Computing Closed Observable Sublanguages Superior to the Supremal Closed Normal Sublanguage in Supervisory Control of DES

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Abstract: In this paper, an algorithm for computing closed observable sublanguages is given, which will converge within m steps (the number of discrete events in system). In addition, no matter how the events in system are sequenced, the closed observable sublanguages got from the algorithm always contains the supremal closed normal sublanguage.

Key words: discrete event systems; supervisory control; observable language

1 Introduction

In supervisory control theory of discrete event systems (DES)[1], DES are modeled by controlled automata, and their behaviors are described by the associated formal languages. Control is exercised by a supervisor, whose action is to enable or disable events so that the controlled system generates some prespecified desired languages. In some cases, the supervisor may also be constrained to observe only events in a specified set of observable events[2]. It has been shown that such a supervisor exists if and only if the language to be synthesized is both controllable and observable[2]. Hence, the formal synthesis problem for a supervisor is posed as one of synthesizing the largest possible sublanguage of a specified "desirable" or "iegal" language. However, this largest possible sublanguage is not necessarily unique. Therefore, a slightly stronger version of observability, called normality, has been introduced[2]. It has been shown that normality implies observability, and a unique supremal normal sublanguage is guaranteed to exist. Hence the supervisor can be designed to synthesize the supremal controllable and normal sublanguage. In general, the supremal normal sublanguage may greatly constrain the behaviors of the closed system. In order to reduce this constraint, we propose an algorithm for computing observable sublanguages. When the events of system sequence in different ways, the algorithm may give different observable sublanguages. But they all contain the supremal normal sublanguage. Hence, using these languages for designing supervisor can reduce the constraint caused by the supremal normal sublanguage, and improve the performance of the closed system.

In the followings, we will present the iterative algorithm, and prove its properties.

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2 Computing Observable Sublanguage

Let automata G denote controlled DES, Σ is its finite set of events, and $|\Sigma|=m$. $L(G)$ is a fixed language over Σ, called the behaviors of G, which represents "feasible" or "physically possible" behaviors.

Let $L\subseteq L(G), L=\overline{L}$, represent desirable or legal behavior. According to[1,2], $\Sigma=\Sigma_c\cup\Sigma_{uc}=\Sigma_o\cup\Sigma_{uo}$, where Σ_c denotes the set of controllable events, Σ_o denotes the set of observable events, Σ_{uc} denotes the set of uncontrollable events, and Σ_{uo} denotes the set of unobservable events. The observable projection $P:\Sigma^*\to\Sigma_o^*$ is

$$P(\varepsilon)=\varepsilon,\ P(s\sigma)=\begin{cases}p(s)\sigma, & \text{if } \sigma\in\Sigma_o,\\ p(s), & \text{if } \sigma\in\Sigma_{uo}.\end{cases}$$

That is, P is a projection whose effect on a string $s\in\Sigma^*$ is just to erase the elements of s which belong to Σ_{uo}, so that $P(s)\in\Sigma_o^*$. The inverse projection of P, denoted P^{-1}, is $P^{-1}(s)=\{t|P(t)=s\}$, $P^{-1}(H)=\{s|P(s)\in H\}$, where H is a language over Σ_o. Given closed $L\subseteq L(G)$, if $\forall\ s,t\in L$, $P(s)=P(t)$; $\forall\ \sigma$, $s\sigma\in L$, $t\sigma\in L(G)$, $\Rightarrow t\sigma\in L$, then L is said to be observable with respect to G and P. Given a language K over Σ, if $K=P^{-1}P(K)\cap L(G)$, then K is said to be normal with respect to G and P. In other words, given normal language $K\subseteq L(G)$, $\forall\ s\in K$, $t\in L(G)$, if $P(s)=P(t)$, then $t\in K$.

Considering that Σ is a finite set of events, we can derive the following algorithm from this general property to compute the closed observable sublanguage of L.

Algorithm Let $K_1=L$, $i=1$,

① compute $\Omega_i(K_i)=K_i-\{P^{-1}P[P^{-1}P(K_i\sigma_i\cap K_i)\cap K_i\sigma_i\cap L(G)-K_i]\cap K_i\sigma_i\}\Sigma^*$,

let $K_{i+1}=\Omega_i(K_i)$,

if $i=m$, then terminate,

otherwise, let $i=i+1$, go to ①.

Obviously, this algorithm only inerates m steps. In next section we will prove that the K_{m+1} is closed observable sublanguage of L, and it contains the closed supremal normal sublanguage of L.

Lemma 1 Given language sequence $K_i(i=1,2,\cdots,m+1)$ got from the above algorithm, then K_i is a monotonic and reductive sequence, and $K_i=\overline{K}_i$.

Proof According to[3], $\overline{B}-A\Sigma^*$ is a closed language (where B and A are the languages over Σ), so $K_i=\overline{K}_i$. The reductive monotonicity is obvious.

Lemma 2 $\forall\ s,t\in K_2$, $s\neq t$, $P(s)=P(t)$, $s\sigma_1=K_2$, $t\sigma_1\in K_2$, $t\sigma_1\in L(G)$, then $t\sigma_1\in K_2$.

Proof Since $K_2=\Omega_1(K_1)=K_1-\{P^{-1}P[P^{-1}P(K_1\sigma_1\cap K_1)\cap K_1\sigma_1\cap L(G)-K_1]\cap K_1\sigma_1\}\Sigma^*$, let

$$K_2=K_1-K_{11}\sigma_1\Sigma^* \tag{1}$$

Since s, $t\in K_2$, according to Lemma 1, s, $t\in K_1$, so $s\sigma 1$, $t\sigma_1\in K_1\sigma_1$; since $t\sigma_1\in L(G)$, then $t\sigma_1\in K_1\sigma_1\cap L(G)$; since $s\sigma_1\in K_2$, according to Lemma 1, $s\sigma_1\in K_1$, so $s\sigma_1\in K_1\sigma_1\cap K_1$; given

$P(s)=P(t)$, then $P(s\sigma_i)=P(t\sigma_1)$, so $t\sigma_1\in P^{-1}P(t\sigma_1)=P^{-1}P(s\sigma_1)\subseteq P^{-1}P(K_1\sigma_1\cap K_1)$,

$$t\sigma_1 \in P^{-1}P(K_1\sigma_1 \cap K_1) \cap K_1\sigma_1 \cap L(G). \qquad (2)$$

Assume $t\sigma_1\notin K_2$, according to (1), $t\sigma_1\notin K_1$ or $t\sigma_1\in K_1\cap K_{11}\sigma_1\Sigma^*$.

① if $t\sigma_1\notin K_1$,

from (2) $t\sigma_1\in P^{-1}P(K_1\sigma_1\cap K_1)\cap K_1\sigma_1\cap L(G)-K_1$, $P(t\sigma_1)\in P[P^{-1}P(K_1\sigma_1\cap K_1)\cap K_1\sigma_1\cap L(G)-K_1]$. Since $P(s\sigma_1)=P(t\sigma_1)$, so $s\sigma_1\in P^{-1}P(s\sigma_1)\subseteq P^{-1}P[P^{-1}P(K_1\sigma_1\cap K_1)\cap K_1\sigma_1\cap L(G)-K_1]$, obviously $s\sigma_1\in\{P^{-1}P[P^{-1}P(K_1\sigma_1\cap K_1)\cap K_1\sigma_1\cap L(G)-K_1]\}\Sigma^*$, according to (1) $s\sigma_1\in K_{11}\sigma_1\Sigma^*$. Since $s\sigma_1\in K_2$, according to (1), $s\sigma_1\in K_1$ and $s\sigma_1\notin K_{11}\sigma_1\Sigma^*$. Contradiction!

② if $t\sigma_1\in K_1\cap K_{11}\sigma_1\Sigma^*$, that is $t\sigma_1\notin K_2$

since $t\in K_2$, according to (1), $t\in K_1$, $t\notin K_{11}\sigma_1\Sigma^*$, that is,

$$t \notin K_{11}\sigma_1 \cap K_{11}\sigma_1(\Sigma^* - \varepsilon). \qquad (3)$$

Since $t\sigma_1\in K_{11}\sigma_1\Sigma^*$, let $t\sigma_1\in K_{11}\sigma_1\cup K_{11}\sigma_1(\Sigma^*-\varepsilon)$, if $t\sigma_1\in K_{11}\sigma_1(\Sigma^*-\varepsilon)$, then $t\in K_{11}\sigma_1\Sigma^*$.

Contradiction!

If $t\sigma_1\in K_{11}\sigma_1$, that is, $t\sigma_1\in P^{-1}P[P^{-1}P(K_1\sigma_1\cap K_1)\cap K_1\sigma_1\cap L(G)-K_1]\cap K_1\sigma_1$, so $P(t\sigma_1)\in P[P^{-1}P(K_1\sigma_1\cap K_1)\cap K_1\sigma_1\cap L(G)-K_1]$, since $P(s\sigma_1)=P(t\sigma_1)$, so $P(t\sigma_1)\in P[P^{-1}P(K_1\sigma_1\cap K_1)\cap K_1\sigma_1\cap L(G)-K_1]$, since $s\sigma_1\in P^{-1}P(s\sigma_1)$, $s\sigma_1\in K_1\sigma_1$ (proved above), so $s\sigma_1\in\{P^{-1}P[P^{-1}P(K_1\sigma_1\cap K_1)\cap K_1\sigma_1\cap L(G)-K_1]\cap K_1\sigma_1\}\Sigma^*$, that is $s\sigma_1\in K_{11}\sigma_1\Sigma^*$. Similarly, $s\sigma_1\in K_2$, according to (1), $s\sigma_1\in K_1$, $s\sigma_1\notin K_{11}\sigma_1\Sigma^*$. Contradiction! So $t\sigma_1\notin K_{11}\sigma_1\Sigma^*$. Contradiction again!

Then $t\sigma_1\in K_1$.

Lemma 3 $\forall\ s,\ t\in K_{i+1},\ s\neq t,\ P(s)=P(t),\ s\sigma_i=K_{i+1},\ t\sigma_i\in L(G),\ (i=1,2,\cdots,m)$, then $t\sigma_i\in K_{i+1}$.

Proof Similar to the proof in Lemma 2.

Lemma 4 $\forall\ s,\ t\in K_3,\ s\neq t,\ P(s)=P(t),\ s\sigma_1=K_3,\ t\sigma_1\in L(G)$, then $t\sigma_1\in K_3$.

Proof Given $K_3=\Sigma_2(K_2)$, let

$$K_3 = K_2 - K_{22}\sigma_2\Sigma^*. \qquad (4)$$

Assume $t\sigma_1\notin K_3$, according to (4), then $t\sigma_1\notin K_2$ or $t\sigma_1\in K_2\cap K_{22}\sigma_2\Sigma^*$.

① if $t\sigma_1\notin K_2$,

Since s,t, $s\sigma_1\in K_3$, according to Lemma 1, s,t, $s\sigma_1\in K_2$. Given $P(s)=P(t)$, $t\sigma_1\in L(G)$, according to Lemma 2, $t\sigma_1\in K_2$. Contradiction!

② if $t\sigma_1\in K_2\cap K_{22}\sigma_2\Sigma^*$,

let $K_{22}\sigma_2\Sigma^*=K_{22}\sigma_2\cup K_{22}\sigma_2(\Sigma^*-\varepsilon)$, since $t\sigma_1\in K_{22}\sigma_2\Sigma^*$, $t\sigma_1\in K_{22}\sigma_2(\Sigma^*-\varepsilon)$, $t\sigma_1\in K_{22}\sigma_2\Sigma^*\sigma_1$, so, $t\in K_{22}\sigma_2\Sigma^*$; Since $t\in K_3$, from (4), $t\in K_2$, $t\notin K_{22}\sigma_2\Sigma^*$. Contradiction! So $t\sigma_1\in K_3$.

Lemma 5 $\forall\ j,\ 1\leqslant j\leqslant m;\ \forall\ s,\ t\in K_{i+1}, j\leqslant i\leqslant m,\ s\neq t,\ P(s)=P(t)$, if $s\sigma_j\in K_{i+1}$, $t\sigma_j\in L(G)$, then $t\sigma_j\in K_{i+1}$.

Proof According to Lemma 2, when $j=1$, $\forall\, s,t\in K_{i+1}$, $i=1$, $s\neq t$, $P(s)=P(t)$, $s\sigma_j\in K_{i+1}$, $t\sigma_j\in L(G)$, then $t\sigma_j\in K_{i+1}$. According to Lemma 4 and Lemma 3, when $j=1,2$, $\forall\, s,t\in K_{i+1}$, $(j\leqslant i\leqslant 2)$, $s\neq t$, $P(s)=P(t)$, $s\sigma_j\in K_{i+1}$, $t\sigma_j\in L(G)$, then $t\sigma_j\in K_{i+1}$.

Assumption ⟨1⟩ $\forall\, j=1,2,\cdots,l$, $1\leqslant l\leqslant m-1$, $\forall\, s,t\in K_{i+1}(j\leqslant i\leqslant 1)$, $s\neq t$, $P(s)=P(t)$, $s\sigma_j\in K_{i+1}$, $t\sigma_j\in L(G)$, then $s\sigma_j\in K_{i+1}$.

It need to prove: $\forall\, j=1,2,\cdots,l+1$, $l+1\leqslant m-1$, $\forall\, s,t\in K_{i+1}(j\leqslant i\leqslant 1+1)$, $s\neq t$, $P(s)=P(t)$, $s\sigma_j\in K_{i+1}$, $t\sigma_j\in L(G)$, then $t\sigma_j\in K_{i+1}$. That is, it need only to prove: $\forall\, j=1,2,\cdots,l+1$, $l+1\leqslant m-1$, $\forall\, s,t\in K_{i+2}$, $s\neq t$, $P(s)=P(t)$, $s\sigma_j\in K_{i+2}$, $t\sigma_j\in L(G)$, then $t\sigma_j\in K_{i+2}$.

Assumption ⟨2⟩ $t\sigma_j\notin K_{i+2}$

Given $K_{i+1}=\Omega_{i+1}(K_{i+1})$, let

$$K_{i+2}=K_{i+1}-K_{i+1,l+1}\sigma_{i+1}\Sigma^*, \tag{5}$$

So $t\sigma_j\notin K_{i+1}$ or $t\sigma_j\in K_{l+1}\cap K_{l+1,l+1}\sigma_{l+1}\Sigma^*$.

① if $t\sigma_j\notin K_{l+1}$,

Since s,t, $s\sigma_j\in K_{l+2}$, according to Lemma 1, s,t, $s\sigma_j\in K_{l+1}$, given $t\sigma_j\in L(G)$, from assumption ⟨1⟩, $t\sigma_j\in K_{l+1}$. Contradiction!

② if $t\sigma_j\in K_{l+1}\cap K_{l+1,l+1}\sigma_{l+1}\Sigma^*$,

let $K_{l+1,l+1}\sigma_{l+1}\Sigma^*=K_{l+1,l+1}\sigma_{l+1}\cup K_{l+1,l+1}\sigma_{l+1}(\Sigma^*-\varepsilon)$, if $1\leqslant j\leqslant l$, then $t\sigma_j\in K_{l+1,l+1}\sigma_{l+1}(\Sigma^*-\varepsilon)$, $t\sigma_j\in K_{l+1,l+1}\sigma_{l+1}\Sigma^*\sigma_j$, so, $t\in K_{l+1,l+1}\sigma_{l+1}\Sigma^*$. But given $t\in K_{l+2}$, from (5), $t\in K_{l+1}$ and $t\notin K_{l+1,l+1}\sigma_{l+1}\Sigma^*$. Contradiction!

If $j=l+1$, according to Lemma 3, $t\sigma_j\in K_{l+2}$. Contradiction with assumption ⟨2⟩. In one word, $t\sigma_j\in K_{l+2}$, Lemma 5 is proved.

Theorem 1 K_{m+1} is the closed observable sublanguage of L.

Proof According to Lemma 1, $K_{m+1}$is the closed sublanguage of L. According to Lemma 5, $\forall\, 1\leqslant j\leqslant m$, $\forall\, s,t\in K_{m+1}$, $s\neq t$, $P(s)=P(t)$, if $s\sigma_j\in K_{m+1}$, $t\sigma_j\in L(G)$, then $t\sigma_j\in K_{m+1}$, which is consistent with the definition about closed observable language. Hence, K_{m+1} is observable.

Let K_{cn} denote the supremal closed normal sublanguage of L, and K_{co} denote K_{m+1}. We can get the following theorem.

Theorem 2 $K_{cn}\subseteq K_{co}$.

Proof Assume $K_{cn}\not\subseteq K_{co}$, since $\varepsilon\in k_{co}$, $\exists\, s\sigma_i\notin K_{co}$, but $s\sigma_j\in K_{cn}(1\leqslant i\leqslant m)$. Since $K_1=L$, $K_{cn}\subseteq L$, so

$$K_{cn}\subseteq K_1. \tag{6}$$

Given $K_{co}=K_{m+1}$, so $s\sigma_i\in K_1$, $s\sigma_i\notin K_{m+1}$. Given

$$K_{j+1}=K_j-K_{jj}\sigma_j\Sigma^*(1\leqslant j\leqslant m), \tag{7}$$

we can let $s\sigma_i\in K_j$, $s\sigma_i\notin K_{j+1}$; From (7), $s\sigma_i\in K_{ii}\sigma_j\Sigma^*$; According to Lemma 1, $K_j=\overline{K}_j$, $\exists\, t,t\sigma_j\in\overline{s\sigma i}$, and $t,t\sigma_j\in K_j$, $t_o\in K_j$, $t_o\neq t$, $P(t_o)=P(t)$, $t_o\sigma_j\in L(G)$ such that

$$t_0\sigma_j\notin K_j. \tag{8}$$

Since $s\sigma_i \in K_{\infty}$, $K_{\infty}=\overline{K_{\infty}}$, so $t\sigma_j \in K_{\infty}$, and since $P(t_0\sigma_j)=P(t\sigma_j)$, K_{∞} is normal, then

$$t_0\sigma_j \in K_{\infty}; \tag{9}$$

from (6), $t_0\sigma_j \in K_1$. According to (8) and (9), let $t_0\sigma_j \in K_j$, $t_0\sigma_j \notin K_{j1+1}(1 \leqslant j1 < j)$, then $t_0\sigma_j \in K_{j1,j1}\sigma_{j1}\Sigma^*$, let $t_0\sigma_j = t_0\sigma_{j1}\Sigma^* u_0\sigma_j$; Since $K_{j1}=\overline{K}_{j1}$, t_1, $t_1\sigma_{j1} \in K_{j1}$, $\exists$ $t_{11} \in K_{j1}$, $t_{11} \neq t_1$, $P(t_{11}) = P(t_1)$, $t_{11}\sigma_{j1} \in L(G)$, such that $t_{11}\sigma_{j1} \notin K_{j1}$; From (9) and $K_{\infty}=\overline{K_{\infty}}$, $t_1\sigma_{j1} \in K_{\infty}$; Since $P(t_{11}\sigma_{j1})=P(t_1\sigma_{j1})$, so $t_{11}\sigma_{j1} \in K_{\infty}$, $t_{11}\sigma_{j1} \in K_1$.

Similarly, let $t_{11}\sigma_{j1} \in K_{j2}$, $t_{11}\sigma_{j1} \notin K_{j2+1}$, $1 \leqslant j2 < j1$, $t_{11}\sigma_{j1} \in K_{j2,j2}\sigma_{j2}\Sigma^*$, $t_{11}\sigma_{j1} = t_2\sigma_{j2}u_1\sigma_{j1}$, t_2, $t_2\sigma_{j2} \in K_{j2}$, $\exists$ $t_{22} \in K_{j2}$, $t_{22} \neq t_2$, $P(t_{22})=P(t_2)$, $t_{22}\sigma_{j2} \in L(G)$ such that $t_{22}\sigma_{j2} \in K_{\infty}$, $t_{22}\sigma_{2j} \in K_1$. ...... Procced in such way, since $j \leqslant m$, within m steps, $\exists$ $t_{ll}\sigma_{jl} \notin K_{jl}$, $t_{ll}\sigma_{jl} \in K_{\infty}$, $t_{ll}\sigma_{jl} \in K_1$, $jl=2$, so $t_{ll}\sigma_2 \in K_{11}\sigma_1\Sigma^*$. Similarly, $t_{ll}\sigma_2 = s_0\sigma_1u_1\sigma_2$, $\exists$ $s_{00} \in K_1$, $s_{00} \neq s_0$, $P(s_{00})=P(s_0)$, $s_{00}\sigma_1 \in L(G)$, $s_{00}\sigma_1 \notin K_1$; But since $s_0\sigma_1 \in K_{\infty}$, $P(s_0\sigma_1)=P(s_{00}\sigma_1)$, K_{00} is normal, so $s_{00}\sigma_1 \in K_1$.

Contradiction! Hence, $K_{\infty} \subseteq K_{\infty}$.

The following example can be used to illustrate $K_{\infty} \subset K_{\infty}$.

Example Let $\Sigma=\{\alpha_1,\alpha_2,\alpha_3,\alpha_4\}$, $\Sigma_0=\{\alpha_1,\alpha_2\}$,

and $L(G)=\overline{(\varepsilon+\alpha_3+\alpha_4)\alpha_1+\alpha_2+\alpha_1\sigma_4}$,

$L=\varepsilon+\alpha_1+\alpha_3+\alpha_4+\alpha_3\alpha_1+\alpha_4\alpha_1$.

Obviously, $L=\overline{L}$. From the algorithm, get $K_{m+1}=L$, that is $K_{\infty}=L$;

$K_{\infty}=\varepsilon+\alpha_3+\alpha_4$, so $K_{\infty} \subset K_{\infty}$.

Theorem 3 If L is a closed observable language, no matter how the events in Σ are sequenced, then $K_{m+1}=L$.

Proof The theorem can be easily proved by proving its inverse proposition.

Proposition 1 Let $K=\overline{K}$, $K \subseteq L(G)$, then

$$K \cap \{K\sigma \cap P^{-1}P[(L(G)-K) \cap K\sigma]\}\Sigma^*$$
$$= K \cap \{P^{-1}P[P^{-1}P(K\sigma \cap K) \cap K\sigma \cap L(G) - K] \cap K\sigma\}\Sigma^*$$

According to this proposition, we can substitute $\Omega(K_i)=K_i-\{K_i\sigma_i \cap P^{-1}P[(L(G)-K_i) \cap K_i\sigma_i]\}\Sigma^*$ for the old one to simplify the algorithm.

3 Conclusion

The language K_{m+1} got from the algorithm is not unique. Because the events in Σ may be sequenced in different way. In addition, this algorithm keeps the regularity of L. Since each kind of operation concerned with the algorithm keeps the regularity of its language. It is an obvious advantage that this algorithm is not involved with analyzing and proving convergence.

Future research tasks might include finding algorithm to compute maximal observable sublanguages which contain the supremal normal sublanguage and developing method to compute the controllable and observable sublanguages by making use of this algorithm.

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计算DES监控中优于闭可识别语言的闭可观子语言

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摘要：本文给出一种求闭能观子语言的算法,该算法经 m 步(系统中离散事件个数)收敛;并且证明无论对系统中事件如何排序,由该算法求出的闭能观子语言总包含最大闭可识别子语言.

关键词：离散事件(DES);监控;可观语言

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