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# A Nonlinear Controller Design Approach and Its Application to a Biochemical Reactor*

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**Abstract**: This paper addressed a design problem for a class of nonlinear control systems via a two-step approach. The simulation result of a biochemical reactor example shows that the method proposed is applicable to practical problems.

**Key words**: nonlinear systems; two-step design; biochemical process

## 1 Introduction

In the global linearization methodology to an affine nonlinear system, $\dot{x}=f(x)+g(x)u$, one must solve a set of partial differential equations in order to get a nonlinear transformation.

In this paper, we will develop another kind of transformation problems for a class of nonlinear systems, and then the nonlinear controller to the system is derived via a two-step design approach. Finally, an example of a biochemical reactor is given to show the applicability of the method proposed.

## 2 Nonlinear State Transformation

Consider the following class of nonlinear systems:

S1:
$$\dot{x}_1 = f_1(x_1, x_2), \tag{1}$$
$$\dot{x}_2 = f_2(x_1, x_2, u), \tag{2}$$

where $x=\{x_1^{\mathrm{T}}\ x_2^{\mathrm{T}}\}^{\mathrm{T}}\in \mathrm{R}^n$, it is assumed that $n$ is an even number and $\dim(x_1)=\dim(x_2)=n/2$. When $n$ is an odd number, a dummy state (e.g. $\dot{x}_i=0$) could be added to $x_1$ in order to make $n+1$ be even. Here the key of this assumption is that the control action $u\in \mathrm{R}^r$ only acts on less than half state variables, e.g. $x_2$. And we assume that the nonlinear functions, $f_1(x_1,x_2)$, $f_2(x_1,x_2,u)$, are at least twice differentiable with respect to $x_1, x_2, u$. For the convenience, denote $f_1(x_1, x_2)=f_1(x)$, $f_2(x_1,x_2,u)=f_2(x,u)$.

Note the special structure of the system, consider the following nonlinear state transformation ($z=T(x)=[T_1^{\mathrm{T}}(x)\ T_2^{\mathrm{T}}(x)]^{\mathrm{T}}$):

$$z_1 = T_1(x) = x_1, \tag{3}$$
$$z_2 = T_2(x) = \dot{x}_1 = f_1(x), \tag{4}$$

where $z$ is the new state vector in the mapping space.

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**Theorem 1** The nonlinear system (1)～(2) can be transformed, by the nonlinear state transformation (3)～(4), into the following linear controllable canonical form:

$$\dot{z} = Az + Bv, \tag{5}$$

S2:

where

$$A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ I \end{bmatrix},$$

$$z = [z_1^{\mathrm{T}} \; z_2^{\mathrm{T}}]^{\mathrm{T}},$$

$$v = \frac{\partial f_1}{\partial x_1} f_1(x_1, x_2) + \frac{\partial f_1}{\partial x_2} f_2(x_1, x_2, u). \tag{6}$$

**Proof** According to Eqs. (3)～(4), the proof is straightforward.

**Theorem 2** The nonlinear state transformation $T(x \to z)$ is invertible if and only if

$$\det \left[\frac{\partial f_1}{\partial x_2}\right] \neq 0. \tag{7}$$

## 3 Nonlinear Feedback Control Algorithm

### 3.1 Step-by-Step Design Approach for Nonlinear Controller

The main idea of the step-by-step design approach was proposed by Takamatsu et al.[1]. This approach divides the controller design by several steps using different simpler models, respectively.

Step 1 It is not difficult to design a linear control law for the linear controllable canonical model Eq. (5) as follows:

$$v = Kz. \tag{8}$$

And according to the relations Eqs. (3)～(4), we can obtain

$$v = v(x), \tag{9}$$

where $v(x)$ is some nonlinear function of the system state $x$.

Step 2 In this step, we will use the result obtained in step 1 to develop a nonlinear control law for the original system (1), (2) based on the Eq. (6) of theorem 1 as follows:

$$u = u(v, x) = u(v(x), x) = u(x). \tag{10}$$

### 3.2 Nonlinear Feedback Control Law

Eq. (10) is the solution of Eq. (6), which will be the desired feedback control law in the nonlinear expression. However, the explicit form of the control algorithm, Eq. (10), is mostly impossible to be available, except for the special case when the control action $u$ appears linearly. Here we will use discretization method to give a control algorithm[1]. From Eq. (6) we take the Taylor series around one instant $k-1$ before, and approximating with only the first order, we have

$$\left[\frac{\partial f_1}{\partial x_2}(k)\right]^{-1} \left(v_k - \frac{\partial f_1}{\partial x_1}(k) f_1(x_k)\right)$$
$$= f_2(x_{k-1}, u_{k-1}) + \frac{\partial f_2}{\partial x}(k-1)(x_k - x_{k-1}) + \frac{\partial f_2}{\partial u}(k-1)(u_k - u_{k-1}), \tag{11}$$

i.e.,

$$M(x_{k-1},u_{k-1})u_k = f(x_k,x_{k-1},u_{k-1}), \tag{12}$$

where

$$M(x_{k-1},u_{k-1}) = \frac{\partial f_2}{\partial u}(k-1),$$

$$f(x_k,x_{k-1},u_{k-1}) = -f_2(k-1) + \frac{\partial f_2}{\partial u}(k-1)u_{k-1} + \frac{\partial f_2}{\partial x}(k-1)(x_{k-1}-x_k)$$
$$+ \left(\frac{\partial f_1}{\partial x_2}(k)\right)^{-1}(v_k - \frac{\partial f_1}{\partial x_2}(k) - f_1(k)).$$

Eq. (12) consists of a set of linear algebric equations. The existence conditions of the control law is given in Takamatsu et al. [1]

## 4 Nonlinear Control of a Biochemical Reactor

In this section an example of a biochemical reactor cited from Takamatsu et al. [2] is taken into consideration for nonlinear controller design. The nonlinear model of the process is as follows:

$$\frac{da}{dt} = 0.1353\exp(-\frac{0.001a}{1-a})\frac{S}{0.25+S}$$
$$-0.2945a(a+0.385)(\frac{S}{0.001+S}-\frac{S}{0.25+S}),$$
$$\frac{dc}{dt} = 83.53(7.6-c) - (\frac{258.98aS}{0.001+S}+3.3)X_c,$$
$$\frac{dX_c}{dt} = \frac{0.2945aSX_c}{0.001+S} - UX_c,$$
$$\frac{dS}{dt} = -\frac{0.0409aSX_c}{0.001+S} - \frac{0.04SX_c}{1.15+S} + U(1-S).$$

By applying the result above, Fig. 1 shows only the result of the key variable, $X_c$, under the nonlinear control, linear control, and non-control, when its initial state away from the steady state by 0.3. Fig. 2 shows the result that in a normal operation around the steady state, a sudden shut of the control valve occurs for one hour, after it repaired, the designed nonlinear control, linear control, and non-control are running again. The results in two figures give that the performance by nonlinear control is better than that by linear control or non-control.

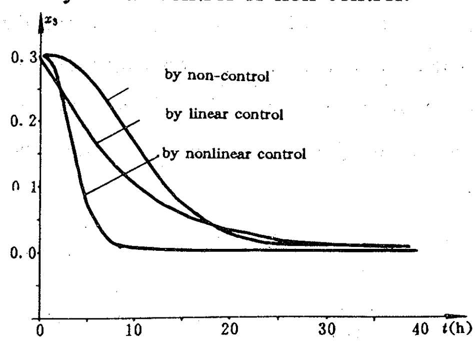


Fig. 1 A simulation with $x_3(0)=0.3$

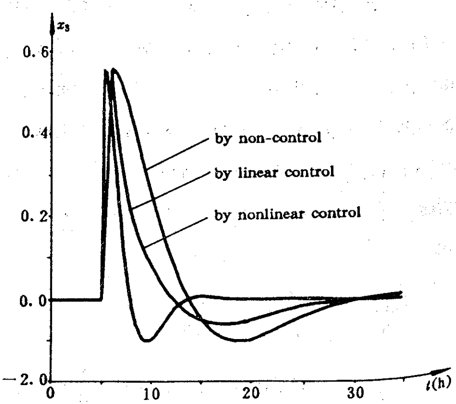


Fig. 2 A simulation with a sudden shutdown of control value for one hour at $t=5$.

## 5 Conclusions

This paper concerns with the state transformation of a class of nonlinear systems, and a two-step design approach for nonlinear controller. The simulation results of the biochemical reactor show that the design approach proposed in this paper is applicable.

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# 一种非线性控制器设计法及其在生化反应器中的应用

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**摘要**：本文讨论一类非线性系统的控制器设计问题.文中给出一种基于非线性模型结构的状态变换,讨论了一种非线性控制器的两步设计法,并得到相应的非线性控制算法.最后以一个生化反应器为例,通过仿真说明文中给出的控制器算法的可行性.

**关键词**：非线性系统；非线性变换；控制器设计；两步设计法；生化反应过程

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