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Robust Stability Margin for a Class of Plants with Structured and Unstructured Uncertainties

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Abstract: This paper studies the robust stability problem for plants under structured and unstructured perturbations. Our goal is to obtain the unstructured perturbation bound for a class of plants. Our results are nontrivial extensions of those in [1].

Key words: structured uncertainties; unstructured uncertainties; robust stability; polytopic plants; edge plants

1 Introduction

In the design of robust controllers, there are at least two types of system uncertainties[2]: one is modeled as structured uncertainties, which are represented as the variation of parameters and suitable for describing high frequency uncertainties. The other is modeled as unstructured uncertainties, i. e., a nomial plant is given and the perturbations are restricted within a norm-bounded set. This model is suitable for describing low frequency uncertainties. These two types of models were studied independently in the past. Only recently are there papers dealing with perturbations of mix-type[1~3].

[3] proposed a new model which contained both the above-mentioned types of uncertainties: A nominal plant, whose structured uncertainty is represented as proper rational matrices, is given and it's subjected to unstructured perturbation which is additive or multiplitive. And [1] gave a method to campute the robust stability unstructured perturbation margin for such new models.

Notice that in [1] the structured uncertainty is represented as interval matrices, which can only describe a small class of uncertainties, because the coefficients of the interval polynomials are required to be independent. In general, these coefficients vary dependently in a given polytope, and this class of uncertainties can be represented as polytopic matrices[4,5]. In this paper, the nominal plant is given as a family of polytopic matrices. Our goal is to give a simple method to compute the robust stability margin for such a nominal plant under additive perturbation.

Throughout the paper, all systems are assumed to be single-input single-output. The proofs are given in appendix.

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2 Preliminaries

Given two polytopes N and D, $N \subset R^{p+1}$, $D \subset R^{q+1}$, consider following families of polytopic polynomials:

$$N(s) = \{n(s): n(s) = n_0 + n_1 s + \cdots + n_p s^p, \quad (n_0, \cdots, n_p)^T \in N\}, \tag{2.1}$$

$$D(s) = \{d(s): d(s) = d_0 + d_1 s + \cdots + d_q s^q, \quad (d_0, \cdots, d_q)^T \in D\}. \tag{2.2}$$

Corresponding to the extremes of N and D. there exist extremal polynomials $N_v(s)$ and $D_v(s)$, i. e., the polynomials in $N(s)$ and $D(s)$ whose coefficients are extremes of N and D respectively. So corresponding to the edges of N and D, there exist edge polynomials $N_e(s)$ and $D_e(s)$, i. e., the polynomials in $N(s)$ and $D(s)$ whose coefficients belong to the edges of N and D respectively.

Let $A = N \times D \subset R^{p+q+2}$, notice A is also a polytope. Consider a family of Plant transfer functions:

$$G_0 = \{g(s) = \frac{n(s)}{d(s)}: n(s) \in N(s), d(s) \in D(s), p < q, (n_0, \cdots, n_p, d_0, \cdots, d_q)^T \in A\}. \tag{2.3}$$

We define the edge plants of G_0 as follow:

$$G_{0e} = \{g(s) = \frac{n(s)}{d(s)}: n(s) \in N_e(s), d(s) \in D_e(s)\}. \tag{2.4}$$

We call a polynomial stable, if all its zeros are in the left half plane. It's well-known that when we want to check the stabily of a family of polytopic polynomials, It's sufficient to check the family of its edge plants.

Lemma 2.1[4] Suppose N in (2.1) is a polytope, then $N(s)$ is stable if and only if $N_\infty(s)$ is stable.

The lemma below transform the problem of computing the largest perturbation bound into the problem of checking the stability of a family of polynomials, which is the basis of this paper.

Lemma 2.2[1] Suppose $g(s) = n(s)/d(s)$ is a proper rational stable function, $\deg(d(s)) = q$. Then $\|g\|_\infty < 1$ if and only if

i) $|n_q| < |d_q|$, where n_q, d_q are the coefficients of s^q in $n(s)$, $d(s)$ respectively.

ii) $d(s) + e^{j\theta} n(s)$ is stable for all $\theta \in [0, 2\pi]$.

Finally, the following lemma plays a crucial role in the proofs of our main results.

Lemma 2.3[7] suppose T is a linear transformation from R^n to R^m. A is a polytope in R^n, then TA is a polytope in R^m.

3 Main Results

Suppose a linear time-invariant SISO plant is given, its transfer function is $g(s)$. Consider the case when the plant is subjected to perturbation Δp without feedback loop (Fig. 1). In [6] there exist such a result.

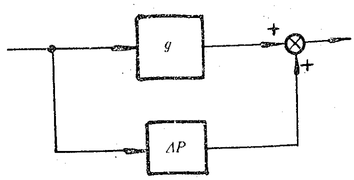

Fig. 1 Plant with unstructured uncertainties; without feedback

Lemma 3.1[6] Suppose $g(s)$ is a proper rational stable tansfer function, then for all perturbations Δp satisfying $\|\Delta p\|_\infty < \alpha$, the closed-loop in Fig. 1 remains stable if and only if

$$\alpha \leqslant \|\Delta g\|_\infty^{-1}. \tag{3.1}$$

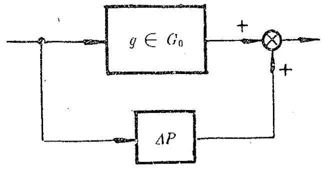

Fig. 2 Plant with both unstructured and structured uncertainties; without feedback

In general, there exist structured uncertainties in the plant itself. Suppose the structured uncertainties are reprensented as (2.3). Then we have this result:

Theorem 3.1 Given a stable proper plant G_0 (2.3), for all perturbations Δp satisfying $\|\Delta p\|_\infty < \alpha$, the closed loop in Fig. 2 remains stable if and only if

$$\alpha \leqslant 1/(\max_{g \in G_{0e}} \|g\|_\infty). \tag{3.2}$$

The above theorem treats the case when controllers are not taken into consideration, when a fixed stabilizing controller for G_0 is connected to the plant, we can get the following result.

Theorem 3.2 Given a strictly proper plant G_0 (2.3) (no necessarily stable) and its stabilizing controller c. For all perturbations Δp satisfying $\|\Delta p\|_\infty < \alpha$, the closed loop in Fig. 3 remains stable if and only if

$$\alpha \leqslant 1/\max_{g \in G_{0e}} \|c(s)(1 + g(s)c(s))^{-1}\|_\infty \tag{3.3}$$

Finally, we consider the case when the stabilizing controller is also under perturbations. We assume it has structured uncertainties described also by a family of polytopic matrices, i. e.,

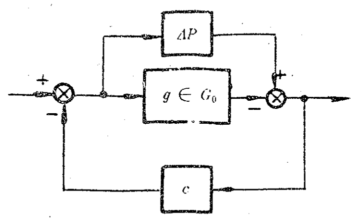

Fig. 3 Plant with both unstructured and structured uncertainties; with feedback

$$C_0 = \{c(s) = \frac{n_c(s)}{d_c(s)} : n_c(s) = n_{c0} + n_{c1}s + \cdots + n_{cp}s^{p'},$$
$$d_c(s) = d_{c0} + d_{c1}s + \cdots + d_{cq}s^{q'},$$
$$(n_{c0}, \cdots, n_{cp}, d_{c0}, \cdots, d_{cq})^{\mathrm{T}} \in B \subset R^{p'+q'+2},\ B \text{ is a polytope}\}. \tag{3.4}$$

We can also get the edge controller C_{oe} of C_o. We have:

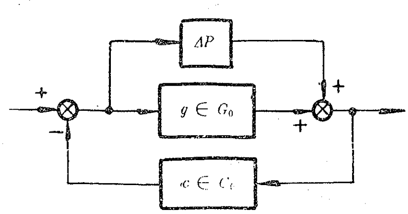

Fig. 4 Plant with both unstructured and structured uncertainties; controller with structured uncertainties

Theorem 3.3 Given a family of strictly proper plants G_0 (2.3) and a family of stabilizing controller C_0 (3.4). For all perturbations Δp satisfying $\|\Delta p\|_\infty < \alpha$, the closed loop in Fig. 4 remains stable if

$$\alpha \leqslant 1/\max_{\substack{g \in G_{0e} \\ c \in C_{0e}}} \|c(s)(1 + g(s)c(s))^{-1}\|_\infty. \tag{3.5}$$

Notice in theorem 3.3, only sufficient condition is given.

4 Conclusions

This paper considered the robust stability problem for plants with structured uncertainties under unstructured uncertainty perturbations from three different cases and obtained the robust stability margins, which makes it possible to compute the unstructured perturbation bound for a class of plants. The results are not only suitable in SISO case but also in multi-input single-output and single-input multi-output cases. Furthermore, we can consider the case when both the polytopic plants and the polytopic controllers are subjected to unstructured perturbations.

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Appendix:

A Proof of Theorem 3.1

By Lemma 2.2, $\max_{g\in G_0}\|g\|_\infty\leqslant\alpha^{-1}$ if and only if $d(s)+\alpha e^{j\theta}n(s)$ is stable for all $g\in G_0$ and $\theta\in[0,2\pi]$. Let

$$M_\theta=\{d(s)+\alpha e^{j\theta}n(s): g(s)=\frac{n(s)}{d(s)}\in G_0\} \text{ for all } \theta\in[0,2\pi]$$

and let T_θ be a linear transformation from $A=\begin{bmatrix}D\\N\end{bmatrix}$ to $D+\alpha e^{j\theta}N$. By Lemma 2.3 $T_\theta A$ is a polytope and its extremes are $T_\theta A_v$, where A_v is the set of extremes of A. Thus M_θ is a family of polytopic polynomials. It's easy to see that its edge polynomials are

$$M_{\theta e}=\{d(s)+\alpha e^{j\theta}n(s): g(s)=\frac{n(s)}{d(s)}\in G_{0e}\}.$$

By Lemma 2.1, M_θ is stable if and only if $M_{\theta e}$ is stable. So $\max_{g\in G_{0e}}\|g\|_\infty\leqslant\alpha^{-1}$ if and only if $\max_{g\in G_0}\|g\|_\infty\leqslant\alpha^{-1}$. And by Lemma 3.1 we can complete the proof.

B Proof of Theorem 3.2

Notice $c(s)(1+g(s)c(s))^{-1}=\frac{n_c(s)d(s)}{n(s)n_c(s)+d(s)d_c(s)}$, by Lemma 2.2, $\|c(s)(1+g(s)c(s))^{-1}\|_\infty<\alpha^{-1}$ for all $g(s)\in G_0$ is equivalent to that $n(s)n_c(s)+d(s)d_c(s)+\alpha e^{j\theta}n_c(s)d(s)$ is stable for all $\theta\in[0,2\pi]$ and $g(s)\in G_0$. Let

$$R_\theta=\{n_c(s)d(s)+d_c(s)+\alpha e^{j\theta}n_c(s)d(s): g(s)\in G_e\}.$$

Because $c(s)$ is fixed, the coefficients of the polynomials in R_θ can be obtained by the coefficients of the polynomals in G_0 by a linear transformation. By Lemma 2.3, R_θ is a family of polytopic polynomials, and its edge polynomials are

$$R_{\theta e}=\{n_c(s)d(s)+(d_c(s)+\alpha e^{j\theta}n_c(s))d(s): g(s)\in G_{0e}\}$$

So by Lemma 2.1, R_θ is stable if and only if $R_{\theta e}$ is stable. Therefore $\max_{g\in G_0}\|c(s)(1+g(s)c(s))^{-1}\|_\infty\leqslant\alpha^{-1}$ if and only if $\max_{g\in G_{0e}}\|c(s)(1+g(s)c(s))^{-1}\|_\infty\leqslant\alpha^{-1}$. Also by Lemma 3. 1, we complete the proof.

C Proof of Theorem 3. 3

Before we prove theorem 3. 3, we need a lemma. Its proof can be found in the appendix of [5].

Lemma A. 1[5] suppose $A\subset R^n$, $B\subset R^m$ are polytope. $f: A\times B\subset R^{n+m}\to R^s$ is a bilinear mapping. Let v be the set of all extremes of $A\times B$, then

$$\mathrm{conv}[f(A\times B)]=\mathrm{conv}[f(v)]$$

where conv[A] denotes the convex hull of A, $f(A)$ denotes $\{f(a): a\in A\}$.

Now we prove theorem 3. 3. Notice in the proof of theorem 3. 2, if $c(s)$ is not fixed, then R_θ is not necessarily a family of polytopic polynomials. But since C_0 is a family of polytopic matrices, it's easy to show by lemma 2. 2 that

$$C' = \left\{\frac{n_c(s)}{d_c(s)+\alpha e^{j\theta}n_c(s)}: \frac{n_c(s)}{d_c(s)}\in C_0\right\}$$

is a family of polytopic matrices ad its coefficient polytope is a linear transformation of the coefficient polytope of C_0. So by Lemma A. 1, conv[R_θ]=conv[$G_{0v}C_v$], that is to say, the convex hull of R_θ is a family of polytopic polynomials, and its edge can be determined by G_{0v} and C_v(or C_{0v}). The rest is similar to the proof of Theorem 3. 2 by substituting conv[R_θ] for R_θ.

一类结构不确定系统鲁棒稳定之非结构摄动边界

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摘要: 本文研究同时具有结构及非结构不确定性对象的鲁棒稳定性问题，目的在于得到一类对象的非结构摄动边界的计算方法. 本文结果是文献[1]的非平凡推广.

关键词: 结构不确定; 非结构不确定; 鲁棒稳定; 多面体对象; 边对象

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