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Matrix Pade Typé Approximations and Model Reduction of Multivariable Control Systems*

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Abstract: In this paper a new mixed method of reduction——matrix Padé type approximation method is introduced. This method chooses an arbitrary matrix polynomial as denominator of a reduced model, then determines numerator of the reduced model by matrix Padé type approximation. The example given in this paper shows that the reduced model derived from our method is a good approximation to the original system. The computational procedure of this method is very concise.

Key words: matrix Padé type approximation; model reduction; multivariable control system; transfer function matrix

1 Introduction

The exact analysis of most systems of higher order is both tedious and costly. Thus the problem of reducing a higher order system to its lower order models is considered important in analysis, systhesis and simulation of practical systems such as the aircraft systems and the chemical processing control systems. Rational approximation is considered an important approach in linear multivariable systems. In fact, the problem of model reduction may be stated as follows: Given some information about a system with a rational transfer function (of high order), find a system with a lower order rational transfer function that in some sense approximates the original system.

Lots of methods of reduction[7,8] are based on the retention of the dominant poles of the system in the reduced model. The most important feature of these methods is that the reduced model is always stable (unstable) if the original system is stable (unstable). However, most of these methods assume that the system is described in state-vector form, and we must compute the eigenvalues and eigenvectors of the high order state matrix. Thus these methods are computationally very cumbersome.

Another popular approach to the reduction problem is based on partial sequences of Markov parameters and time-moments, continued-fractions and Padé approximation[1,4,5]. Although these rational approximation methods play an important part and have been widely used in many domains, they have a very serious disadvantage: the reduced model may be unstable (stable) even if the original system is stable (unstable).

Many mixed mehtods, which retain the advantages of both the stability-equation and the

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Padé approximation mehtod, have been put forward (see [6,11,12,13]). In this paper, we first introduce the new concept: matrix Padé type approximation, based on which we develop a method that also combines the desirable features of the Padé approximation method and the modal methods. Illustrative example will show that this is a convenient and powerful reduction method.

2 Matrix Padé Type Approximation (MPTA)

We first introduce the definition of matrix Padé type approximation (MPTA).

Let $f(z)$ be given by a power series with matrix coefficients

$$f(z) = \sum_{k=0}^{\infty} c_k z^k, \quad c_k \in C^{p\times m}. \tag{2.1}$$

Let $v(x)$ be an arbitrary quasi-monic polynomial of degree k

$$v(x) = \sum_{i=0}^{k} b_i x^i, \quad b_i \in C^{m\times m}, \quad b_k \text{ is nonsingular.} \tag{2.2}$$

Set

$$w(z) = a_0 + a_1 z + \cdots + a_{k-1} z^{k-1}, \tag{2.3}$$

where
$$a_i = \sum_{j=0}^{k-i-1} c_j b_{i+j+1}, \quad i = 0,1,\cdots,k-1. \tag{2.4}$$

Define
$$\tilde{w}(z) = z^{k-1} w(z^{-1}), \quad \tilde{v}(z) = z^k v(z^{-1}),$$

then

$$f(z) - \tilde{w}(z)[\tilde{v}(z)]^{-1} = O(z^k), \quad z \to 0, \tag{2.5}$$

we call $\tilde{w}(z)[\tilde{v}(z)]^{-1}$ the ${}^r(k-1/k)$ right-handed matrix Padé type approximant (RMPTA) of $f(z)$.

Similarly, if we replace (2.4) by

$$a_i = \sum_{j=0}^{k-i-1} b_{i+j+1} c_j, \quad i = 0,1,\cdots,k-1, \tag{2.6}$$

we have

$$f(z) - [\tilde{v}(z)]^{-1}\tilde{w}(z) = O(z^k), \quad z \to 0. \tag{2.7}$$

We call $[\tilde{v}(z)]^{-1}\tilde{w}(z)$ the ${}^l(k-1/k)$ left-handed matrix Padé type approximant (LMPTA) of $f(z)$.

In general, ${}^r(k-1/k)$ RMPTA does not equal ${}^l(k-1/k)$ LMPTA, which can be illustrated by the following example.

Example 1

$$f(z) = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} + c_1 z + c_2 z^2 + \cdots, \quad \forall\, c_i \in C^{2\times 2}, i = 1,2\cdots,$$

Choose
$$v(z) = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} z - \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},$$

then
$${}^r(0/1)_f(z) = \begin{pmatrix} 1 & z \\ 1 & z \end{pmatrix}; \quad {}^l(0/1)_f(z) = \begin{pmatrix} 1+z & 0 \\ 1 & 0 \end{pmatrix}.$$

Obviously,

$$^{r}(0/1)_f(z) \neq {}^{l}(0/1)_f(z).$$

As in the scalar case, we can obtain many algebraic properties and convergent results of matrix Padé type approximant. Here we do not discuss them in detail. Interested reader may refer to book[2].

3 Reductions of Multivariable Systems

Consider a multivariable system expressed by

$$Y(s) = H(s)U(s), \tag{3.1}$$

where $H(s)$ is the transfer function matrix of the system, which is writen in the form

$$\begin{aligned} H(s) &= N(s)[D(s)]^{-1} \\ &= [A_0 + A_1 s + \cdots + A_{n-1}s^{n-1}][B_0 + B_1 s + \cdots + B_n s^n]^{-1} \end{aligned} \tag{3.2}$$

$$A_i(i = 0,1,\cdots,n-1) \in C^{p\times m}, \quad B_i(i = 0,1,\cdots,n) \in C^{m\times m}.$$

Now we use right-handed matrix Padé type approximation to produce the reduced models. Let

$$v(s) = \sum_{j=0}^{l} b_j s^j, \quad b_j \in C^{m\times m}, \quad b_l \text{ is nonsingular} \tag{3.3}$$

be an arbitrarily given polynomial matrix, and take the matrix polynomial $\tilde{v}(s)$ as the denominator of the reduced model. Then compute the numerator of the reduced model by the matrix Padé type approximation. The computational procedure for RMPTA is summarized as follows

Step 1 Expand $H(s)$ into a power series

$$H(s) = \sum_{i=0}^{\infty} c_i s^i, \quad c_i \in C^{p\times m} \tag{3.4}$$

with

$$\begin{cases} c_i = (A_i - \sum_{j=0}^{i-1} c_j B_{i-j})B_0^{-1}, \\ c_{-1} = 0, \text{ and } B_i = 0 \text{ for all } i > n. \end{cases} \tag{3.5}$$

Step 2 Use (2.3)~(2.6), we can get an $^{r}(l-1/l)$ MPTA of $H(s)$. The reduced model is given by

$$H_l(s) = {}^{r}(l-1/l)_H. \tag{3.6}$$

The procedure for LMPTA can be obtained similarly.

Remark Only in a neighborhood of the original point $s=0$ Padé type approximation $H_l(s)$ $(l=0,1,\cdots)$ is a good approximation of $H(s)$, because the approximant $H_l(s)$ is obtained by letting the first l terms of the Taylor expansion series of $H_l(s)$ at $s=0$ be the same as those of $H(s)$, this can be seen from (2.5) or (2.7). However, if s is large, the error $H(s) \sim H_l(s)$ may be large in magnitude.

Example 2

Consider the transfer function matrix

$$H(s) = \begin{pmatrix} 2s^3 + 51s^2 - 186s - 960 & s^3 + 36s^2 + 128s - 252 \\ 3s^3 - 4s^2 - 29s & 2s^3 + 13s^2 - 59s - 42 \end{pmatrix} \cdot \begin{pmatrix} -3s^4 - 14s^3 - 119s^2 - 812s - 960 & 4s^4 + 31s^3 - 8s^2 - 239s - 252 \\ -s^4 - 4s^3 - 73s^2 - 390s & s^4 - s^3 - 81s^2 - 181s - 42 \end{pmatrix}^{-1} \tag{3.7}$$

From (3. 5) the $c_i's$ can be evaluated. The first $c_i's$ are

$$\begin{cases} c_0 = \begin{pmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{pmatrix}, \\ c_1 = \begin{pmatrix} -0.6520834 & -4.825595 \\ -0.3760417 & -0.6485119 \end{pmatrix}, \\ c_2 = \begin{pmatrix} 2.334868 & 0.5096516 \\ 9.449852 & -0.361371 \end{pmatrix}. \end{cases} \tag{3.8}$$

Select

$$v_1(s) = \begin{pmatrix} s+1 & 3s+2 \\ -2s & 1 \end{pmatrix}, \tag{3.9}$$

$$v_2(s) = \begin{pmatrix} -24s^2-15s+1 & 4s^2+7s+3 \\ -10s & -2s^2-s+1 \end{pmatrix}, \tag{3.10}$$

$$v_3(s) = \begin{pmatrix} -192s^3-124s^2+s-3 & -36s^3-29s^2+3s+4 \\ -78s^2+s-1 & -6s^3-25s^2-8s+1 \end{pmatrix}, \tag{3.11}$$

we get the reduced models

$$H_1(s) = \begin{pmatrix} 1 & 3 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} s+1 & 2s+3 \\ -2 & s \end{pmatrix}^{-1}, \tag{3.12}$$

$$H_2(s) = \begin{pmatrix} 0.6500015s-24.0 & 14.04286s+4.0 \\ -0.9749994s & -1.207143s-2.0 \end{pmatrix} \cdot \begin{pmatrix} s^2-15s-24 & 3s^2+7s+4 \\ -10s & s^2-s-2 \end{pmatrix}^{-1}, \tag{3.13}$$

$$H_3(s) = \begin{pmatrix} 9.96s^2+1.2s-192.0 & 1.7959s^2-23.4286s-36.0 \\ -0.36s^2-5.8s & 2.93877s^2-7.57143s-6.0 \end{pmatrix} \cdot \begin{pmatrix} -3s^3+s^2-124s-192 & 4s^4+3s^2-29s-36 \\ -s^3+s^2-78s & s^3-8s^2-25s-6 \end{pmatrix}^{-1}. \tag{3.14}$$

respectively. The unit-step response of two components (y_1 and y_2) of the output Y are shown in Fig. 1 and Fig. 2.

If some poles of $H(s)$ are known, it can be meaningful to utilize this information. In the above example, the poles of $H(s)$ are $-1, -2, \cdots, -8$, the denominator polynomial $\tilde{v}_1(s)$, $\tilde{v}_2(s)$ and $\tilde{v}_3(s)$ have poles $-2, -3$; $-1, -2, -3, -8$ and $-1, -2, -3, -4, -6, -8$ respectively.

These figures indicate that the results are acceptable, that is to say, the reduced models give good approximation to the orig-

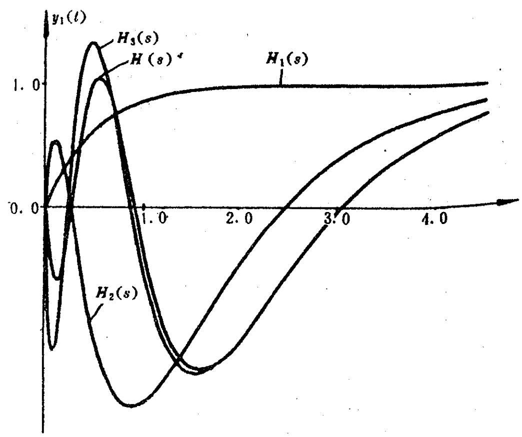

Fig. 1 Unit-step responses of the first output of example 2

nal one. The main advantages of this method are

1) The computational procedure is simple.

2) If the polynomial $v(s)$ is chosen such that all its zeros are in the left complex plane, then the reduced model is stable. Especially, if the original system is stable, and $D_1(s)$ is a divisor of $D(s)$, by choosing $v(s)=\tilde{D}_1(s)$, we can obtain a stable reduced model which retains some eigenvalues (may not be dominant ones) of the original system. This is an important feature of matrix Padé type approximation method.

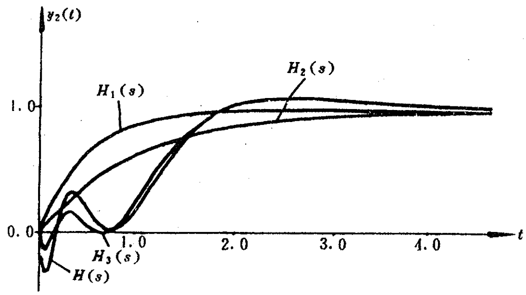

Fig. 2 Unit-step responses of the second output of example 2

3) By properly choosing $v(s)$, the reduced model may retain a number of large magnitude poles of the system in the reduced model. This point has been emphasized by Shamash in [12].

It should be pointed out that if we choose $v(s)=\sum_{k=1}^{l}(s-s_k)\ I$ (I is the $m\times m$ identity matrix), where $s_k(k=1,\cdots,l)$ are dominant eigenvalues of $H(s)$, then our method is equivalent to the method proposed in [12].

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矩阵 Padé 型逼近及多变量控制系统的模型简化

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摘要:本文给出了一个新的模型简化混合方法:矩阵 Padé 型方法,该方法首先任选一个矩阵多项式作为简化模型之分母,然后由矩阵 Padé 型逼近求出其分子. 文中的数值例子表明由我们的方法得到的简化模型是原系统的一个好的近似. 该法的计算步骤十分简洁.

关键词:矩阵 Padé 型逼近;模型简化;多变量控制系统;传递函数阵

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庄国忠 1966 年生. 1984 年考入南京大学数学系计算数学专业,1988 年毕业,并获学士学位. 1988 年至 1991 年 3 月就读于中国科学院计算中心,专攻方向为计算数学与数学软件,获硕士学位,毕业后留所工作. 目前主要研究领域为:逼近论(含数值逼近),数学软件以及控制与系统理论.

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