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Deadlock-Free Modular State Feedback Control of Discrete Event Systems

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Abstract: In this paper we discuss the problem of synthesizing deadlock-free modular state feedback controllers for discrete event systems. By introducing the D-invariant relation between automata pairs, we show that for the case the control objective is expressed in terms of the intersection of two predicates, a necessary and sufficient condition for the modular state feedback controller to be deadlock-free is that the component subcontrollers are all deadlock-free and the corresponding pair of automaton meets a D-invariant relation. A design procedure for deadlock-free modular state feedback controller is also presented.

Key words: discrete event systems; automata; modular state feedback control; deadlock

1 Introduction

Modular supervisory control of Discrete Event Systems (DES) was studied first by Ramadge and Wonham[1], and developed later in [2,3]. It is an efficient way to overcome computational complexity in control of DES. By "Modularity" it means that the desired system behavior, i. e., the specification, is given in terms of a set of independent subspecifications. If one synthesizes the component subcontrollers independently and then merges them in the form of controller conjunction or disjunction, one gets the so called modular controller. This kind of synthesis offers us the merits of lower computational and hardware requirements, as well as the convenience of controller maintenance and redesign[3].

The main problem related to modular supervisory control is, when the component subcontrollers have some desired properties, how to guarantee that the modular controller behaves in the same way. In [1,3] the problem of nonblocking modular control are discussed, and in [5] the computational problems that arise in nonblocking modular control and so called SCPB[4] are discussed.

Another problem related to modular synthesis is deadlock. This happens in the case that the modular controller is formed by subcontroller conjunction. This fact was first observed by Professor Wonham as reported by Y. Li in [6] and a special solution to deadlock-free modular supervisory control of vector DESs was given. However, no further result has been obtained later on. To our best knowledge our study is the first systematic attempt to solve this problem for general DESs.

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2 Preliminaries and Problem Formulation

2.1 The Model

An automaton G is a five tuple $G=(\Sigma,Q,\delta,q_0,Q_m)$, where Σ is the event set, Q is the state set, δ is the transition function, q_0 is the initial state, and Q_m is the set of marking states.

Assign $\Sigma_c\subset\Sigma$ be the set of controllable events. Here controllability of an event means that it can be prevented from occuring by an outside control agency. Let Σ_u be the set of uncontrollable events. Define $\gamma:\Sigma_u\subset\gamma\subset\Sigma$ be a control pattern. If there is $\sigma\in\Sigma$ such that $\gamma(\sigma)=1$, then σ is permitted to occur by γ, otherwise σ is prevented from occuring by γ. Let $\Gamma=\{\gamma:\Sigma\rightarrow\{0,1\}\wedge\forall\sigma\in\Sigma_u,\gamma(\sigma)=1\}$ be the set of control patterns. The control of DESs is exercised by switching the control patterns according to the observed history of the controlled system.

The system G coupled with the structure of Γ is called a Controlled DES, and it is formally defined as:

$$G_c=(\Gamma\times\Sigma,Q,\delta_c,q_0)$$

where

$$\delta_c(\gamma,\sigma,q)=\begin{cases}\delta(\sigma,q), & \text{if } \delta(\sigma,q) \text{ is defined and } \gamma(\sigma)=1,\\ \text{undefined}, & \text{otherwise.}\end{cases}$$

A control $f\in\Gamma^Q$ is called a state feedback. The controlled DES G_c controlled by f is the system

$$G^f=(\Sigma,Q,\delta^f,q_0)$$

where

$$\delta^f(\sigma,q)=\begin{cases}\delta(\sigma,q), & \text{if } \delta(\sigma,q) \text{ is defined and } f_\sigma(q)=1,\\ \text{undefined}, & \text{otherwise.}\end{cases}$$

2.2 Predicates and Control Invariance

Let $P:2^Q\rightarrow\{0,1\}$ be a function defined on the nonempty state set Q. It is also called a predicate characterizing Q. Let $\mathbf{P}$ be the set of all predicates defined on Q. $\forall P_1,P_2\in\mathbf{P}$, define operations $(\sim,\wedge,\vee)$ as follows:

$$(\sim P)(q)=1 \text{ if and only if } P(q)=0,$$
$$(P_1\wedge P_2)(q)=1 \text{ if and only if } P_1(q)=P_2(q)=1,$$
$$P_1\vee P_2=\sim((\sim P_1)\wedge(\sim P_2)).$$

It has been proved[2] that $(\mathbf{P};\sim,\wedge,\vee)$ is a Boolean Algebra.

There are cases the control objectives are expressed in terms of predicates, i.e. to guarantee all the reachable states of the controlled system meet certain requirements as asserted by the given predicates. Since $(\mathbf{P};\sim,\wedge,\vee)$ is a boolean algebra, the aforementioned control problem is identical to an invariance problem in conventional control theory, i.e., to keep the state trajectories of the controlled system remain at a state subset $Q'\in 2^Q$.

The following predicate transformations are used to describe properties of the given predicates[2].

$$\mathrm{wlp}_\sigma(P)(q) = \begin{cases} 1 & \text{if } \delta(\sigma,q) \text{ is defined and } P(\delta(\sigma,q)) = 1 \text{ or } \delta(\sigma,q) \text{ is undefined}, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathrm{sp}_\sigma(P)(q) = \begin{cases} 1 & \text{if } \exists\, q' \in P,\ \delta(\sigma,q') \text{ is defined and } q = \delta(\sigma,q'), \\ 0, & \text{otherwise.} \end{cases}$$

Definition 1[2] P is called control-invariant with respect to (wrt) G, if for some state feedback $f \in \Gamma^Q$, there is

$$P \leqslant \mathrm{wlp}_\sigma^f(P) \quad \forall\, \sigma \in \Sigma$$

where

$$\mathrm{wlp}_\sigma^f(P) = \mathrm{wlp}_\sigma \vee \sim f^\sigma.$$

If P is control-invariant and $P(q_0) = 1$, then it is shown [2] that at all the reachable states of the closed loop system, P will keep true. This fact is equivalently characterized by a control-independent property of P.

Definition 2[2] P is called Σ_u-invariant wrt G if

$$P \leqslant \mathrm{wlp}_\sigma(P) \quad \forall\, \sigma \in \Sigma_u.$$

Proposition 1[2] P is control-invariant if and only if it is Σ_u-invariant.

2.3 Basic Operations

Definition 3 $G_1 \subset G$ is called realizable if G_1 is formed by only deleting controllable arcs from the transition diagram of G.

Definition 4. Given $G = (\Sigma, Q, \delta, q_0)$ and $G_i = (\Sigma, Q_i, \delta_i, q_{0i})$ $(i=1,2)$, where $Q_i \subset Q$, $\delta_i = \delta | Q_i$ (the restriction of δ on Q_i) and $q_{0i} = q_0$, then

$$G_1 \cap G_2 = (\Sigma, Q_1 \cap Q_2, \delta_1 \wedge \delta_2, q_0),$$

$$G_1 \cup G_2 = (\Sigma, Q_1 \cup Q_2, \delta_1 \vee \delta_2, q_0).$$

The above operations are called automata conjunction and disjunction respectively.

Definition 5 A controlled DES $G^f \subset G$ is called deadlock-free if one of the following conditions hold:

1) $\forall\, q \in Q^f$ (the reachable state set of G^f), $\exists\, \sigma \in \Sigma_c$ such that $\delta^f(\sigma, q)$ is defined or

2) $\forall\, q \in Q^f (\subset Q)$, $\exists\, \sigma \in \Sigma_u$ such that $\delta(\sigma, q)$ is defined.

2.4 The Problem

Example 1 Let $G = (\Sigma, Q, \delta, q_0, Q_m)$, $Q_m = Q$ be the controlled plant, $L(G) = (\sigma_1\sigma_2((\beta_1\beta_2)^* + (\beta_3\beta_4)^*)$. Assume that β_1 and β_3 are controllable and that $L(S_1) = \sigma_1\sigma_2(\beta_1\beta_2)^*$ and $L(S_2) = \sigma_1\sigma_2(\beta_3\beta_4)^*$. It is easily checked that S_1 and S_2 are deadlock-free but $S_1 \wedge S_2$ is deadlocked.

This simple example demonstrates the following facts. First, since S_1 and S_2 are all closed, S_1 and S_2 are nonconflicting. So nonconflicting cannot guarantee deadlock-freeness. Secondly, the deadlock-freeness of component subcontrollers can not guarantee the deadlock-freeness of the modular controller.

Before we proceed any further, we state the following proposition on the relation between nonblocking and deadlock-freeness in supervisory control of DES. The proof is direct.

Proposition 2 A nonblocking DES $G=(\Sigma,Q,\delta,q_0,Q_m)$ is deadlock-free if and only if $\forall q \in Q_m$, $\Sigma(q)\neq\emptyset$. Here $\Sigma(q)$ is the active event set of G at state q, i. e. , $\Sigma(q)=\{\sigma\in\Sigma,\delta(\sigma,q)$ is defined$\}$.

3 Deadlock-Free Modular State Feedbacks

3.1 D-Invariance

Definition 6 Given G and $G_i\subset G(i=1,2)$. (G_1, G_2) is called D-invariant if $\forall q\in Q_1\cap Q_2$, $\Sigma_{G_1}(q)\cap\Sigma_{G_2}(q)\neq\emptyset$.

Notice that D-invariance is stronger than that of deadlock-freeness (see Definition 5). Further we note that the D-invariant relation is reflexive, symmetric, but generally not transitive.

The following theorem is the first result of this paper. It states a necessary and sufficient condition for the deadlock-freeness of modular state feedback controllers.

Theorem 1 Given G and $G_i(i=1,2)$, $G_i(i=1,2)$ is a realizable subautomaton of G, then $G_1\cap G_2$ is deadlock-free if and only if (G_1, G_2) is D-invariant.

Proof Sufficiency: Since (G_1, G_2) is D-invariant, $\forall q\in Q_1\cap Q_2$, $\Sigma_{G_1}(q)\cap\Sigma_{G_2}(q)\neq\emptyset$. So there is a $v\in\Sigma_{G_1\cap G_2}(q)$ such that $\delta_{1\wedge 2}=\delta_1\wedge\delta_2$ is well defined at (v,q). From Definition 5, $G_1\cap G_2$ is deadlock-free.

Necessity: Suppose that $G_1\cap G_2$ is deadlock-free but (G_1, G_2) is not D-invariant. So there is $q\in Q_1\cap Q_2$ s. t. $\Sigma_{G_1}(q)\cap\Sigma_{G_2}(q)\neq\emptyset$. Subsequently there is no $\sigma\in\Sigma_c$ such that $\delta_{1\wedge 2}$ is defined. Since $G_1\cap G_2$ is deadlock-free, there exists a $\sigma\in\Sigma_u$ such that $\delta(\sigma,q)$ is defined. By the assumption that G_1, G_2 are all realizable, it follows that $\delta_1(\sigma,q)$ and $\delta_2(\sigma,q)$ are all defined. This leads to the fact that $\delta_{1\wedge 2}(\sigma,q)$ is defined, a contradiction to $\Sigma_{G_1}\cap\Sigma_{G_2}(q)\neq\emptyset$.

Remark 1 The D-invariance relation is defined on pairs of given automata rather than on their corresponding state sets. This is because a control-invariant state subset corresponds to a number of realizable subautomata of the given automaton. However we can show that there is a maximal realizable subautomaton that corresponds to the control-invariant state subset.

Remark 2 With graph-theoretic terms we know that the transition diagram of a realizable subautomaton of a given automaton is exactly a subgraph of the transition diagram of the given automaton by deleting some controllable arcs.

It should be emphasized that without the realizability assumption of $G_i(i=1,2)$, Theorem 1 is not necessary in general. In fact, for the case of arbitrary $G_i(i=1,2)$, we have

Corollary 1 A sufficient condition for $G_1\cap G_2$ to be deadlock-free is that (G_1,G_2) is D-invariant.

We point out here that for the sake of computational efficiency and ease of modular synthesis, Theorem 1 is generally used.

Corresponding to Theorem 1, we give a definition on deadlock-free conjunction of component subcontrollers.

Definition 7 Given two component state feedback subcontrollers f_1, f_2, for G which realize the subautomata G_1 and G_2 respectively are called D-invariant if $\forall q\in Q_1\cap Q_2$, $\Sigma_{G_1}\cap\Sigma_{G_2}(q)$

$=\varnothing$.

With difinition 7, we have

Theorem 2 Given complete and deadlock-free subcontrollers f_1, f_2, their conjunction $f=f_1\wedge f_2$ is complete and deadlock-free if and only if (G_1,G_2) is D-invariant.

4 D-Invariant Subautomaton of A Given Automaton

In this section we will show that for a given G and $G_i(i=1,2)\subset G$, if we fix G_1, then there is $G_2^d\subset G_2$, called the maximal D-invariant subautomaton of G_2 wrt G_1.

Let $D_{G_1}(G_2)=\{G_2^{(i)}\mid G_2^{(i)}\subset G_2,\ (G_1,\ G_2^{(i)})$ is D-invariant$\}$ then there is

Proposition 3 $D_{G_1}^{G_2}$ is closed under disjunction operation.

The proof of this proposition is direct. The implication of it is that $D_{G_1}(G_2)$ contains a maximal element. Denote this maximal element as $\sup D_{G_1}(G_2)$.

Combine the result in [7] and Proposition 3, we reach the following conclusion:

Proposition 4 Let $CD_{G_1}(G_2)=\{G_2^{(i)}\mid G_2^{(i)}\subset G_2,\ G_2^{(i)}$ is realizable, and $(G_2^{(i)},\ G_1)$ is D-invariant$\}$, then $CD_{G_1}(G_2)$ is closed under disjunction operation.

Similarly we denote the maximal element in $CD_{G_1}(G_2)$ as $\sup CD_{G_1}(G_2)$.

5 Computational Procedures

5.1 Synthesizing Deadlock-Free State Feedback Controller

The following proposition shows that for a given G, the class of realizable and deadlock-free subautomata of a given automaton contains a maximal element.

Porposition 5 Let $G_{RD}=\{G^{(i)}\mid G^{(i)}\subset G,\ G^{(i)}$ is realizable and deadlock-free$\}$, then G_{RD} is closed under disjunction operation.

Based on Proposition 4 and 5, the following algorithm is proposed.

Algorithm RD

Step 1 Construct $G_R^{(i)}\subset G$ such that $G_R^{(i)}$ is realizable;

Step 2 For all states q in the state set of $G_R^{(i)}$, if $\Sigma_{G_R^{(i)}}(q)=\emptyset$ then $q\in Q_{bad}$, otherwise $q\in Q_{good}$;

Step 3 If $Q_{bad}\neq\emptyset$, then perform a forbidden state control synthesis by letting $Q_{bad}=Q_f$[7]; Otherwise go to Step 5; Let the resultant subautomaton be $G_{RD}^{(i)}$;

Step 4 Let $i=i+1$, go to Step 1;

Step 5 Stop.

5.2 Design Procedure DMSFC

There are cases that control objectives are expressed in terms of predicate conjunctions. In this paper we only consider the situation that $P=P_1\wedge P_2$. For the case that the conjunction is composed of more than two predicates, the solution is much more difficult.

For $P=P_1\wedge P_2$, it is equivalent to construct state feedback controller f such that $G^f=G^{f_1}\cap G^{f_2}$ if it ever exists. The following algorithm, called DMSFC (Deadlock-Free Modular State Feedback Controller), are used to accomplish the above mentioned synthesis problem.

Algorithm DMSFC

Step 1 Construct G_{10}, G_{20} such that G_{10} and G_{20} are all realizable and deadlock-free;

Step 2 Compute $\sup CD_{G_{10}}(G_{20})$;

Step 3 Construct state feedbacks f_1 and f_2 such that $G^{f_1}=G_{10}$, $G^{f_2}=\sup CD_{G_{10}}(G_{20})$;

Step 4 Let $f=f_1 \wedge f_2$.

It is easy to show that this algorithm will converge if G is of finite state.

5.3 Computing $\sup CD_{G_{10}}(G_{20})$

In algorithm DMSFC we left $\sup CD_{G_{10}}(G_{20})$ untreated. The following is an algorithm to compute it.

Let $G_0=G_{10} \wedge G_{20}$ then G_0 is realizable but not necessarily deadlock-free. We construct $G_{0m} \subseteq G_0$ such that G_{0m} is the maximal realizable and deadlock-free subautomaton of G_0. We expand G_{0m} in such a manner:

1) Compute G_{20}-G_0, the subtraction of G_0 from G_{20};

2) Add all the states and transitions of G_{20}-G_0 to G_{0m} if they are reachable from any state in G_{0m}. Let the resultant subautomaton be G_{2m}, then we have

Proposition 6 $G_{2m}=\sup CD_{G_0}(G_{20})$.

6 Conclusion

Modular supervisory control is an important synthesis method in DES control since it offers the advantages of computational efficiency and the easiness of controller maintenance as well as convenience of task changes. However, as regarding to the global properties of modular supervisory control such as deadlock-freeness, the problem is far from satisfactorily solved.

The main motivation of this paper is to consider deadlock problem in modular state feedback control of DESs. We have shown that our approach keeps certain degree of modularity while maintaining the deadlock-freeness of the global system. However, if the involved tasks are more than 2, then the question is still unanswered.

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离散事件系统的无死锁模块化状态反馈控制

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摘要:本文讨论离散事件系统的无死锁模块化状态反馈问题.首先我们定义自动机的交与并运算,然后通过引入自动机对的D-不变关系,我们证明当控制目标是两个谓词的交时,模块化状态反馈控制器是无死锁的充要条件是各子控制器是无死锁的且相应的控制器满足D-不变关系.我们证明了一个给定的自动机相对于另一自动机的D-不变子自动机类有最大元存在,并由此给出一个综合算法.

关键词:离散事件系统;自动机;模块化状态反馈控制;死锁

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