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Robust Stabilization for a Kind of Uncertain Systems

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Abstract: In this paper a robust stabilization problem is studied for plants with both structured and unstructural uncertainties. An H_∞ robust performance problem is investigated, where parameter uncertainties in system input and output matrices are considered. It is shown that the problem is equivalent to a standard H_∞ design problem for an extended system of the plant with a scaling parameter. Using this result, a solution of the robust stabilization problem is derived by application of existing H_∞ technique.

Key words: robust stabilization; uncertain systems; H_∞ control

1 Introduction

Robust stabilization problem have received a great deal of attention in the past years. It is well known that most effective method to solve the problem is H_∞ design approach. For unstructural uncertainty described in frequency domain, robust stabilization problem is equivalent to an H_∞ sub-optimal design problem[1] which can be solved by solutions of two Riccati equations[2,3]. For structural uncertainty described in the state space such as $E\Delta F$, an effective method dealing with robust stability is quadratic stabilization[4;5], and it is shown recently in [5] that the quadratic stabilization problem can be reduced to an H_∞ sub-optimal design problem.

In this paper, attention is focused to robust stabilization problem for linear plants with both structural uncertainty and parameter perturbations. It will be shown that the solution can be obtained by solving an equivalent H_∞ robust performance design problem for an extended system of plant with parameter perturbations. Among various methods dealing with H_∞ robust performance problems, the ARI (Algebraic Riccati Inequality) method seems to be most simple and effective. Through the method a conservative result is obtained since a fixed Lyapnov function $V(x)$ is required for all parameter perturbations in plant. Static state feedback controllers for this problem are designed via ARI method in Xie et al. [6,7] and Shen et al. [8,9]. And linear dynamic controller is discussed in Xie et al. [10]. However, these results can not be applied to our robust stabilization problem, because a dynamic controller is required as well as perturbations in the disturbance input matrix should be considered in order to solve the problem. In [10] only the dynamic controller is discussed and in [9] only the perturbation is involved.

In the first part of this paper the robust stabilization problem is reduced to the robust perfor-

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mance design problem which has the perturbation in the disturbance input matrix. Then it is shown that the H_∞ robust performance design problem can be transformed to a standard H_∞ sub-optimal design problem. Finally, a solution of the robust stabilization is obtained by solving the standard problem.

2 Preliminary

The plant considered in this paper has both structural and unstructural uncertainties, and is given by (1).

$$\bar{P} = P(s,\Sigma)(I + \Delta P(s)), \tag{1}$$

$\Delta P(s)$ denotes the unstructural uncertainty with the properties

$$\|\Delta P(s)\|_\infty \leqslant \varepsilon, \quad \Delta P(s) \in RH_\infty, \tag{2}$$

$P(s,\Sigma)$ represents the plant with parameter perturbations such as

$$P(s,\Sigma) = \left[\begin{array}{c:c} A+\Delta A & B+\Delta B \\ \hdashline C+\Delta C & D+\Delta D \end{array}\right], \tag{3}$$

$$\begin{bmatrix} \Delta A & \Delta B \\ \Delta C & \Delta D \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \Sigma [F_1 \quad F_2], \quad \Sigma \in \Omega$$

where E_1, E_2, F_1 and F_2 are given matrices, and unknown matrix $\Sigma \in \Omega$ is square with appropriate dimension.

$$\Omega = \{\Sigma \mid \Sigma^T \Sigma \leqslant I\}. \tag{4}$$

When $\Delta P = 0$ and $\Sigma = 0$, the plant $P_0(s) = P(s,0)$ is called nominal plant.

Remark There are many literatures which discusse (1) with $\Sigma = 0$ or $\Delta P = 0$. When $\Sigma = 0$, (1) denote plant with uncertainty in high frequency range, this is usual case in which a lower order model was used for designing controller. When $\Delta P = 0$, (1) denote plant with uncertain parameters, for example, a manipulator with different loads or under different operating environment, aircraft under different fighting conditions, etc.. In fact, these two type uncertainties must be considered simultaneously, because we have to use a lower order model for simplicity, and we have to consider different operating environment in practice.

Robust stabilization problem (RSP) Given the plant (1), find a feedback controller $K(s)$ such that the closed loop system is stable for any $\Sigma \in \Omega$ and $\Delta P(s)$.

When parameter perturbation is given as Σ_1, it is well known that a closed loop system with a controller $K(s)$ is stable for any $\Delta P(s)$ if and only if

$$\|K(s)P(s,\Sigma_1)[I + K(s)P(s,\Sigma_1)]^{-1}\|_\infty < \varepsilon^{-1}.$$

So that, we have the following fundamental Lemma.

Lemma 2.1 Let $\gamma = \varepsilon^{-1}$. The closed loop system in Fig. 1 is robust stable if and only if

$$\|K(s)P(s,\Sigma)[I + K(s)P(s,\Sigma)]^{-1}\|_\infty < \gamma \tag{5}$$

for any $\Sigma \in \Omega$.

Hence, the robust stabilization problem becomes to find a controller $K(s)$ which satisfies (5). It will be shown later that the problem can be transformed to the following H_∞ robust performance design problem.

H_∞ robust performance design problem (RPP) Given a generalized plant $G(s,\Sigma)$, find a controller $K(s)$ such that the closed loop system is stable and

$$\|T_{zw}(s,\Sigma)\|_\infty < \gamma \tag{6}$$

for any $\Sigma \in \Omega$, where T_{zw} denotes transfer function from w to z.

The generalized plant is given by

$$G(s,\Sigma) = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \left[\begin{array}{c:cc} A + \Delta A & B_1 + \Delta B_1 & B_2 + \Delta B_2 \\ \hdashline C_1 & 0 & D_{12} \\ C_2 + \Delta C & D_{21} + \Delta D_1 & D_{22} + \Delta D_2 \end{array}\right], \tag{7}$$

$$\begin{bmatrix} \Delta A & \Delta B_1 & \Delta B_2 \\ \Delta C & \Delta D_1 & \Delta D_2 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \Sigma [F_a \quad F_1 \quad F_2], \quad \Sigma \in \Omega. \tag{8}$$

Let the state space representation of the closed loop system be given by

$$\dot{x}_c = A_c(\Sigma)x_c + B_c(\Sigma)w, \tag{9}$$

$$z = C_c(\Sigma)x_c. \tag{10}$$

Lemma 2. 2 Suppose that the state space representation of the closed loop system with a controller $K(s)$ is given by (9), (10). If there exists a positive definite matrix P such that

$$A_c^T(\Sigma) + PA_c(\Sigma) + \gamma^{-2}PB_c(\Sigma)B_c^T(\Sigma)P + C_c^T(\Sigma)C_c(\Sigma) < 0 \tag{11}$$

for any $\Sigma \in \Omega$, then $K(s)$ is a solution of the RPP.

Proof The robust stability follows immediately from (11). The validity of the norm condition can be shown by considering $\frac{\mathrm{d}}{\mathrm{d}t}x^TPx$ with $x(0)=x(\infty)=0$. Q. E. D.

Lemma 2. 2 gives a conservative result to check $K(s)$ being an H_∞ robust performance controller. But it is a feasible way based on the Algebraic Riccati Inequality ARI. In this paper, we focused our attention on the solutions of H_∞ robust performance problem which satisfy the condition (11), and call them H_∞ robust sub-optimal controllers. Note that the set of H_∞ robust performance problem which satisfy the condition (11), and call them H_∞ robust sub-optimal controllers is a subset of H_∞ robust performance controllers.

Now, let us consider the relation between the RSP and the RPP. Define a generalized plant $\overline{G}(s,\Sigma)$ of the plant (1) by

$$\overline{G}(s,\Sigma) = \begin{bmatrix} 0 & I \\ P(s,\Sigma) & -P(s,\Sigma) \end{bmatrix}. \tag{12}$$

Lemma 2. 3 If there exist an H_∞ robust sub-optimal controller $K(s)$ for the generalized plant $\overline{G}(s,\Sigma)$, then $K(s)$ is a solution of the robust stabilization problem for the plant $\overline{P}$.

Proof The result follows from Lemma 2. 1 and Lemma 2. 2 with the fact that

$$K(s)P(s,\Sigma)[I + K(s)P(s,\Sigma)]^{-1} = \mathrm{LFT}(\overline{G}(s,\Sigma);K(s)), \tag{13}$$

where LFT denotes linear fractional transformation. Q. E. D.

3 H_∞ Robust Sub-Optimal Controller Design

In this section we discuss a design of an H_∞ robust sub-optimal controller for the plant $G(s,\Sigma)$. The controller obtained here is of strictly proper. Consider an ARI given by

$$A^{\mathrm{T}}X + XA + \gamma^{-2}XBB^{\mathrm{T}}X + C^{\mathrm{T}}C < 0 \tag{14}$$

with the Hamiltonian matrix

$$H = \begin{bmatrix} A & \gamma^{-2}BB^{\mathrm{T}} \\ -C^{\mathrm{T}}C & -A^{\mathrm{T}} \end{bmatrix}. \tag{15}$$

Define the matrix function AR by

$$\begin{aligned} \mathrm{AR}(H,X) &= [X \ \ -I]H\begin{bmatrix} I \\ X \end{bmatrix} \\ &= A^{\mathrm{T}}X + XA + \gamma^{-2}XBB^{\mathrm{T}}X + C^{\mathrm{T}}C. \end{aligned} \tag{16}$$

Consider the ARI with perturbations in matrices A and B.

$$\begin{gathered} (A + \Delta A)^{\mathrm{T}}X + X(A + \Delta A) + \gamma^{-2}X(B + \Delta B)(B + \Delta B)^{\mathrm{T}}X + C^{\mathrm{T}}C < 0, \\ [\Delta A \ \Delta B] = E\Sigma[F_a \quad F_b]. \end{gathered} \tag{17}$$

For a given $\lambda > 0$, define matrix by

$$H_\lambda = \begin{bmatrix} A_\lambda & \gamma^{-2}B_\lambda B_\lambda^{\mathrm{T}} \\ -C_\lambda^{\mathrm{T}}C_\lambda & -A_\lambda^{\mathrm{T}} \end{bmatrix}, \tag{18}$$

where

$$A_\lambda = A + (\lambda\gamma)^{-2}BF_b^{\mathrm{T}}F_a, \tag{19}$$

$$B_\lambda = [BR \ \lambda\gamma E \ E], \tag{20}$$

$$C_\lambda = \begin{bmatrix} C \\ \lambda^{-1}F_a \end{bmatrix}, \tag{21}$$

$$R^2 = I + (\lambda\gamma)^{-2}F_b^{\mathrm{T}}F_b. \tag{22}$$

Lemma 3.1 Assume that $F_bF_b^{\mathrm{T}} = I$. Let $X > 0$ be a solution of $\mathrm{AR}(H,X) < 0$. Then, X is also a solution of ARI (17) if and only if there exists an $\lambda > 0$ such that

$$\mathrm{AR}(H_\lambda, X) < 0. \tag{23}$$

Proof Necessity. Assume that (17) holds for any $\Sigma \in \Omega$. (17) can be rewritten as follows.

$$\begin{aligned} &\mathrm{AR}(H,X) + XE\Sigma(F_a + \gamma^{-2}F_bB^{\mathrm{T}}X) + (F_a + \gamma^{-2}F_bB^{\mathrm{T}}X)^{\mathrm{T}}\Sigma^{\mathrm{T}}E^{\mathrm{T}}X \\ &\quad < -\gamma^{-2}XE\Sigma\Sigma^{\mathrm{T}}E^{\mathrm{T}}X. \end{aligned} \tag{24}$$

So, for any $\xi \neq 0$, we have

$$\xi^{\mathrm{T}}\mathrm{AR}(H,X)\xi + 2\xi^{\mathrm{T}}XE\Sigma(F_a + \gamma^{-2}F_bB^{\mathrm{T}}X)\xi < -\gamma^{-2}\xi^{\mathrm{T}}XE\Sigma\Sigma^{\mathrm{T}}E^{\mathrm{T}}X\xi, \quad \forall\ \Sigma \in \Omega. \tag{25}$$

From Lemma 3.1 in [11], there exist a $\Sigma(\xi) \in \Omega$ such that

$$\max_{\Sigma \in \Omega} |\xi^{\mathrm{T}}XE\Sigma(F_a + \gamma^{-2}F_bB^{\mathrm{T}}X)\xi| = \xi^{\mathrm{T}}XE\Sigma(\xi)(F_a + \gamma^{-2}F_bB^{\mathrm{T}}X)\xi \tag{26}$$

and

$$\Sigma^{\mathrm{T}}(\xi)\Sigma(\xi) = I.$$

Thus, from (25)

$$\begin{aligned} &\xi^{\mathrm{T}}\mathrm{AR}(H,X)\xi + 2\xi^{\mathrm{T}}XE\Sigma(F_a + \gamma^{-2}F_bB^{\mathrm{T}}X)\xi \\ &\leqslant \xi^{\mathrm{T}}\mathrm{AR}(H,X)\xi + 2\xi^{\mathrm{T}}XE\Sigma(\xi)(F_a + \gamma^{-2}F_bB^{\mathrm{T}}X)\xi \\ &< -\gamma^{-2}\xi^{\mathrm{T}}XEE^{\mathrm{T}}X\xi. \end{aligned} \tag{27}$$

Hence, we have

$$\xi^{\mathrm{T}}[\mathrm{AR}(H,X) + \gamma^{-2}XEE^{\mathrm{T}}X]\xi < -2\xi^{\mathrm{T}}XE\Sigma(F_a + \gamma^{-2}F_bB^{\mathrm{T}}X)\xi, \quad \forall\ \Sigma \in \Omega. \tag{28}$$

So that, using a technique similar to that used in the proof of Theorem 3. 3 in [11], it follows that there exist an $\varepsilon>0$ such that

$$\begin{aligned}&\varepsilon^2 XEE^{\mathrm{T}}X+\varepsilon[\mathrm{AR}(H,X)+\gamma^{-2}XEE^{\mathrm{T}}X]\\&\quad+(F_a+\gamma^{-2}F_bB^{\mathrm{T}}X)^{\mathrm{T}}(F_a+\gamma^{-2}F_bB^{\mathrm{T}}X)<0.\end{aligned}\tag{29}$$

Therefore, (23) follows from (29) with $\lambda=\sqrt{\varepsilon}$.

Sufficiency. The proof follows immediately by considering

$$\begin{aligned}\mathrm{AR}(H_\lambda,X)&=\mathrm{AR}(H,X)+\gamma^{-2}XEE^{\mathrm{T}}X+\lambda^2XEE^{\mathrm{T}}X\\&\quad\lambda^{-2}(F_a+\gamma^{-2}F_bB^{\mathrm{T}}X)^{\mathrm{T}}(F_a+\gamma^{-2}F_bB^{\mathrm{T}}X)\\&=\mathrm{AR}(H,X)+\gamma^{-2}XEE^{\mathrm{T}}X+XE\Sigma(F_a+\gamma^{-2}F_bB^{\mathrm{T}}X)\\&\quad+(F_a+\gamma^{-2}F_bB^{\mathrm{T}}X)^{\mathrm{T}}\Sigma^{\mathrm{T}}E^{\mathrm{T}}X+M(\Sigma),\end{aligned}\tag{30}$$

where

$$M(\Sigma)=[XE\Sigma-(F_a+\gamma^{-2}F_bB^{\mathrm{T}}X)^{\mathrm{T}}][XE\Sigma-(F_a+\gamma^{-2}F_bB^{\mathrm{T}}X)^{\mathrm{T}}]^{\mathrm{T}}\geqslant 0.\quad \text{Q. E. D.}$$

Let $K(s)$ be a controller described by

$$\dot{\eta}=A_c\eta+B_cy,\tag{31}$$

$$u=C_c\eta.\tag{32}$$

For given $\lambda>0$ and $\gamma>0$, define $k=\gamma\lambda$ and $R_1^2=I+k^{-2}F_1^{\mathrm{T}}F_1$.

Theorem 3. 1 Assume that $F_1F_1^{\mathrm{T}}=I$. $K(s)$ given by (31), (32) is an H_∞ robust sub-optimal controller for the plant $G(s,\Sigma)$ if and only if there exit a $\lambda>0$ and a positive definite matrix X such that

$$\mathrm{AR}(\hat{H}_\lambda,X)<0,\tag{33}$$

where the Hamiltonian matrix $\hat{H}_\lambda$ is given by

$$\hat{H}_\lambda=\begin{bmatrix}\hat{A}_\lambda & \gamma^{-2}\hat{B}_\lambda\hat{B}_\lambda^{\mathrm{T}}\\ -\hat{C}_\lambda^{\mathrm{T}}\hat{C}_\lambda & -\hat{A}_\lambda^{\mathrm{T}}\end{bmatrix},$$

$$\hat{A}_\lambda=\begin{bmatrix}A & 0\\ 0 & A_c\end{bmatrix}+\begin{bmatrix}I & 0\\ 0 & B_c\end{bmatrix}\begin{bmatrix}k^{-2}B_1F_1^{\mathrm{T}}F_a & B_2+k^{-2}B_1F_1^{\mathrm{T}}F_2\\ C_2+k^{-2}D_{21}F_1^{\mathrm{T}}F_a & D_{22}+k^{-2}D_{21}F_1^{\mathrm{T}}F_2\end{bmatrix}\begin{bmatrix}I & 0\\ 0 & C_c\end{bmatrix},$$

$$\hat{B}_\lambda=\begin{bmatrix}I & 0\\ 0 & B_c\end{bmatrix}\begin{bmatrix}B_1R_1 & kE_1 & E_1\\ D_{21}R_1 & kE_2 & E_2\end{bmatrix},$$

$$\hat{C}_\lambda=\begin{bmatrix}C_1 & D_{12}\\ \lambda^{-1}F_a & \lambda^{-1}F_2\end{bmatrix}\begin{bmatrix}I & 0\\ 0 & C_c\end{bmatrix}.$$

Proof Let a state space description of the plant $G(s,\Sigma)$ be given by

$$\dot{x}=(A+\Delta A)x+(B_1+\Delta B_1)w+(B_2+\Delta B_2)u,\tag{34}$$

$$z=C_1x+D_{12}u,\tag{35}$$

$$y=(C_2+\Delta C)x+(D_{21}+\Delta D_1)w+(D_{22}+\Delta D_2)u.\tag{36}$$

Then, the closed loop system with the controller $K(s)$ is given by a state space equation with the state vector $x_c=[x^{\mathrm{T}}\quad \eta^{\mathrm{T}}]^{\mathrm{T}}$

$$\dot{x}_c=(\bar{A}+\Delta\bar{A})x_c+(\bar{B}+\Delta\bar{B})w,\tag{37}$$

$$z=\bar{C}x_c,\tag{38}$$

$$[\Delta\bar{A}\quad \Delta\bar{B}]=\bar{E}\Sigma[\bar{F}_a\quad \bar{F}_b]\tag{39}$$

where

$$\overline{A}=\begin{bmatrix}A & B_2C_2\\ B_cC_2 & A_c+B_cD_{22}C_c\end{bmatrix},\quad \overline{B}=\begin{bmatrix}B_1\\ B_cD_{21}\end{bmatrix},$$

$$\overline{C}=[C_1\quad D_{12}C_c],$$

$$\overline{E}=\begin{bmatrix}E_1\\ B_cE_2\end{bmatrix},\quad \overline{F}_a=[F_a\quad F_2C_c],\quad \overline{F}_b=F_1.$$

Hence, the desired results follows from Lemma 3. 1 with the following definitions.

$$\hat{A}_\lambda=\overline{A}+k^{-2}\overline{B}\overline{F}_b^{\mathrm{T}}\overline{F}_a, \tag{40}$$

$$\hat{B}_\lambda=[\overline{B}R\quad k\overline{E}\quad \overline{E}], \tag{41}$$

$$\hat{C}_\lambda=\begin{bmatrix}\overline{C}\\ \lambda^{-1}\overline{F}_a\end{bmatrix}, \tag{42}$$

$$\hat{R}^2=I+k^{-2}\overline{F}_b^{\mathrm{T}}\overline{F}_b. \tag{43}$$

In this theorem a necessary and sufficient condition is given in terms of ARI for $K(s)$ being an H_∞ robust sub-optimal controller. Next theorem gives an equivlent condition with which the standard H_∞ controller design method is available to obtain the controller.

Theorem 3. 2 Given $\lambda>0$. There exists a positive definite matrix X satisfying $\mathrm{AR}(\hat{H}_\lambda, X)>0$ if and only if $K(s)$ satisfies

$$\|\mathrm{LFT}(P_\lambda,K)\|_\infty<\gamma, \tag{44}$$

where

$$P_\lambda=\left[\begin{array}{c:cc}A+k^{-2}B_1F_1^{\mathrm{T}}F_a & [B_1R_1\quad kE_1\quad E_1] & B_2+k^{-2}B_1F_1^{\mathrm{T}}F_2\\ \hdashline \begin{bmatrix}C_1\\ \lambda^{-1}F_2\end{bmatrix} & 0 & \begin{bmatrix}D_{12}\\ \lambda^{-1}F_a\end{bmatrix}\\ C_2+k^{-2}D_{21}F_1^{\mathrm{T}}F_a & [D_{21}R_1\quad kE_2\quad E_2] & D_{22}+k^{-2}D_{21}F_1^{\mathrm{T}}F_2\end{array}\right]. \tag{45}$$

Proof Note that a state space realization of $\mathrm{LFT}(P_\lambda,K)$ is given by

$$\mathrm{LFT}(P_\lambda,K)=\left[\begin{array}{c:c}\hat{A}_\lambda & \hat{B}_\lambda\\ \hdashline \hat{C}_\lambda & 0\end{array}\right], \tag{46}$$

where $\hat{A}_\lambda$, $\hat{B}_\lambda$ and $\hat{C}_\lambda$ are given (40)～(43). Thus, the theorem follows immediately from Lemma 2. 2 in Zhou et al. [12]. Q. E. D.

Corollary There exists a strictly proper H_∞ robust sub-optimal controller $K(s)$ for the plant $G(s,\Sigma)$ if and only if there exist a $\lambda>0$ such that $K(s)$ is an H_∞ sub-optimal controller for the false plant P_λ given by (45) with a scaling parameter λ.

4 Obtaining a Solution to Robust Stabilization Problem

In this section we discuss the RSP defined in section 2 for plant (1) with uncertainties. The robust stabilizing controller obtained here is again of strictly proper. To obtain the controller we assume that

A1) (A,B) is stabilizable.

A2) $F_2F_2^{\mathrm{T}}=I$.

In view of Lemma 2. 3, it is clear that an H_∞ robust sub-optimal controller for the generalized plant $\overline{G}(s,\Sigma)$ gives a desired controller for the RSP.

$$\overline{G}(s,\Sigma) = \begin{bmatrix} 0 & I \\ P(s,\Sigma) & -P(s,\Sigma) \end{bmatrix} = \left[\begin{array}{c:cc} A+\Delta A & B+\Delta B_1 & -B+\Delta B_2 \\ \hdashline 0 & 0 & I \\ C+\Delta C & D+\Delta D_1 & -D+\Delta D_2 \end{array}\right], \tag{47}$$

where

$$\begin{bmatrix} \Delta A & \Delta B_1 & \Delta B_2 \\ \Delta C & \Delta D_1 & \Delta D_2 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \Sigma [F_1 \quad F_2 \quad -F_2], \quad \Sigma \in \Omega.$$

From Theorems 3. 1 and 3. 2 a desired controller is obtained by finding an H$_\infty$ sub-optimal controller $K(s)$ for the false plant P_λ with λ

$$P_\lambda = \left[\begin{array}{c:cc} A+k^{-2}BF_2^{\mathrm{T}}F_1 & [BR_1 \quad kE_1 \quad E_1] & -BR_2^2 \\ \hdashline \begin{bmatrix} 0 \\ \lambda^{-1}F_1 \end{bmatrix} & 0 & \begin{bmatrix} I \\ -\lambda^{-1}F_2 \end{bmatrix} \\ C+k^{-2}DF_2^{\mathrm{T}}F_1 & [DR_1 \quad kE_2 \quad E_2] & -DR_2^2 \end{array}\right], \tag{48}$$

where $R_1^2 = I + k^{-2}F_1^{\mathrm{T}}F_1$, $R_2^2 = I + k^{-2}F_2^{\mathrm{T}}F_2$ and $k = \lambda\gamma$.

Given matrices F and K, define Hamiltonian matrices $H_\lambda(F)$ and $H_\lambda(K)$ by

$$H_\lambda(F) = \begin{bmatrix} \widetilde{A} & BR_1^2B^{\mathrm{T}} + (1+k^2)E_1E_1^{\mathrm{T}} \\ -\widetilde{C}^{\mathrm{T}}\widetilde{C} & -\widetilde{A}^{\mathrm{T}} \end{bmatrix}, \tag{49}$$

$$H_\lambda(K) = \begin{bmatrix} \overline{A} & \overline{B}\overline{B}^{\mathrm{T}} \\ -k^{-2}F_1^{\mathrm{T}}F_1 & -\overline{A}^{\mathrm{T}} \end{bmatrix}, \tag{50}$$

where

$$\widetilde{A} = A + B(k^{-2}F_2^{\mathrm{T}}F_1 - R_2^2)F,$$
$$\widetilde{C}^{\mathrm{T}} = [F^{\mathrm{T}} \quad \lambda^{-1}(F_1 - F_2F)^{\mathrm{T}}],$$
$$\overline{A} = A + KC + k^{-2}(B + KD)F_2^{\mathrm{T}}F_1,$$
$$\overline{B} = [(B+KD)R_1 \quad k(E_1 + KE_2) \quad E_1 + KE_2].$$

Theorem 4. 1 Assume that $(A+k^{-2}BF_2^{\mathrm{T}}F_1,\ C+k^{-2}DF_2^{\mathrm{T}}F_1)$ is detectable for a given $\lambda > 0$. There exists a strictly proper H$_\infty$ sub-optimal controller for the plant P_λ if and only if there exist $X > 0$ and $Y > 0$ which satisfy the following conditions:

a) There exists F such that

$$\mathrm{AR}(H_\lambda(F), X) < 0. \tag{51}$$

b) There exists K such that

$$\mathrm{AR}(H_\lambda(K), Y) < 0. \tag{52}$$

c) $N = \gamma^2 Y - X > 0$.

Proof Let the controller $K(s)$ has the following state space realization.

$$K(s) = \begin{bmatrix} A_c & B_c \\ C_c & 0 \end{bmatrix}. \tag{53}$$

It is easy to show that for given $\gamma > 0$ and $\lambda > 0$, there exists $K(s)$ such that $\|\mathrm{LFT}(P_\lambda, K)\|_\infty <$

γ if and only if a strictly proper $\hat{K}(s)$ satisfies $\|\mathrm{LFT}(\hat{P}_\lambda, \hat{K})\|_\infty < \gamma$, where

$$\hat{K}(s) = \begin{bmatrix} A_c + B_c D R_2^2 C_c & B_c \\ C_c & 0 \end{bmatrix}, \tag{54}$$

$$\hat{P}_\lambda = \left[\begin{array}{c:ccc} A + k^{-2} B F_2^{\mathrm{T}} F_1 & [BR_1 \quad kE_1 \quad E_1] & & -BR_2^2 \\ \hdashline \begin{bmatrix} 0 \\ \lambda^{-1} F_1 \end{bmatrix} & 0 & & \begin{bmatrix} I \\ -\lambda^{-1} F_2 \end{bmatrix} \\ C + k^{-2} D F_2^{\mathrm{T}} F_1 & [DR_1 \quad kE_2 \quad E_2] & & 0 \end{array}\right]. \tag{55}$$

Since (A,B) is stabilizable, $(A + k^{-2} B F_2^{\mathrm{T}} F_1, BR_2^2)$ is stabilizable for any $\lambda > 0$. Hence, the theorem follows from the result given in Sampei et al. [3], with the detectablity assumption. Q. E. D.

A check of detectablity of $(A + k^{-2} B F_2^{\mathrm{T}} F_1, C + k^{-2} D F_2^{\mathrm{T}} F_1)$ is necessary in above theorem when a standard H_∞ theory is applied to our problem. If the nominal plant P_0 has no zeros in the right half plane this check is not necessary, since $(A + k^{-2} B F_2^{\mathrm{T}} F_1,\ C + k^{-2} D F_2^{\mathrm{T}} F_1)$ is detectable for any λ.

The following theorem gives a solution to find a robust stabilizing controller using a standard H_∞ design method.

Theorem 4. 2 Consider a strictly proper controller $K(s)$ and the plant given by (1). If there exists a $\lambda > 0$ such that the conditions in Theorem 4. 1 hold, then a robust stabilizing controller is given by

$$K(s) = \begin{bmatrix} A_c & B_c \\ C_c & 0 \end{bmatrix}, \tag{56}$$

$$C_c = F,$$

$$B_c = -\gamma^2 N^{-1} Y K,$$

$$A_c = A - B_c (C + D R_2^2 C_c + k^{-2} D F_2^{\mathrm{T}} F_1) - B R_2^2 C_c - N^{-1} M,$$

where

$$\begin{aligned} M = & - C_c^{\mathrm{T}} R_2^2 B^{\mathrm{T}} X + C_c^{\mathrm{T}} (R_2^2 C_c - \lambda^{-2} F_2^{\mathrm{T}} F_1) + H \\ & + \gamma^{-2} N \{ (B_c D - B) R_1^2 B^{\mathrm{T}} + k^2 (B_c E_2 - E_1) E_1^{\mathrm{T}} + (B_c E_2 - E_1) E_1^{\mathrm{T}} \} X, \\ H = & - \mathrm{AR}(H_\lambda(F), X) > 0. \end{aligned}$$

Proof The proof follows immediately by Lemma 2. 3, Theorem 4. 1 and the Corollary 1 in Sampei et al. [3] Q. E. D.

5 Conclusion

In this paper we have developed an equivalence of robust performance design problem with perturbation in the disturbance input matrix and a standard H_∞ sub-optomal problem. The equivalence is used to solve robust stabilization problem for the plant with both structural and unstructural uncertainties.

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一种不确定系统的鲁棒稳定控制

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摘要：本文讨论了同时具有结构性和非结构性不确定性的系统的鲁棒稳定控制问题. 首先考虑输入和输出阵同时存在参数摄动系统的 H_∞ 鲁棒性能准则设计问题，本文证明了此问题等价于对于适当广义对象的 H_∞ 标准设计问题. 基于这一结果，利用 H_∞ 设计方法给出了鲁棒稳定控制问题的一个解.

关键词：鲁棒稳定；不确定系统；H_∞ 控制

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