第 10 卷第 6 期
1993 年 12 月

控制理论与应用
CONTROL THEORY AND APPLICATIONS

Vol. 10, No. 6
Dec., 1993

The Global Structure of Discrete-Time Nonlinear Control Systems with Symmetries*

LIU Xiaoping

(Department of Automatic Control, Northeast University of Technology·Shenyang, 110006, PRC)

Abstract: The paper defines the concept of symmetry for discrete-time nonlinear control systems, which parallels the corresponding concept in continuous-time nonlinear control systems. It shows that under some conditions, nonlinear control systems with symmetries admit global decomposition in terms of lower dimensional subsystems and feedback loops.

Key words: discrete-time systems; symmetry; structure decomposition

1 Introduction

The notion of symmetry of a dynamical system has been a subject of long-standing interest in Physics and Mathematics. Roughly speaking, a dynamical system possesses a symmetry if its dynamics are invariant under a (coordinate) transformation or a family of (coordinate) transformations[1,2]. The existence of such a symmetry implies usually that the system can be decomposed into subsystems of lower dimension, or that the system can be reduced to a (quotient) system of lower dimension. In this way the knowledge of the existence of symmetries can be very useful for the qualitative understanding, or even the explicit description of the dynamics of a system.

For a class of continuous-time control systems, through the use of differential geometric techniques, the notion of symmetry has been defined. For example, Schaft has given the definition of symmetry for Hamiltonian systems with input and output, applied it to optimal control problems and proven that the existence of symmetries may simplify the solution of optimal control problems[3,4]. This notion of symmetry is further explored by Grizzle and Marcus[5] for general nonlinear control systems, by using in particular families of symmetries generated by the action of a Lie group. Moreover, in these papers, the role of symmetries in obtaining a local or global decomposition of a system into smaller subsystems is emphasized.

Despite the important role of symmetry in studying the structure of continuous-time nonlinear control systems, nothing of the kind can be said to have occurred for their discrete-time counterpart. Given the pervasiveness of digital techniques in control applications, it would seem to be especially important, and useful, to extend the notion of symmetry to a class of discrete-time nonlinear control systems. The goal of this paper is to take the first step in this direction. A con-

* It was supported by the National Natural Science Foundation of China and the Foundation of Complex Systems Control Laboratory.

Manuscript received Jan. 11, 1992, revised Sept. 24, 1992.

cept of symmetry is defined for general discrete-time nonlinear control systems. Similar to continuous-time nonlinear control systems with symmetries, it is shown, under various technical conditions, that discrete-time nonlinear control systems with symmetries admit global decompositions in terms of lower dimensional subsystems and feedback loops.

Similar to [5], this paper will employ the language of differential geormetry. Some good engineering references for this material are [6,7]; standard mathematical texts are [2,8,9]. The following pedagogical example is meant to serve as both a "preview of things to come" and a source of examples illustrating some of the aspects of the results.

Example 1.1 Consider a discrete-time nonlinear control system described by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1 \\ x_4(k+1) \end{bmatrix} = \begin{bmatrix} x_1(k)[x_3(k)^2+x_4(k)^2]^{-3/2}+u_1 \\ x_2(k)[x_3(k)^2+x_4(k)^2]^{-3/2}+u_2 \\ x_3(k)[x_1(k)^2+x_2(k)^2]^{-3/2}+u_3 \\ x_4(k)[x_1(k)^2+x_2(k)^2]^{-3/2}+u_4 \end{bmatrix} = f[x(k),u(k)]. \tag{1.1}$$

Let g and g' belong to SO(2), that is, g and g' are two different 2×2 orthogonal matrices with determinant 1. Then one can prove that (1.1) is not invariant under

$$x \mapsto \begin{bmatrix} g & 0 \\ 0 & g' \end{bmatrix} x,$$

that is,

$$\begin{bmatrix} g & 0 \\ 0 & g' \end{bmatrix} f[x(k),u(k)] \neq f\left\{\begin{bmatrix} g & 0 \\ 0 & g' \end{bmatrix} x(k),u(k)\right\}, \tag{1.2}$$

but instead f satisfies

$$\begin{bmatrix} g & 0 \\ 0 & g' \end{bmatrix} f[x(k),u(k)] = f\left\{\begin{bmatrix} g & 0 \\ 0 & g' \end{bmatrix} x(k), \begin{bmatrix} g & 0 \\ 0 & g' \end{bmatrix} u(k)\right\}. \tag{1.3}$$

So this invariance involves the controls also. However, if one does a state-dependent change of the input basis by

$$\begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} = \begin{bmatrix} x_1(k)[x_3(k)^2+x_4(k)^2]^{-1/2} & -x_2(k)[x_3(k)^2+x_4(k)^2]^{-1/2} \\ x_2(k)[x_3(k)^2+x_4(k)^2]^{-1/2} & x_1(k)[x_3(k)^2+x_4(k)^2]^{-1/2} \end{bmatrix} \begin{bmatrix} v_1(k) \\ v_2(k) \end{bmatrix}, \tag{1.4a}$$

$$\begin{bmatrix} u_3(k) \\ u_4(k) \end{bmatrix} = \begin{bmatrix} x_3(k)[x_1(k)^2+x_2(k)^2]^{-1/2} & -x_4(k)[x_1(k)^2+x_2(k)^2]^{-1/2} \\ x_4(k)[x_1(k)^2+x_2(k)^2]^{-1/2} & x_3(k)[x_1(k)^2+x_2(k)^2]^{-1/2} \end{bmatrix} \begin{bmatrix} v_3(k) \\ v_4(k) \end{bmatrix}, \tag{1.4b}$$

then (1.1) becomes as follows

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1 \\ x_4(k+1) \end{bmatrix} = \begin{bmatrix} x_1(k)r_2(k)^{-3/2}+r_2(k)^{-1/2}[x_1(k)v_1(k)-x_2(k)v_2(k) \\ x_2(k)r_2(k)^{-3/2}+r_2(k)^{-1/2}[x_2(k)v_1(k)+x_1(k)v_2(k) \\ x_3(k)r_1(k)^{-3/2}+r_1(k)^{-1/2}[x_3(k)v_3(k)-x_4(k)v_4(k) \\ x_4(k)r_1(k)^{-3/2}+r_1(k)^{-1/2}[x_4(k)v_3(k)+x_3(k)v_4(k) \end{bmatrix}$$

$$= f'[x(k);v(k)]. \tag{1.5}$$

which does satisfy

$$\begin{bmatrix} g & 0 \\ 0 & g' \end{bmatrix} f'[x(k),v(k)] = f'\left\{\begin{bmatrix} g & 0 \\ 0 & g' \end{bmatrix} x(k),v(k)\right\}, \tag{1.6}$$

where $r_1(k)=x_1(k)^2+x_2(k)^2$; $r_2(k)=x_3(k)^2+x_4(k)^2$. By using the coordinate transformation

$$x_1(k) = y_1(k)y_3(k); \quad x_2(k) = y_1(k)[1 - y_3(k)^2]^{1/2},$$
$$x_3(k) = y_2(k)y_4(k); \quad x_4(k) = y_2(k)[1 - y_4(k)^2]^{1/2},$$

(1. 5) becomes

$$y_1(k+1) = y_1(k)y_2(k)^{-3}w_2(k),$$
$$y_2(k+1) = y_2(k)y_1(k)^{-3}w_1(k),$$
$$y_3(k+1) = [y_3(k) + y_2(k)^2y_3(k)v_1(k) - y_2(k)^2][1 - y_3(k)^2]^{1/2}v_2(k)]w_2(k)^{-1},$$
$$y_4(k+1) = [y_4(k) + y_1(k)^2y_4(k)v_3(k) - y_1(k)^2][1 - y_4(k)^2]^{1/2}v_4(k)]w_1(k)^{-1},$$

with

$$w_1(k) = \{[1 + y_1(k)^2v_3(k)]^2 + y_1(k)^4v_4(k)\}^{1/2},$$
$$w_2(k) = \{[1 + y_2(k)^2v_1(k)]^2 + y_2(k)^4v_2(k)\}^{1/2}.$$

In this coordinate one sees that the system (1. 5) has a natural cascade decomposition into a system parameterized by $y_1(k)$ and $y_2(k)$ feeding forward into a system parameterized by $y_3(k)$ and $y_4(k)$. From this, one gets that the original system (1. 1) has the decomposition in terms of subsystems and feedback loops.

The rest of the paper is organized as follows. Section 2 gives the definitions of symmetries used in the paper. Section 3 investigates the global structure of systems with symmetries. It is shown, under some assumptions, that discrete-time nonlinear control systems with symmetries admit global decompositions in terms of lower dimensional subsystems and feedback loops. Section 4 contains the conclusion and comments.

2 Definitions

This section will fix the notation employed for a discrete-time nonlinear control system and provide the definitions of symmetry.

Definition 2. 1[10,11] A discrete-time nonlinear control system Σ is a 3-tuple $\Sigma(B,M,f)$, where $\pi: B\to M$ is a smooth fibre bundle and $f: B\to M$ is a smooth mapping. The points of M are the states of the system, the fibres of B are the input spaces. The system's dynamics are defined by $x(k+1)=f[x(k),u(k)]$ for $u(k)\in\pi^{-1}[x(k)]$.

Example 2. 1 For example 1. 1, M is $R^4-\{0\}$, B is $M\times U$ for $U=R^4$, and f is given by (1. 1).

Note that B in exmaple 1. 1 is a trivial bundle (being globally the product of M with U). This occurs if and only if the control spaces are state-independent.

Definition 2. 2[8] Let M is a smooth manifold. A left action of a Lie Group G on M is a smooth mapping $\Phi: G\times M\to M$ such that a) for all $x\in M$, $\Phi(e,x)=x$ and b) for every g, $h\in G$, $\Phi(g,\Phi(h,x))=\Phi(gh,x)$ for all $x\in M$.

For further investigations, we denote for each $g\in G$ the mapping $\Phi(g,\cdot): M\to M$ by Φ_g, and the mapping $\Phi(\cdot,x): G\to M$, with $x\in M$, by Φ_x. Note that because $(\Phi_g)^{-1}=\Phi_{g^{-1}}$, Φ_g is a diffeomorphism. It is easy to prove that example 1. 1 gives a 2-dimensional Lie group action on $R^4-\{0\}$.

We use that following additional terminology.

Definition 2.3[8] Let Φ be an action of G on M. For $x \in M$, the orbit of x is given by

$$G.x = \{\Phi_g(x) \mid g \in G\}.$$

The space of orbits is denoted by M/G. The projection $p: M \to M/G$ is defined by $x \to Gx$. The action is free if for each $x \in M$, the map Φ_g is one-to-one. The action is proper if $\Phi': G \times M \to M \times M$, defined by $\Phi'(g,x) = (x, \Phi(g,x))$ is a proper mapping, that is, if $K \in M \times M$ is compact, then $\Phi'^{-1}(K)$ is compact.

Definition 2.4 Let $\Sigma(B,M,f)$ be a discrete-time nonlinear control system. Let G be a Lie group such that $\Theta: G \times B \to B$ and $\Phi: G \times M \to M$ are group actions. This pair of group actions is denoted by (G,Θ,Φ). Then, (G,Θ,Φ) is a symmetry for $\Sigma(B,M,f)$ if for each $g \in G$,

$$\pi \circ \Theta_g = \Phi_g \circ \pi, \tag{2.1a}$$

$$f \circ \Theta_g = \Phi_g \circ f. \tag{2.1b}$$

An important special case of the above occurs when the symmetry lies "entirely on the state space".

Definition 2.5 Let $B = M \times U$ for some manifold U. (G,Φ) is a state-space symmetry of $\Sigma(B,M,f)$ If (G,Θ,Φ) is a symmetry of Σ for $\Theta_g = (\Phi_g, Idu)$: $(x,u) \mapsto (\Phi_g(x), u)$, that is

$$f(\Phi_g x, u) = \Phi_g(f(x,u)). \tag{2.2}$$

Remark 2.1 State-space symmetries can be defined globally only for systems in which B is a trivial bundle since, otherwise, the input spaces are state-dependent.

Example 2.2

a) Let $\Sigma(B,M,f)$ be the system consisting of $M = R^4 - \{0\}$, $B = M \times R^4$ and f as in (1.1). Let $G = \mathrm{diag}\{g, g'\}$ with $g, g' \in SO(2)$ and define $\Phi_g: M \to M$ by

$$\Phi_g(x) = \begin{bmatrix} g & 0 \\ 0 & g' \end{bmatrix} x,$$

and $\Phi_g: B \to B$ by

$$\Phi_g(x,u) = \left\{ \begin{bmatrix} g & 0 \\ 0 & g' \end{bmatrix} x, \begin{bmatrix} g & 0 \\ 0 & g' \end{bmatrix} u \right\},$$

one easily proves that (2.1b) holds, see (1.3). Hence, Σ has symmetry (G,Θ,Φ).

b) Let B, M, G and Φ as above and let f' be as in (1.5). Define $\Theta_g: B \to B$ by $\Theta_g(x,u) = (\Phi_g(x), u)$. Then it follows from (1.6) that (2.2) holds. Hence, Σ' has state-space symmetry (G,Φ).

Definition 2.6[10,11] a feedback r is a bundle isomorphism from B to B; i.e. r is a diffeomorphism such that $\pi \circ r = \pi$. In local trivializing coordinates (x,u) for B, one has $r(x,u) = (x, r_x(u))$. Since r is non-singular, feedback can be viewed simply as a state-dependent change of the input coordinates.

Definition 2.7 A system $\Sigma(B,M,f)$ is feedback equivalent to a system $\Sigma'(B,M,f')$ if there exists a feedback r such that $f' = f \circ r$.

Example 2.3 Let Σ and Σ' be as in Exmaple 2.2. Define $r: B \to B$ by (1.4). Then, as in

Example 1. 1, it is easy to verify that Σ' is feedback equivalent to Σ.

3 Global Structures

This section contains two important results. The first is that if $\Sigma(M\times U, M, f)$ has state-space symmetry (G, Φ) such that Φ acts freely and properly and M is diffeomorphic to $M/G\times G$, then Σ has a global decomposition into a system evolving on M/G feeding forward into a system on G. The second is that, under the above conditions, if $\Sigma(M\times U, M, f)$ has symmetry (G, Θ, Φ), then it is feedback equivalent to one with state-space symmetry (G, Φ). This in turn gives the global structure of such a system.

For the global analysis of the structure induced by the existence of a symmetry, the following assumptions will be made.

Assumptions 3. 1 a) B is trivial, i. e. $B=M\times U$ with U a manifold;

b) Φ is free and proper;

c) M is diffeomorphic to $M/G\times G$.

Remark 3. 1 a) It follows from [5] that Θ also is free and proper if Φ is free and proper.

b) Assumptions 3. 1 c) is equivalent to assumption that $p: M\to M/G$ admits a cross section, that is, a smooth map $\sigma: M/G\to M$ such that $p\circ\sigma=$identity on M/G.

3. 1 State-Space Symmetries

Suppose $\Sigma(M\times U, M, f)$ is a control system with state-space symmetry (G, Φ), where Φ is free and proper. Since Φ is free and proper, M/G is a well-posed manifold with submersive smooth projection $p: M\to M/G$. (G, Φ) being a symmetry of Σ gives that $\Phi_g f(x, u)=f(\Phi_g(x), u)$ for all $g\in G, x\in M$ and $u\in U$. It is easy to verify the following proposition.

Proposition 3. 1 Let (G, Φ) be a state-space symmetry for a discrete-time nonlinear control system $\Sigma(M\times U, M, f)$ with Φ being free and proper. Then there exists a smooth $f': M/G\times U\to M/G$ defined by $f'(p(m), u)=pf(m, u)$.

It follows from Proposition 3. 1 that $\Sigma'(M/G\times U, M/G, f')$ is a well-defined control system on M/G, and it called as a quotient system. Note that solutions of the quotient system Σ' correspond to the "transverse part" of solutions of original system Σ. If it is possible to construct another system, say Σ_g which when driven by the states of Σ' and control sequence $u(k)$ generated that part of the solution of Σ "along the orbits", then this would result in a cascade decomposition of Σ. In fact this can be done globally if $p: M\to M/G$ admits a cross section. The basic idea is that a solution of Σ can be described by specifying how it moves from orbit-to-orbit and then how it moves along the orbits. The former is given by Σ', the latter is to be determined.

Let $x(0)\in M$, $u(\cdot)$ be a input control, $x(\cdot)$ the solution of Σ corresponding to $u(\cdot)$ and $y(\cdot)=p(x(\cdot))$, the corrsponding solution of Σ' having $y(0)=p(x(0))$. Then $y(\cdot)$ should satisfy

$$y(k+1)=f'(y(k), u(k)). \tag{3.1}$$

Now assume that $p: M\to M/G$ admits a cross section, denoted by σ, that is, M is equivalent to

$M/G\times G$, and define $d(k)$ in M by $d(k)=\sigma(y(k))$. Since $p(d(k))=y(k)=p(x(k))$, and Φ is free and proper, one can write $x(k)=\Phi(g(k),d(k))$ for a unique $g(k)\in G$. The goal now is to find a difference equation for $g(k)$. It follows from the dynamics of Σ that

$$f[x(k),u(k)]=f[\Phi(g(k),d(k)),u(k)]=x(k+1)=\Phi[(g(k+1)\ d(k+1)]. \tag{3.2}$$

Φ being free and proper implies that $\Phi_m: G\to M$ is a diffeomorphism onto its range. Hence (3.2) can be solved uniquely for $g(k+1)$ to give

$$g(k+1)=\Phi^{-1}_{d(k+1)}f\{\Phi[g(k),d(k)],u(k)\}. \tag{3.3}$$

Finally, using the fact that $d(k)=\sigma(y(k))$, one gets

$$g(k+1)=\Phi^{-1}_{\sigma(y(k+1))}f\{\Phi[g(k),\sigma(y(k))],u(k)\}. \tag{3.4}$$

In summary, the following has been proven.

Theorem 3.1 Suppose $\Sigma(B,M,f)$ is a discrete-time nonlinear control system with state-space symmetry (G,Φ). Then, under assumptions 3.1, Σ is isomorphic to the system

$$y(k+1)=f'(y(k),u(k)),$$
$$g(k+1)=\Phi^{-1}_{\sigma(y(k+1))}f\{\Phi[g(k),\sigma(y(k))],u(k)\}.$$

which evolves on $M/G\times G$.

Example 4.1 Let $\Sigma(B,M,f')$ and (G,Φ) be as in Example 2.2 b). Φ is proper because G is compact and is easily checked to be free. M/G is equal to R^2-$\{0\}$ and M is diffeomorphic to $M/G\times G=[R^2\text{-}\{0\}]\times R^2$. Hence, Σ admits a global decomposition and example 1.1, in fact, has given the global structure of this system.

3.2 Symmetries

This subsection extends the results obtained for systems with state-space symmetries to systems with more general symmetries. The key step involves showing that a system with a symmetry is (under certain conditions) feedback equivalent to a system with a state-space symmetry.

The following proposition is immediate (cf. [4]).

Proposition 3.2 Suppose that $\Sigma(B,M,f')$ has symmetry (G,Θ,Φ). Let Ψ be another G-action on B satisfying $\pi\circ\Psi_g=\Phi_g\circ\pi$, for any $g\in G$. Then Σ is feedback equivalent to some system having a symmetry (G,Θ,Φ) if and only if there exists a feedback $r: B\to B$ satisfying $f\circ\Theta_g\circ r=f\circ r\circ\Psi_g$, for any $g\in G$.

Proof Sufficiency: If there exists a feedback $r: B\to B$ satisfying $f\circ\Theta_g\circ r=f\circ r\circ\Psi_g$, for each $g\in G$, then

$$\Phi_g\circ f'(b')=\Phi_g\circ f\circ r(b')=f\circ\Theta_g\circ r(b')=f\circ r\circ\Psi_g(b')=f'\circ\Psi_g(b'),$$

for all $b'\in B$, that is $\Phi_g\circ f'=f'\circ\Psi_g$. Hence Σ is feedback equivalent to Σ' with symmetry (G,Ψ,Θ).

Necessarity If Σ is feedback equivalent to Σ' with symmetry (G,Ψ,Φ), then there exists a feedback r such $f'=f\circ r$. Since (G,Θ,Φ) and (G,Ψ,Θ) are symmetries of Σ and Σ', respectively, one has

$$f\circ\Theta_g\circ r(b')=\Phi_g\circ f\circ r(b')=\Phi_g\circ f'(b')=f'\circ\Psi_g(b')=f\circ r\circ\Psi_g(b').$$

A sufficient condition for the existence of a feedback r is stated in the following.

Proposition 3. 3 Suppose $\Sigma(B,M,f)$ has symmetry (G,Θ,Φ). Let Ψ be another G-action on B satisfying $\pi\circ\Psi_g=\Phi_g\circ\pi$, for each $g\in G$. Furthermore Suppose that M is diffeomorphic to $M/G\times G$. Then there exists a feedback r satisfying $\Theta_g\circ r=r\circ\Psi_g$, for each $g\in G$.

Proof Proposition 3. 2 implies that it suffices to construct a bundle isomorphism $r:B\to B$ such that

$$\Theta_g\circ r=r\circ\Psi_g,\quad \text{for all } g\in G. \tag{3.5}$$

Define r by $r=\Theta^1\circ(\Psi^1)^{-1}$ where $\Theta^1:M/G\times G\times U\to B$ by $(y,g,u)\to\Theta_g(\sigma(y),u)$ and $\Psi^1:M/G\times G\times U\to B$ by $(y,g,u)\to\Psi_g(\sigma(y),u)$. From [5], it follows that Θ^1 and Ψ^1 are all diffeomorphisms. Hence r is a diffeomorphism. It remains to be shown that a) $\pi\circ r=\pi$ and b) (3. 5).

Fix $b\in B$ and let $(y,g,v)=(\Psi^1)^{-1}(b)$. Then

$$\begin{aligned}\pi\circ r(b)&=\pi\circ\Theta^1\circ(\Psi^1)^{-1}(b)=\pi\circ\Theta^1(y,g,v)=\pi\circ\Theta_g(\sigma(y),v)\\&=\Phi_g\circ\pi(\sigma(y),v)=\Phi_g(\sigma(y))=x=\pi(b).\end{aligned}$$

This gives a).

For b), let $b\in G$ and observe that

$$\Theta_h\circ r(b)=\Theta_h\circ\Theta^1\circ(\Psi^1)^{-1}(b)=\Theta_h\circ\Theta^1(y,g,v)=\Theta_h\circ\Theta_g(\sigma(y),v)=\Theta_{hg}(\sigma(y),v),$$

and

$$\begin{aligned}r\circ\Psi_h(b)&=\Theta^1\circ(\Psi^1)^{-1}\circ\Psi_h\circ\Psi^1(y,g,v)=\Theta^1\circ(\Psi^1)^{-1}\circ\Psi_h\circ\Psi_g(\sigma(y),v)\\&=\Theta^1\circ(\Psi^1)^{-1}\circ\Psi_{hg}(y,g,v)=\Theta^1\circ(\Psi^1)^{-1}(b')=\Theta^1(y,hg,v)=\Theta_{hg}(\sigma(y),v).\end{aligned}$$

Let now (G,Θ,Φ) be a symmetry for Σ. Consider the action Ψ of Lie group G on $M\times U$ defined by

$$\Psi_g(x,u)=(\Phi_g(x),u),\quad x\in M,\quad u\in U.$$

It is clear that $\pi\circ\Psi_g=\Phi_g\circ\pi$. Hence, by Propositon 3. 2 and 3. 3 there exists a feedback r such that Σ is feedback equivalent to the control system $\Sigma'(M\times U,M,f')$ with $f'=f\circ r$, having symmetry (G,Ψ,Φ), i. e. state-space symmetry (G,Φ). Combining Theorem (3. 1) and Proposition 3. 3 one can obtain the following theorem.

Theorem 3. 2 Suppose that $\Sigma(M\times U,M,f)$ has symmetry (G,Θ,Φ). Then, under assumptions 3. 1, there exists a system Σ' with state-space symmetry (G,Φ) to which Σ is feedback equivalent. Hence, Σ admits a decomposition in terms of subsystems and feedback loops.

Example 3. 2 Let $\Sigma(B,M,f)$ and (G,Θ,Φ) be as in Example 2. 2. Φ is free and proper as discussed in Example 3. 1; also, $M/G=R^2-\{0\}$ and $p:M\to M/G$ admits a cross section. Hence, Σ has a global representation in terms of a feedback loop and a system on M/G feeding forward into a system on G. The feedback function is given in Example 2. 3.

4 Conclusion and Comments

In this paper, the concept of symmetry has been defined for a class of discrete-time nonlinear control system. It has been shown that discrete-time nonlinear control systems with symmetries, under a few technical conditions, admit global decomposition in terms of lower dimensional subsystems and feedback loops.

The results in this paper are parallel in several ways to those for continuous-time systems in Grizzle and Marcus[5]. However, there are several important differences. The most important difference between the discrete and continuous-time systems is the definitions of symmetries, for continuous-time case (G,Θ,Φ) is a symmetry if $T\Phi_g^{\cdot} f = f \cdot \Theta_g$.

Two directions of research are presently being pursued in order to increase the applicability of this work. The first involves local structure of discrete-time nonlinear control systems with symmetries. The second extension involves applications to optimal control problems. These results will be reported at a latter date.

References

[1] Nejmeije- H. and Ven Der Schaft, A. J.. Partial Symmetries for Nonlinear Systems. Math. Syst. Theory, 1985, 18: 79—96

[2] Shayman, M. A.. A Symmetry Group for the Matrix Riccati Equation. Syst. Contr. Lett., 1982, 2:17—24

[3] Ven Der Schaft, A. J.. Symmetries and Conversation laws for Hamiltonian Systems with Input and Outputs: A Generalization of Neother's Theorem. Syst. Contr. Lett., 1981, 1:108—115

[4] Ven Der Schaft, A. J.. Symmetries in Optimal Control. SIAM. J. Contr. Optimaiz., 1987, 25(2):245—259

[5] Grizzle, J. W. and Marcus, S. I.. The Structure of Nonlin · Control Systems Possessing Symmetries. IEEE Trans. Automat. Contr., 1985, AC-30:248—258

[6] Brockett, R. W.. Nonlinear Systems and Differential Geometry. Proc. IEEE, 1976, 64:61—72

[7] Brockett, R. W.. Lie Algebras and Lie Groups on Control Theory. In Geometric Methods in System Theory, D. Q. Mayne and R. W. Brockett, Eds. Dordrecht, The Netherlands: Reidel, 1973

[8] Abraham, R. and Marsden, J. E.. Foundations of Mechanics. MA: Benjamin/Cummings, 2nd ed. 1978

[9] Spivak, M.. A Comprehensive Introduction to Differential Geometry, Berkeley, CA: Publish or Perish, 1979

[10] Grizzle, J. W.. Local Input-output Decoupling of Discrete-time Nonlinear Systems. I. J. Contr. 1986, 43(5):1517—1530

[11] Grizzle, J. W.. Controlled Invariance for Discrete-time Nonlinear Systems with an Application to the Disturbance Decoupling Problem. IEEE Trans. Automat. Contr., 1985, AC-30(9):868—874

离散对称非线性控制系统的全局结构

刘晓平

(东北工学院自动控制系·沈阳,110006)

摘要: 本文给出离散非线性控制系统的对称性概念,并证明了在一定条件下,对称离散非线性控制系统可通过反馈及坐标变换化成具有更简单的结构形式.

关键词: 离散系统; 对称性; 结构分解

本文作者简介

刘晓平 1962年生. 博士生导师. 分别于1984年、1987年、1989年在东北工学院自动控制系获学士、硕士、博士学位. 毕业后留校任教,并于1992年晋升为教授. 曾在最优控制,对策论,奇异系统及非线性系统等方面在国内外杂志及会议上发表论文50余篇. 目前主要从事非线性系统的研究.