reliability analysis, by determining the degrees of affect of different components to behavior of control system, it will be known what component is important, and the faliure of this component will have significant effect to properties of the control system. Therefore, determination of degrees of importance of components is important for the design of fault-tolerant control systems with high reliability.

Study of sensitivities of parameters has been discussed by many articles[8~10]. But, they are limited to small changes in parameters. For large parameter changes, the affect to properties of control system still can not be explained. In this paper, degrees of static and dynamic importance of a component are presented, and they denote the amplitude change of static and dynamic response respectively when a unit step input of a component is applied. In the following section, a CAD algorithm is presented, which can compute degrees of static and dynamic importance of r

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components simultaneously. In the third section, an example shows the computation of degrees of importance of components in a process control system. The theoretical results are verified by experiment showing the effectiveness of this CAD algorithm.

For the sake of simplicity, in the later part of this paper, q_j will be used to represent component $q_j (j=1,\cdots,r)$.

2 Main Results

2.1 Definition

For a linear time-invariant controllable system

$$\dot{x}(t) = A(q_j)x(t) + B(q_j)u, \tag{1}$$

where $x(t) \in R^n$, $u \in R^m$, $A \in R^{n\times n}$, $B \in R^{n\times m}$, with unit step input signals

$$u_k = 1, \quad (k = 1,\cdots,m). \tag{2}$$

The response of the control system is $x_i(t)\ (i=1,\cdots,n)$. Obviously, the unit step response of $x_i(t)$ reflect the properties of the control system.

According to Taylor formula, when the change value of q_j is Δq_j, then in a neighborhood of q_j, there exists $x^{(n)}(t)$, and can be formulated as

$$\Delta x_i(t) = \left(\frac{\partial x_i(t)}{\partial q_j} q_j\right)\frac{\Delta q_j}{q_j} + \frac{\partial^2 x_i(t)}{\partial q_j^2}\frac{1}{2}\Delta q_j^2 + \cdots. \tag{3}$$

If t_s denotes the time, at which the unit step response reaches its steady state, then

$$\frac{\partial x_i(t)}{\partial q_j} = \text{const} \quad \text{when } t \geqslant t_s \tag{4}$$

and

$$\frac{\partial x_i^{(h)}(t)}{\partial q_j^{(h)}} = 0 \quad \text{when } t \geqslant t_s, h \geqslant 2, \tag{5}$$

therefore, in the notation of eq. (3), obviously

$$\Delta x_i(t) = \left(\frac{\partial x_i(t)}{\partial q_j} q_j\right)\frac{\Delta q_j}{q_j}. \tag{6}$$

The value of $\left(\frac{\partial x_i(t_s)}{\partial q_j} q_j\right)$ is the changed static value of state variable x_i when $\frac{\Delta q_j}{q_j}=1$. Obviously, the larger the value of $\left|\frac{\partial x_i(t_s)}{\partial q_j} q_j\right|$ is, the more important q_j is. Hence the definition of degree of static importance of a component is given as follows.

Definition 1 For some component q_j in a control system, degree of static importance (DSI) of q_j is defined as

$$\text{DSI} = \left(\frac{\partial x_i(t_s)}{\partial q_j} q_j\right)/x_i(t_s).$$

Similarily, the defination of degree of dynamic importance (DDI) of a component can be given in the similar way.

Definition 2 For some component q_j in a control system, degree of dynamic importance of q_j is defined as

$$\text{DDI} = \left(\int_0^{t_s}\left|\frac{\partial x_i(t)}{\partial q_j}\right| q_j \mathrm{d}t\right)/\left(\int_0^{t_s}|x_i(t)|\,\mathrm{d}t.\right.$$

Before discuss the computation of degrees of importance, one explanation must be made. The degrees of importance are defined under the supposition of unit step input signal action. The reason is that the process of step response is linearly related to input signals in time-domain. Under the action of step input signals with unit value, response of the control system can simultaneously reflect the static and dynamic properties of the system, and also reflect the degrees of static and dynamic importance of components simultaneously. In a practical system, the large changes of q_j may result in the non-linearity of the system. Then the accuracy of DDI of q_j depends on the values of $\frac{\partial x_i^{(h)}(t)}{\partial q_j^{(h)}}(h \geqslant 2)$, and DSI of q_j can be accuratly determined by definition 1 because (4) and (5) are still held.

2.2 Algorithm

For given unit step input signals, eq. (1) can be rewriten

$$\dot{x}(t) = A(q_j)x(t) + w(q_j), \tag{7}$$

where $w(q_j) \in \mathrm{R}^n$, $w_i \in \sum_{k=1}^{m} b_{ik}u_k, i = 1, \cdots, n$. Because the control system is controllable, there must exists a non-singular transfer

$$x(t) = T(q_j)z(t), \tag{8}$$

then

$$\dot{z}(t) = \bar{A}z(t) + \bar{W}, \tag{9}$$

where

$$\bar{A} = \begin{bmatrix} 0 & 1 & & & \\ & \ddots & \ddots & & 0 \\ & & \ddots & \ddots & \\ 0 & & & 0 & 1 \\ -\bar{a}_1(q_1) & & \cdots & & -\bar{a}_n(q_j) \end{bmatrix}, \quad \bar{W} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

For any parameter q_j, following equation is obtained from eq. (8)

$$\frac{\partial x(t)}{\partial q_j} = \frac{\partial T(q_j)}{\partial q_j} z(t) + T(q_j) \frac{\partial z(t)}{\partial \bar{a}(q_j)} \frac{\partial \bar{a}(q_j)}{\partial q_j} \tag{10}$$

where $\frac{\partial x(t)}{\partial q_j} \in \mathrm{R}^n$, $\frac{\partial T(q_j)}{\partial q_j} \in \mathrm{R}^{n \times n}$, $\frac{\partial z(t)}{\partial \bar{a}(q_j)} \in \mathrm{R}^{n \times n}$, $\frac{\partial \bar{a}(q_j)}{\partial q_j} \in \mathrm{R}^n$.

Obviously, if $\frac{\partial x(t)}{\partial q_j}$ is obtained from eq. (10), the degrees of static and dynamic importance of q_j are obtained simultaneously. Now, for computing $\frac{\partial x(t)}{\partial q_j}$, formulas for computing $\bar{a}(q_j)$, $\frac{\partial \bar{a}(q_j)}{\partial q_j}$, $T(q_j)$ and $\frac{\partial T(q_j)}{\partial q_j}$ have been introduced[8~10].

$z(t)$ can be directly obtained by solving eq. (9). Differentiating eq. (9) with respect to $\bar{a}_1$, a typical sensitivity model is

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial z(t)}{\partial \bar{a}_1}\right) = \bar{A}\frac{\partial z(t)}{\partial \bar{a}_1} - z_1(t), \tag{11}$$

where $z_1(t)=[0 \quad 0 \quad z_1(t)]^{\mathrm{T}}$ and $\dfrac{\partial z(0)}{\partial a_1}=0$, then, $\dfrac{\partial z}{\partial a_1}$ can be directly obtained by the simulation of eq. (11), and following results are ensured.

$$\frac{\partial z_n}{\partial \bar{a}_k}=-\sum_{i=1}^{n} a_i\left(\frac{\partial z_n}{\partial \bar{a}_{k-1}}\right)-z_{k-1}, \quad (k=2,\cdots,n),$$

$$\frac{\partial z_i}{\partial \bar{a}_k}=\frac{\partial z_{i+1}}{\partial \bar{a}_{k-1}}, \quad (i=1,\cdots,n-1, j=1,\cdots,n). \tag{12}$$

In the notation of eq. (8), simulation results also give the step response $x(t)$. For components $q_j(j=1,\cdots,r)$, it is obvious that degrees of static and dynamic importance of all components can be simultaneously calculated by the algorithm. A flow chart of this CAD algorithm is shown in Fig. 1.

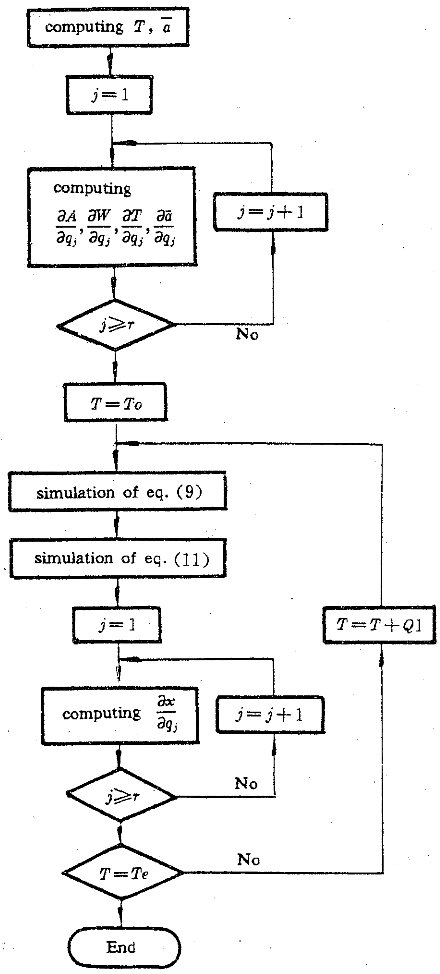

Fig. 1 Flow chart of CAD algorithm for computing degrees of importance of r components

3 An Example

A water level control experimental apparatus in laboratory shown in Fig. 2 is used for vertification.

State variable $x_i(k=1,2,3)$ separately denotes the height of levels, and the dynamic equation is

$$\dot{x}=\begin{bmatrix} \dfrac{-1.25-0.735q_2}{1+0.735q_1q_2} & \dfrac{-0.8q_1}{1+0.735q_1q_2} & \dfrac{0.9512q_1-0.8085}{1+0.735q_1q_2} \\ 2.3625 & -1.25 & 0 \\ 0 & 0.98823 & -1.17647 \end{bmatrix} x + \begin{bmatrix} \dfrac{0.735}{1+0.735q_1q_2} \\ 0 \\ 0 \end{bmatrix} U, \tag{13}$$

where $(q_1 \ q_2)=(0.5 \quad 1.945)$.

Because $x_3(t)$ is an output, degrees of importance of two components $(j=1,2)$ is studied. Fig. 3 shows degrees of importance of both components under unit step input signal. From Fig. 3, it is seen that degrees of importance of q_1 to x_3 are DSI$=0$ and DDI$=0.083$ respectively. This means that the failure of q_1 will not change the static value of x_3. Because the of dynamic importance degree of q_1 is also small, the failure of q_1 will smally affect the

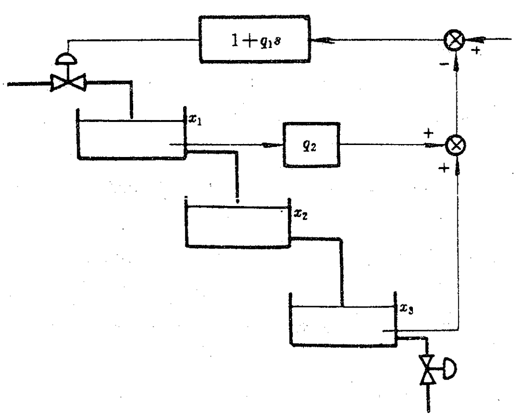

Fig. 2 Water level control apparatus

dynamic response of $x_3(t)$. For different values of q_1, experimental response curves are shown in Fig. 4. These results closely agree with the theoretical conclusion. Also from Fig. 3, it is obtained that degrees of importance of q_2 to x_3 are DSI$=0.6$ and DDI$=0.48$ respectively. It can be seen that degrees of both static and dynamic importance of q_2 are large. Therefore, the failure of q_2 will have significant affect to the properties of control system. For different q_2, experimental step response curves of x_3 are shown in Fig. 5. It is seen that the dynamic response $x_3(t)$ and steady point x_3 are changed largely due to the failure of q_2. These experimental results conform the theoretical conclusion.

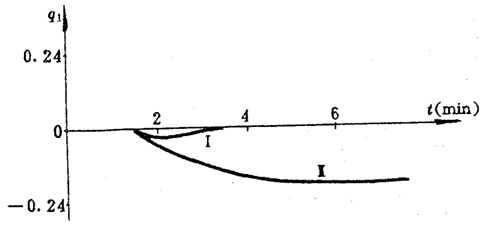

Fig. 3 Degree of importance of components q_j

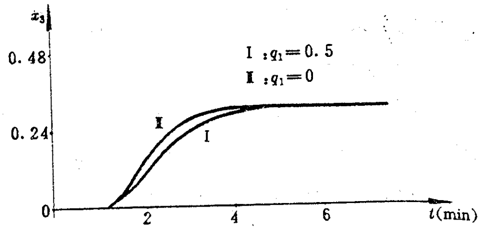

Fig. 4 Experimental step response of $x_3(t)$

4 Conclusion

A concept of degrees of static and dynamic importance of component q_j in control system is introduced and CAD algorithm for simultaneously computing degrees of importance of r components is developed. A process control system example shows that this method is very effective in assessing degrees of importance of a control system. However, the use of these results in design of a fault-tolerant control system with high reliability requires further study.

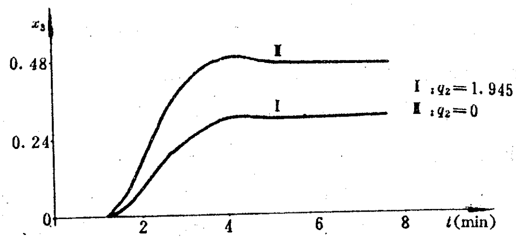

Fig. 5 Experimental step response of $x_3(t)$

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容错控制系统部件重要度计算的CAD算法

葛建华　孙优贤
(浙江大学工业控制研究所·杭州,310027)

摘要：部件(如传感器或执行器)的失效将导致系统性能的下降甚至完全失效.因此,在设计容错控制系统时确定部件重要度是非常重要的.本文提出了部件静态和动态重要度,它们分别表示了部件失效对系统静态和动态性能的影响程度.借助仿真和矩阵转换的运算,本文提出了能同时计算 m 个部件静态和动态重要度的CAD算法,例子验证了该法的有效性.

关键词：重要度；容错控制；传感器；执行器；故障

本文作者简介

葛建华　1964年生.1984年和1987年在浙江大学分别获得工学学士和工学硕士学位.1987年至1988年在宁波机电研究院工作.1988年考取浙江大学博士生,1990年10月提前获得工学博士学位.现为浙江大学博士后.目前从事容错控制理论的研究和应用工作,发表论文10余篇.

孙优贤　1940年生.现任浙江大学工业控制技术研究所副所长、教授;工业控制技术国家重点实验室主任;中国自动化学会理事;中国自动化学会应用委员会副主任 1984年至1987年获德国洪堡研究奖学金.长期从事过程控制理论和应用研究,以及造纸过程的模型化和计算机控制.发表学术论文100多篇,编著5本,获省部级科技进步奖5次,获国家级教学成果奖1项.