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On Local Disturbance Decoupling with Asymptotic Stability for Nonlinear Systems

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Abstract: The problem of disturbance decoupling with stability (DDPS) has been paid quite a lot attention in recent years, and some significant results have been obtained. The main contribution of this paper is that in more general situations where, for example, the number of inputs could be different from that of outputs, the hyperbolic condition could be violated, and etc., necessary and sufficient conditions are given so that the local DDPS problem for a kind of nonlinear control systems can be solved. As an application of the main results, a theorem is shown which simplifies the design of a feedback control law for DDPS for a nonlinear control system obtained by cascading two systems together.

Key words: nonlinear control system; invariant submanifold; cascading system; minimum phase

1 Introduction

In recent years a considerable amount of attention in the field of nonlinear control theory[1~6] has been paid to the problem of feedback stabilization of nonlinear systems and the related ones such as the disturbance decoupling problem with stability (DDPS for short), due to their practical significance and theoretical attraction. In [5] Wegen-Nijmeijer consider the local DDPS for nonlinear systems with equal number of inputs and outputs and with a relative degree at the equilibrium. They introduce the notion of maximal stable distribution Δ_s^*. Under certain hypotheses, they show that the problem of disturbance decoupling with exponential stability is locally solvable if and only if the disturbance channel of the nonlinear system is contained in Δ_s^*. However, Δ_s^* is usually difficult to calculate and its existence strongly depends on the hypotheses that the linearized counterpart of the nonlinear system is controllable and the zero dynamics of the nonlinear system has a hyperbolic equilibrium. For example, let us consider the following system:

$$\begin{cases} \dot{x} = f(x) + G(x)u + e(x)w(t), \\ y = h(x), \end{cases} \tag{1.1}$$

where

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$$f=\begin{bmatrix} x_1 \\ x_1x_2 \\ x_1+x_2-x_3 \\ x_1+x_2-x_3-x_4 \end{bmatrix},\quad G=\begin{bmatrix}1\\1\\1\\2\end{bmatrix},\quad e=\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

and $h(x)=x_1$. The linearized counterpart of the system is controllable but the zero dynamics of the system does not have a hyperbolic equilibrium.

It is easy to show that the largest controlled invariant distribution Δ^* in $\ker\{dh\}$ is $\mathrm{sp}\left\{\frac{\partial}{\partial x_2},\frac{\partial}{\partial x_3},\frac{\partial}{\partial x_4}\right\}$. And both $\Delta_1:=\mathrm{sp}\left\{\frac{\partial}{\partial x_3},\frac{\partial}{\partial x_4}\right\}$, $\Delta_2:=\mathrm{sp}\left\{x_1\frac{\partial}{\partial x_2}+\frac{\partial}{\partial x_3},\frac{\partial}{\partial x_4}\right\}$, are stable controlled invariant distributions in Δ^*. The Δ^* itself is not a stable controlled invariant distribution. Therefore there is no largest stable controlled invariant distribution in Δ^*. However, later on we will show that DDPS is actually solvable for the system.

In this paper, we study the problem of disturbance decoupling with asymptotic stability for nonlinear control systems. As we understand, in the literature the DDPS is generally proposed as DDP plus exponential stability. However, in many cases, the DDPS can be considered solved if the DDP is solved and the state trajectories of the system are bounded when the disturbance is sufficiently small. By the theory on total stability[7], we can replace the requirement of exponential stability by that of asymptotic stability for this purpose. Evidently, when only asymptotic stability is asked for the DDPS, the notion of Δ_s^* becomes too strong. In this paper we solve the problem of local disturbance decoupling with asymptotic stability for a nonlinear control system by using the smallest locally controlled invariant and involutive distribution which contains the disturbance channel. We should point out that in [6] this approach is proposed independently to solve the problem of disturbance decoupling with exponential stablity. As an application of our main results, the case of cascading systems is studied in the paper. A result is given which simplifies the design of a feedback law for DDPS for a nonlinear system obtained by cascading two systems together.

2 Local DDPS for Nonlinear Control Systems

Let us consider the following system

$$\begin{cases} \dot{x}=f(x)+G(x)u(t)+e(x)w(t), \\ y=h(x), \end{cases} \tag{2.1}$$

where $x\in R^n$, $u\in R^m$, $y\in R^l$, $G(x)=(g_1(x)\ g_2(x)\ \cdots\ g_m(x))$, $\dim(G(x)=m$ and $h'(x)=(h_1(x)\ h_2(x)\ \cdots\ h_l(x))$, here "$'$" denotes the transpose.

Suppose $f(0)=0$ and $h(0)=0$. For a smooth feecback control $u=\alpha(x)+\beta(x)v(t)$, (2.1) becomes:

$$\begin{cases} \dot{x}=f(x)+G(x)\alpha(x)+G(x)\beta(x)v+e(x)w(t), \\ y=h(x). \end{cases} \tag{2.2}$$

The DDPS, as suggested in [4], is defined as follows:

Definition 2.1 We say the disturbance decoupling problem with exponential stability

(DDPES) for (2. 1) is solved locally if

1) DDP is locally solved by a pair (α,β) in (2. 2).

2) The equilibrium 0 of (2. 2) is exponentially stable when $v(t)$ and $w(t)$ are equal to zero.

As we have pointed out in the introduction, in many cases, one would consider the DDPS is solved if the output is not affected by the disturbance and the state trajectory of the system is bounded if the disturbance is well bounded. Namely, when $v(t)=0$, for (2. 2):

$\forall\ \varepsilon>0, \exists\ \delta_1>0,\ \delta_2>0$ such that if $\|x_0\|<\delta_1$, $|w(t)|<\delta_2$, then $\|x(t,x_0)\|<\varepsilon \forall\ t\geq 0$.

From the theory on total stability[7], we know that if the equilibrium of (2. 2) is asymptotically stable when $v(t)$ and $w(t)$ is set to zero, then, the bounded state stability is guaranteed. Therefore, we give the following extended definition for DDPS:

Definition 2. 2 We say the disturbance decoupling problem with asymptotic stability (DDPAS) for (2. 1) is solved locally if

1) DDP is locally solved by a pair (α,β) in (2. 2).

2) The equilibrium 0 of (2. 2) is asymptotically stable when $v(t)$ and $w(t)$ are equal to zero.

For the sake of simplicity, we will assume that in (2. 1) the mappings $f(x)$, $G(x)$, $h(x)$ and $e(x)$ are smooth and the noise $w(t)$ is bounded and measurable in a neighborhood of 0. Therefore, the differential equation (2. 2) is always integrable.

In the present approaches of solving DDP, the maximal involutive controlled invariant distribution in the $\ker\{dh(x)\}$, denoted by Δ^*, has played an important role. And Δ^* exists under mild conditions (see e. g. [1]). Throughout this paper, we assume that for (2. 1) Δ^* exist and be nonsingular. Then, there exists a local coordinate system around the equilibrium point such that after a regular state feedback, (2. 1) is locally diffeomorphic to the following system in the new coordinates,

$$\begin{cases} \dot{x}_1 = f_1(x_1) + G_1(x_1)u + e_1(x)w, \\ \dot{x}_2 = f_2(x_1,x_2) + G_2(x_1,x_2)u + e_2(x)w, \\ y = h(x_1). \end{cases} \tag{2. 3}$$

In these coordinates the maximal involutive controlled invariant distribution in $\ker\{dh\}$ is represented by $\Delta^*=\mathrm{sp}\left\{\frac{\partial}{\partial x_2}\right\}$.

For (2. 1) DDP is locally solvable if

H1 $\qquad e(x)\in\Delta^*$, i. e. $e_1(x)=0$ in (2. 3).

On a local chart of the origin, regular state feedback and coordinates transformations form a transformation group. Therefore, when we discuss the DDPS of (2. 1), without loss of generality, we can assume that system (2. 1) is already in the form of (2. 3).

If x_1 is a stabilizable mode of (2. 3), i. e. the subsystem $\dot{x}_1=f_1(x_1)+G_1(x_1)u$ can be stabilized by a state feedback $u=\alpha(x_1)$. Then, as a classical result, we know,

Proposition 2.1 Suppose H1 holds and x_1 is a stabilizable mode, if additionally the noise-free dynamics on the leaf of Δ^* through 0 characterized by

$$\dot{x}_2 = f_2(0, x_2) \tag{2.4}$$

is asymptotically stable, then DDPAS is solvable.

In this case, Δ^* itself is the maximum stable distribution contained in Δ^*.

The hypothesis in Proposition 2.1 is obviously very strong. In this paper, we will solve the much subtler case where (2.4) is not necessarily stable by utilizing the smallest stable distribution $(\Delta_e)_*$ contained in Δ^* which contains $e(x)$.

Let us first introduce some preliminary results which will be needed later. Set $w(t)=0$ and rewrite system (2.3) as follows:

$$\begin{cases} \dot{x} = f(x) + G(x)u, \\ y = h(x). \end{cases} \tag{2.5}$$

We adopt the notion friend from the linear systems theory:

Definition 2.3 Let Δ be an involutive controlled invariant distribution or (f,G)-invariant distribution for (2.5) and denote $F(\Delta)$ the set of all regular state feedback (α,β) such that the distribution Δ is invariant under $f+G\alpha$ and $G\beta$. An element of $F(\Delta)$ is called a friend of Δ.

Lemma 2.2 If Δ_1, Δ_2 are two nonsingular involutive (f,G)-invariant distributions with the property that $\Delta_1\subset\Delta_2$ and $\Delta_2\cap \mathrm{sp}\{G(x)\}=0$, then $F(\Delta_2)\subset F(\Delta_1)$.

Proof Let $(\alpha,\beta)\in F(\Delta_2)$, under a proper coordinate system the closed loop system has the form:

$$\begin{cases} \dot{x}_1 = f_1(x_1) + G_1(x_1)v, \\ \dot{x}_2 = f_2(x_1,x_2) + G_2(x_1,x_2)v, \end{cases} \tag{2.6}$$

where $\Delta_2=\mathrm{sp}\left\{\frac{\partial}{\partial x_2}\right\}$. As $\Delta_1\subset\Delta_2$, let $\Delta_1=\mathrm{sp}\left\{\frac{\partial}{\partial x_{22}}\right\}$, then (2.6) can be rewritten in the following form:

$$\begin{cases} \dot{x}_1 = f_1(x_1) + G_1(x_1)v, \\ \dot{x}_{21} = f_{21}(x_1,x_2) + G_{21}(x_1,x_2)v, \\ \dot{x}_{22} = f_{22}(x_1,x_2) + G_{22}(x_1,x_2)v, \end{cases} \tag{2.7}$$

where $x_2'=(x_{21}',x_{22}')$. As Δ_1 is (f, G)-invariant, there exists (α^*, β^*) such that

$$\frac{\partial}{\partial x_{22}}\begin{pmatrix} f_1(x_1) + G_1(x_1)\alpha^*(x_1,x_2) \\ f_{21}(x_1,x_2) + G_{21}(x_1,x_2)\alpha^*(x_1,x_2)\end{pmatrix} = 0, \quad \frac{\partial}{\partial x_{22}}\begin{pmatrix} G_1(x_1)\beta^*(x_1,x_2) \\ G_{21}(x_1,x_2)\beta^*(x_1,x_2)\end{pmatrix} = 0.$$

Because $\Delta_2\cap\mathrm{sp}\{G(x)\}=0$, $G_1(x_1)$ has full column rank. It is easy to verify that $\frac{\partial}{\partial x_{22}}\alpha^*(x_1,x_2)=0$ and $\frac{\partial}{\partial x_{22}}\beta^*(x_1,x_2)=0$. Therefore, $\frac{\partial}{\partial x_{22}}G_{21}(x_1,x_2)=0$, and then $\frac{\partial}{\partial x_{22}}f_{21}(x_1,x_2)=0$. It implies that $(\alpha,\beta)\in F(\Delta_1)$. Q. E. D.

Lemma 2.3[1] Let x_e be an equilibrium point of the vector field $f(x)$. Suppose that Δ is a nonsingular and involutive controlled invariant distribution and that

$$\Delta \cap \operatorname{sp}\{G(xx)\} = 0, \quad \dim(G) = m.$$

Let $(\alpha_i, \beta_i)(i=1,2) \in F(\Delta)$ with $\alpha_1(x_e)=\alpha_2(x_e)=0$. Let M_{x_e} be the maximal integral submanifold Δ which contains x_e. Then on M_{x_c}

$$\alpha_1(x) = \alpha_2(x).$$

From the previous lemmas, we have seen the importance of the condition $\Delta \cap \operatorname{sp}\{G(x)\}=0$. In this section, we assume for system (2. 1)

H2 $$\Delta^* \cap \operatorname{sp}\{G(x)\}=0.$$

Now we can summarizing the preceding lemmas into the following proposition:

Proposition 2. 4 Suppose H2 holds. Then for system (2. 5), the following statements are true:

1) Each regular involutive controlled invariant distribution Δ contained in Δ^* is also invariant under $f+G\alpha^*$ and $G\beta^*$ for each $(\alpha^*, \beta^*) \in F(\Delta^*)$;

2) If $\Delta \subset \Delta^*$ and Δ is controlled invariant distribution, then, for all regular state feedback $(\alpha, \beta) \in F(\Delta)$ with $\alpha(x_e)=0$, the dynamics of the flow of the vector $f+G\alpha$ on the integral submanifold of Δ through the equilibrium point has the same stability property.

Now we are ready to discuss the DDPS in a fairly general setting. Let us denote $(\Delta_e)_*$ the smallest controlled invariant distribution containing $\operatorname{sp}\{e\}$ and Δ_c the largest local controllability distribution in $\ker\{dh\}$ for (2. 1). Since we assume that for system (2. 1) Δ^* is nonsingular and involutive and that H1 and H2 hold, $(\Delta_e)_*$ is contained in Δ^* and $\Delta_c=0$. Because (2. 1) is diffeomorphic to (2. 3) after an (α, β) transformation when $(\alpha, \beta) \in F(\Delta^*)$, then by Lemma 2. 2

$$(\Delta_e)_* = \langle f, G, e/\operatorname{sp}\{e\}\rangle$$

the smallest distribution invariant under vector fields f, G, e of system (2. 3) and containing $\operatorname{sp}\{e\}$.

In this paper, we pose the regularity assumption on this distribution, i. e. , it is nonsingular and involutive. The algorithm for obtaining the distribution $\langle f, G, e/\operatorname{sp}\{e\}\rangle$ can be found in [1].

Now we state the main result of this section.

Theorem 2. 5 For system (2. 1), assume that H1 and H2 hold. Then DDPAS is solvable if and only if

1) There exists $(\alpha^*, \beta^*) \in F((\Delta_e)_*)$ such that the dynamics of the vector flow $\Phi_t^{f+G\alpha^*}$ on the leaf of $(\Delta_e)_*$ through the zero point is asymptotically stable;

2) The dynamics of (2. 1) modulo $(\Delta_e)_*$ is asymptotically stabilizable.

Proof Under the hypotheses, (2. 1) is diffeomorphic to the following system after a feedback transformation $(\alpha, \beta) \in F((\Delta_e)_*)$

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_{21}) + G_1(x_1, x_{21})u, \\ \dot{x}_{21} = f_{21}(x_1, x_{21}) + G_{21}(x_1, x_{21})u, \\ \dot{x}_{22} = f_{22}(x_1, x_{21}, x_{22}) + G_{22}(x_1, x_{21}, x_{22})u + e_{22}(x)w, \\ y = h(x_1), \end{cases} \tag{2. 8}$$

where $\Delta^* = \mathrm{sp}\{\frac{\partial}{\partial x_{21}}, \frac{\partial}{\partial x_{22}}\}$ and $(\Delta_e)_* = \langle f, G, e/\mathrm{sp}\{e\}\rangle = \mathrm{sp}\{\frac{\partial}{\partial x_{22}}\}$.

1) means that

$$\dot{x}_{22} = f_{22}(0, 0, x_{22}) \tag{2.9}$$

is asymptotically stable. Note that by Proposition 2.4, the stability of this dynamics does not change with different choices of (α, β) in $F((\Delta_e)_*)$.

2) means that there exists a feedback control $u = \alpha(x_1, x_{21})$ such that

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_{21}) + G_1(x_1, x_{21})\alpha_1(x_1, x_{21}), \\ \dot{x}_{21} &= f_{21}(x_1, x_{21}) + G_{21}(x_1, x_{21})\alpha_1(x_1, x_{21}) \end{aligned} \tag{2.10}$$

is asymptotically stable.

The sufficiency of this theorem is a direct consequence of the following lemma:

Lemma 2.6[8] For the following system

$$\dot{x}_1 = f_1(x_1), \tag{2.11a}$$

$$\dot{x}_2 = f_2(x_1, x_2). \tag{2.11b}$$

The equilibrium 0 is asymptotically stable if and only if the equilibrium 0 of (2.11a) is asymptotically stable and when set $x_1 = 0$, the equilibrium 0 of (2.11b) is asymtotically stable.

On the other hand, if there exists a state feedback (α^*, β^*) which solves DDPAS of (2.1), then $e(x) \in \Delta^*$ by H1. Thus, $(\Delta_e)_* \subset \Delta^*$. By Proposition 2.4 $(\alpha^*, \beta^*) \in F((\Delta_e)_*)$ since $(\Delta_e)_* = \langle f, G, e/\mathrm{sp}\{e\}\rangle$. Therefore, 1) and 2) are true according to Proposition 2.4 and Lemma 2.6. Q. E. D.

Now we consider the example shown by (1.1) again. For this system

$$(\Delta_e)_* = \mathrm{sp}\{\frac{\partial}{\partial x_4}\}.$$

It is easy to verify that all conditions of Theorem 2.5 are satisfied. Thus, DDPS is solvable for this system. In fact

$$u = -3x_1 + x_2 + v$$

solves the DDPES.

3 DDPAS for Cascading Nonlinear Control Systems

In this section, we consider the DDPAS problem for a nonlinear system obtained by cascading the following two systems together:

$$\begin{cases} \dot{\xi}_1 = f(\xi_1) + g(\xi_1)u, \\ y = h(\xi_1), \quad \xi_1 \in R^q, \quad y \in R^m, \quad u \in R^m, \end{cases} \tag{3.1}$$

$$\dot{\xi}_2 = F(\xi_2, v) + e(\xi_2)w(t), \quad \xi_2 \in R^s, \quad v \in R^m, \quad w \in R^p. \tag{3.2}$$

By cascading them, we obtain:

$$\begin{cases} \dot{\xi}_1 = f(\xi_1) + g(\xi_1)u, \\ \dot{\xi}_2 = F(\xi_2, y) + e(\xi_2)w(t), \\ y = h(\xi_1), \end{cases} \tag{3.3}$$

where we assume the mappings f, g, h, F and e are smooth and $w(t)$ is bounded and measurable.

Recently, a great deal of attention has been paid to this kind composite systems, for example, [3,4,9].

Obviously, for system (3. 3), the DDP is already solved. Our emphasis should be in finding a feedback control law which stabilizes (3. 3) while the outputs remain decoupled from the disturbance. In this section, we will develop a method which enables us to solve the DDPAS for (3. 3) by mainly studying (3. 2).

We need the following assumptions:

H3 (3. 1) has relative degree $(r_1, \cdots, r_m)$ at zero and is minimum phase. And $\mathrm{sp}\{g_1, \cdots, g_m\}$ is involutive.

H4 For all fixed $v \in R^m$ in (3. 2),

$$(\Delta_e)_* = \langle F(\xi_2, v), e/\mathrm{sp}\{e\}\rangle$$

is nonsingular and involutive and remains the same for all v.

These two assumptions are about the structure of the system. Accordingly, we define

Definition 3. 1 $v(\xi_2)$ is a friend of $(\Delta_e)_*$ $(v(\xi_2) \in F((\Delta_e)_*))$ if $[F(\xi_2, v(\xi_2)), (\Delta_e)_*] \in (\Delta_e)_*$.

Before we present our result, we also need to state the following hypothesis:

H5 (3. 3) is weakly stabilizable, i. e., there exists $u = Q_1\xi_1 + Q_2\xi_2$, such that

$$\sigma\begin{bmatrix} \dfrac{\partial f(0)}{\partial \xi_1} + g(0)Q_1 & g(0)_1 Q_1 \\ \dfrac{\partial F(0, h(0))}{\partial \xi_1} & \dfrac{\partial F(0, h(0))}{\partial \xi_2} \end{bmatrix} \subset \bar{C}^-$$

the closed left half plane. Obviously, this condition is also necessary for the solution of DDPAS.

Now we state our main result of this section:

Theorem 3. 1 Suppose H3～H5 hold. Then, for system (3. 3), the DDPAS is solvable if there exits $v^*(\xi_2) \in F((\Delta_e)_*)$ which $\mathrm{sp}\{dv_*(x)\} \in (\Delta_e)_*^{\perp}$, such that when $w(t)$ is set to zero, the flow on a center mainfold of the close loop system (3. 2) is asymptotically stable and the dynamics on the leaf of $(\Delta_e)_*$ through zero is asymptotically stable.

Proof After a coordinates change, we can write the closed loop system (3. 2) as follows:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} A^+ & 0 & 0 \\ 0 & A^- & 0 \\ 0 & 0 & A^0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} Q_1(x_1, x_2, x_3) \\ Q_2(x_1, x_2, x_3) \\ Q_3(x_1, x_2, x_3) \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} w(t), \tag{3. 4}$$

where $\sigma(A^+) \subset C^+, \quad \sigma(A^0) \subset C^0, \quad \sigma(A^-) \subset C^-.$

On a center manifold $x_1 = \varphi_1(x_3)$, $x_2 = \varphi_2(x_3)$, the flow

$$\dot{q} = A^0 q + Q_3(\varphi_1(q), \varphi_2(q), q) \tag{3. 5}$$

is asymptotically stable. Note that the system itself may not be stable due to the possible existence of A^+.

Since the flow on the leaf of $(\Delta_e)_*$ through zero is asymptotically stable, as a necessary condition,

$$(\Delta_e)_* = \mathrm{sp}\{\frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\}.$$

Then, we can rewrite (3. 4) as:

$$\begin{cases} \dot{x}_1 = A^+ x_1 + Q_1(x_1, x_{21}, x_{31}), \\ \dot{x}_{21} = A_{21}^- x_{21} + Q_{21}(x_1, x_{21}, x_{31}), \\ \dot{x}_{31} = A_{31}^0 x_{31} + Q_{31}(x_1, x_{21}, x_{31}), \\ \dot{x}_{22} = A_{22}^- x_{22} + Q_{22}(x_1, x_2, x_3) + e_1(x)w(t), \\ \dot{x}_{32} = A_{32}^0 x_{32} + Q_{32}(x_1, x_2, x_3) + e_2(x)w(t), \end{cases} \tag{3. 6}$$

where $(\Delta_e)_* = \mathrm{sp}\{\frac{\partial}{\partial x_{22}}, \frac{\partial}{\partial x_{32}}\}$ and

$$\dot{x}_{22} = A_{22}^- x_{22} + Q_{22}(0, x_{22}, x_{32}),$$
$$\dot{x}_{32} = A_{32}^0 x_{32} + Q_{32}(0, x_{22}, x_{32})$$

is asymptotically stable. Since (3. 6) has a triangular structure, there must exists a center manifold in the form:

$$x_1 = \varphi_1(x_{31}), \quad x_{21} = \varphi_{21}(x_{31}), \quad x_{22} = \varphi_{22}(x_{31}, x_{32}).$$

For system (3. 1), if H3 holds, it is well known[3] that after a feedback transformation, it is locally diffeomorphic to the following system:

$$\begin{cases} \dot{z} = f_0(z, \eta_1, \cdots, \eta_r), \\ \dot{\eta}_1 = \eta_2, \\ \vdots \\ \dot{\eta}_r = f_r(z, \eta_1, \cdots, \eta_r) + u, \\ y = \eta_1. \end{cases} \tag{3. 7}$$

Without loss of generality, we assume $r_1 = \cdots = r_m = r$ for the sake of simplicity. And by the hypothesis H3, the zero dynamics

$$\dot{z} = f_0(z, 0)$$

is asymptotically stable.

Now let

$$\begin{cases} \tilde{\eta}_1 = \eta_1 - r_1(x), \\ \tilde{\eta}_i = \eta_i - r_i(x, \eta_1, \cdots, \eta_{i-1}), \quad i = 2, \cdots, r, \end{cases}$$

where $r_1(x) = v^*(\xi_2) = \tilde{v}^*(x)$, $r_{i+1}(x, \eta_1, \cdots, \eta_{i-1}) = \frac{\partial r_i}{\partial x}\tilde{F}(x, \eta_1) + \frac{\partial r_i}{\partial \eta}\dot{\eta}$, $i = 1, \cdots, r-1$.

By the assumptions in Theorem 3. 1, $\mathrm{sp}\{dr_1(x)\} \in (\Delta_e)_*^{\perp}$. And by Lemma 1. 6. 7 in [1], $(\Delta_e)_*^{\perp}$ is also invariant under F. Therefore, we can rewrite (3. 3) as follows

$$\begin{cases} \dot{z} = f_0(z, \tilde{\eta}_1 + r_1(x), \cdots, \tilde{\eta}_r + r_r(x, \tilde{\eta})), \\ \dot{\tilde{\eta}}_1 = \tilde{\eta}_2, \\ \vdots \\ \dot{\tilde{\eta}}_r = f_r(z, \tilde{\eta}_1 + r_1(x), \cdots, \tilde{\eta}_r + r_r(x, \tilde{\eta})) + R(x, \tilde{\eta}) + u, \end{cases}$$

$$\begin{cases} \dot{x}_1 = A^+ x_1 + Q_1(x_1, x_{21}, x_{31}) + P_1(x_1, x_{21}, x_{31}, \tilde{\eta}_1)\tilde{\eta}_1, \\ \dot{x}_{21} = A_{21}^- x_{21} + Q_{21}(x_1, x_{21}, x_{31}) + P_{21}(x_1, x_{21}, x_{31}, \tilde{\eta}_1)\tilde{\eta}_1, \\ \dot{x}_{31} = A_{31}^0 x_{31} + Q_{31}(x_1, x_{21}, x_{31}) + P_{31}(x_1, x_{21}, x_{31}, \tilde{\eta}_1)\tilde{\eta}_1, \\ \dot{x}_{22} = A_{22}^- x_{22} + Q_{22}(x_1, x_2, x_3) + P_{22}(x_1, x_{21}, x_{31}, \tilde{\eta}_1)\tilde{\eta}_1 + e_1(x)w(t), \\ \dot{x}_{32} = A_{32}^0 x_{32} + Q_{32}(x_1, x_2, x_3) + P_{32}(x_1, x_{21}, x_{31}, \tilde{\eta}_1)\tilde{\eta}_1 + e_2(x)w(t). \end{cases} \tag{3.8}$$

By H5, there exists $u = K_1\tilde{\eta}_1 + K_2 x_1$, such that

$$\sigma \begin{bmatrix} A & 0 \\ K_1 & K_2 \\ (P_1(0)0\cdots 0) & A^+ \end{bmatrix} \subset C^-,$$

where

$$A = \begin{bmatrix} 0 & I & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & & I \end{bmatrix}.$$

We claim that $u = K_1\tilde{\eta}_1 + K_2(x_1 - \varphi_1(x_{31})) - f_r - R(x, \tilde{\eta}) + v$ solves the DDPAS for (3.8). Indeed, it can be shown that

$$\tilde{\eta} = 0, \quad x_1 = \varphi_1(x_{31}), \quad x_{21} = \varphi_{21}(x_{31})$$

is a center manifold for the closed loop system (3.8) modulo $(\Delta_e)_*$. And the flow on the center manifold is governed by (3.5). Therefore, the dynamics of (3.8) modulo $(\Delta_e)_*$ is asymptotically stable. By Theorem 2.5, the DDPAS is solved. Q. E. D.

4 Further Discussions

In our results in the preceding sections, the hypothesis that $\Delta^* \cap \mathrm{sp}\{G(x)\} = 0$ plays a very important role. If this hypothesis does not necessarily hold, even though all other hypotheses still hold, the problem becomes much more complicated. In this case, after a proper regular input transformation, i. e. let $u = \beta(x)v$ with $\det\beta(x) \neq 0$, (2.3) locally diffeomorphic to the following system:

$$\begin{cases} \dot{x}_1 = f_1(x_1) + G_1(x_1)v_1, \\ \dot{x}_{21} = f_{21}(x_1, x_{21}) + G_{21}^1(x, x_{21})v_1, \\ \dot{x}_{22} = f_{22}(x_1, x_{21}, x_{22}) + G_{21}^2(x1, x_{21}, x_{22})v_1 + G_{22}^2(x_1, x_{21}, x_{22})v_2 + e_2 w, \\ y = h(x_1), \end{cases} \tag{4.1}$$

where $\Delta^* = \mathrm{sp}\left\{\frac{\partial}{\partial x_2}\right\}$, $(\Delta_e)_* = \mathrm{sp}\left\{\frac{\partial}{\partial x_{22}}\right\}$, and $v_1 = m_1 < m$.

The complexity is reflected in the fact that in this case, conditions 1) and 2) in Theorem 2.5 is no longer necessary, because the largest local controllability distribution in $\{\ker dh\}$ Δ_c is nonzero. We note that the DDPES problem in this case has been discussed in [6]. Greater efforts are needed to fully understand the problem in this general setting.

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Reference

[1] Isidori, A.. Nonlinear Control Systems. Springer-Verlag, Berlin, 1989

[2] Aeyels, D.. Stabilization of a Class of Nonliear Systems by a Smooth Feecback Control. Systems and Control Letters 5, 1985, 289—294

[3] Byrnes, C. I. and Isidori, A.. Asymptotic Stabilization of Minimum Phase Nonlinear Systems. Submitted to IEEE Trans. Automat. Contr.,

[4] Byrnes, C. I. and Isidori, A.. Nonlinear Disturbance Decoupling with Stability. Proc. of the 26th Conference on Decision and Control 1987, 513—518

[5] L. van der Wegen and Nijmeijer, H.. The Local Disturbance Decoupling Problem with Stability for Nonlinear Systems. Systems Control Lett., 1989, 12:139—149

[6] L. van der Wegen. Another Approach to the Local Disturbance Decoupling Problem with Stability for Nonlinear Systems. MTNS-1989, Amsterdam, 1989

[7] Hahn, W. Stability of Motion. Springer-Verlag. Berlin, 1967

[8] Vidyasagar, M.. Decomposition Techniques for Large Scale Systems with Nonadditive Interactions: Stability and Stabilizability. IEEE Trans. Automat. Contr., 1980, AC-25:773—779

[9] Kokotovic, P. and Sussmann, H.. A Positive Real Condition for Global Stabilization of Nonlinear Systems. Systems Control Lett., 1989, 13:125—134

[10] Carr, J.. Applications of Centre Manifold Theory. Springer-Verlag, New York, 1981

[11] Kelly, A.. The Stable, Center-Stable, Center, Center-Unstable, Unstable Manifolds. J. Differential Equation, 1967, 3:546—570

[12] Wonham, W. M.. Linear Multivariable Control: A Geometric Approach. Springer-Verlag, New York, 1979

关于非线性系统带稳定性的局部干扰解耦

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摘要:非线性带稳定性的干扰解耦问题(DDPS)近年来备受人们的注意,并获得了一些成果(参阅[2,3,4,5,6]).本文的主要贡献是,在更一般的情况下(例如,输入-输出个数不同,不要求双曲条件,不要求指数稳定等),给出了一类非线性系统 DDPS 有解的充要条件.

关键词:非线性控制系统;不变子流形;串接系统;最小相位

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