第 11 卷第 1 期
1994 年 2 月

控制理论与应用
CONTROL THEORY AND APPLICATIONS

Vol. 11, No. 1
Feb., 1994

Variable Structure Robust Control for Uncertain MIMO Dynamic Systems*

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Abstract: A design scheme of variable structure control for uncertain MIMO systems which uses only the input and output measurements is presented in this paper. A strictly positive-real model is introduced into the control system. The state variable filters with proper logic switchings is used to perform the variable structure control. It is proved that a sliding mode control can be achieved and the global stability can be guaranteed. And the tracking error tends to zero exponentially. The design scheme is shown to be robust to structured and unstructured uncertainties. Simulation results show the effectiveness of the proposed algorithm.

Key words: variable structure control; robust control; uncertain dynamic system

1 Introduction

The variable structure control (VSC) was first studied in the 1960s[1] and was developed greatly in [5]. It has attracted the attention of many researchers in both theoretical and applied fields. Systematic studies showed that the variable structure control possesses good robustness [2~7]. It is well known that when the sliding mode is achieved the controlled systems are insensitive to the variations of system parameters and independent disturbances. That is why the variable structure control is robust. But usually all state variables should be used to achieve the sliding mode. If some state variables are inaccessible, theoretically we can use an observer to reconstruct all state variables and then use the latter variables to perform the sliding mode. However, system state variables can not be accurately reconstructed if the system parameters are not accurately known. That is to say, the sliding mode control via a state variable observer is sensitive to parameter variations and is therefore not robust. If there are also structure uncertainties besides the parametric uncertainties, the problem will be even more complicated. In general, it is not clear how to design sliding mode control for some systems with structured and unstructured uncertainties. Therefore, it is evident that the design of variable structure control for uncertain dynamic systems by using input and output measurements is a very attractive topic for research.

The application of VSC in model reference adaptive control (MRAC) has also been studied. The earlist work can be traced back to Young[6]. Later, Ambrosino, et al, designed a variable structure model reference system in the absence of unmodeled dynamics[7]. Balestrino, et al, also

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* This research was supported by the National Natural Science Foundation of P. R. China under Grant 68974009.
Manuscript received Mar. 9, 1993, revised Jul. 1, 1993.

applied the VSC to the adaptive control[8]. However, no substantial results on this subject have been obtained as yet, especialy in the presence of unmodeled dynamics. The other applications of VSC in self-tuning and adaptive control has also been studied in [4] and literatures cited therein.

In some recent papers[9~13], model reference adaptive system combined with variable structure control is introduced into the design of control system, and variable structure model reference adaptive control using only input and output measurements is also being considered. However, in papers [9~13], only the minimum-phase systems are considered because their design are based on the Narendra's control scheme. S. V. Emelyanov, et al, proposed a vairable structure control scheme for an uncertain system described by a state variable model[14]. However, in its design the state variable coefficient matrix of the modeled part of the plant is assumed to be known and to be of minimum phase.

There is no doubt that the robustness of feedback control systems can be improved by introducing the variable structure schemes with appropriate logic switchings.

In this paper, a design scheme of variable structure control which uses only the input and output measurements is presented. The controlled plants contain both structured and unstructured uncertainties, and the system can be of non-minimum phase. In our design, a strictly positive-real model is introduced into the control scheme. The state-variable filters with proper logic switchings are used to perform the variable structure contorl. It is proved that the sliding-mode control can be achieved and the global stability can be guaranteed. The results of simulation show the effectiveness of the proposed method.

The outline of this paper is as follows: In Section 2 the description of the system is given. In Section 3 the design scheme for multi-input and multi-output (MIMO) systems is discussed. Section 4 gives some results of simulation. Finally, in Section 5 some concluding remarks are given.

2 System Description

In this paper the controlled plant is described as

$$Y_0(t) = G(s,\alpha)U(t) = G_0(s,\alpha)[I + \mu_1\Delta_1(s,\alpha)]U(t) + W(s)\mu_2\Delta_2(s,\alpha)U(t), \qquad (1)$$

where $Y_0(t)=[y_{01}(t),y_{02}(t),\cdots,y_{op}(t)]^T$, $U(t)=[u_1(t),u_2(t),\cdots,u_q(t)]^T$, $G_0(s,\alpha)=D_0^{-1}(s,\alpha)N_0(s,\alpha)$ denotes the modeled part of the plant. In general, $D_0(s,\alpha)$ and $N_0(s,\alpha)$ are assumed to be $p\times p$ and $p\times q$ matrix polynomials in s for every α. $\Delta_1(s,\alpha)$ and $\Delta_2(s,\alpha)$ represent the multiplicative and additive unmodeled dynamic matrices with approparite dimension in the system. $\alpha\in\Omega$, Ω is a bounded region. μ_1 and μ_2 are positive constants. Eq. (1) describes a multivariable system with q inputs and p outputs. For this system we assume:

A1) $G_0(s,\alpha)$ is a strictly proper rational matrix in s for every $\alpha\in\Omega$.

$$G_0(s,\alpha) = D_0^{-1}(s,\alpha)N_0(s,\alpha),$$
$$D_0(s,\alpha) = I_p s^n + A_1(\alpha)s^{n-1} + \cdots + A_n(\alpha),$$
$$N_0(s,\alpha) = B_0(\alpha)s^{n-1} + B_1(\alpha)s^{n-2} + \cdots + B_{n-1}(\alpha),$$

where $A_i(\alpha)$'s, $B_i(\alpha)$'s are $p\times p$, $p\times q$ constant matrices for every $\alpha\in\Omega$, respectively.

A2) $\Delta_1(s,\alpha)$and $\Delta_2(s,\alpha)$ are stable and causal operators. If $\Delta_i(s,\alpha)$ is linear and time-invariant, then we assume that exists $\bar{a}_i>0$ such that $\Delta_i(s-\bar{a}_i,\alpha)$ is stable for every $\alpha\in\Omega$; If $\Delta_i(s,\alpha)$ is a nonlinear and time-varing operator, then we assume $\|\Delta_i(s,\alpha)U(t)\|\leqslant\sigma_i\|U(t)\|$ for every $\alpha\in\Omega(i=1,2,\sigma_i>0)$.

A3) $W(s)$ is a stable and strictly proper rational $p\times p$ matrix in s.

For simplicity, in the following discussion, we use the abbreviation G_0 for $G_0(s,\alpha)$, and similarly for others functions. The parameters of G_0 are unknown, but are uniformly bounded for all α. G_0 may contain unstable poles and unstable zeros. The purpose of the present paper is to design a variable structure controller by using only the input and output measurements such that the output of plant will be bounded and track a reference output $Y_m(t)$, where $Y_m(t)=W_r(s)\ r(t)$. Here $W_r(s)$ represents a stable reference model, and $r(t)$ is a uniformly bounded external input. In general, for system (1), the dimension of input is different from that of output ($p\neq q$). For simplicity of design we try to make the dimensions of input and output be equal. For this purpose the following measures will be adopted:

2.1 $p<q$.

In this case, (q−p) augmented outputs will be introduced. Let

$$y_{0j}(t)=((s+a_1)^{n-1}/(s+a)^n)u_j(t),\quad (j=p+1,p+2,\cdots,q.\ a>0,a_1>0),$$

$$\bar{Y}_0(t)=[y_{01}(t),\cdots,y_{0p}(t),y_{0,p+1}(t),\cdots,y_{0q}(t)]^{\mathrm{T}},$$

where $u_j(t)$ and $y_{oj}(t)$ are the jth components of $U(t)$ and $Y_0(t)$ respectively. When $j>p$, $y_{oj}(t)$ is the augmented output. Define

$$D_1^{-1}(s)=\mathrm{diag}[1/(s+a)^n,\cdots,1/(s+a)^n],\quad \bar{D}_0^{-1}(s,\alpha)=\mathrm{diag}[D_0^{-1}(s,\alpha),D_1^{-1}(s)],$$

$$N_1(s)=[0,\ n_1(s)I_1],\quad \bar{N}_0(s,\alpha)=[N_0^{\mathrm{T}},\ N_1^{\mathrm{T}}(s)]^{\mathrm{T}},$$

where I_1 is a $(q-p)\times(q-p)$ unit matrix, $n_1(s)=(s+a_1)_{n-1}$. From (1) we have

$$\begin{aligned}\bar{G}(s,\alpha)&=\mathrm{diag}[D_0^{-1}(s,\alpha),D_1^{-1}(s)][N_0^{\mathrm{T}}(s,\alpha),\ N_1^{\mathrm{T}}(s)]^{\mathrm{T}}[I+\mu_1\Delta_1(s,\alpha)]\\&\quad+[(\mu_2W(s)\Delta_2(s,\alpha)^{\mathrm{T}},\ (-D_1^{-1}(s)N_1(s)\mu_1\Delta_1(s,\alpha)^{\mathrm{T}}]^{\mathrm{T}}\\&=\bar{D}_0^{-1}\bar{N}_0(I+\mu_1\Delta_1)+\mu_2\bar{W}\bar{\Delta}_2,\end{aligned}$$

$$\bar{Y}_0(t)=\bar{G}_0(s,\alpha)U(t)=\bar{D}_0^{-1}\bar{N}_0[I+\mu_1\Delta_1]U(t)+\mu_2\bar{W}\bar{\Delta}_2U(t),\tag{2}$$

where

$$\bar{W}(s)=\mathrm{diag}[W(s),((s+a_1)^{n-1}/(s+a)^nI_1],\quad \bar{\Delta}_2=[\Delta_2^{\mathrm{T}}(s,\alpha),([0,I_1]\Delta_1(s,\alpha))^{\mathrm{T}}]^{\mathrm{T}}.$$

This shows that if $p<q$, the system (1) can be transformed into (2) with the same dimensions of input and output by introducing $(q-p)$ augmented outputs which are independent of the original outputs of the plant (1).

2.2 $p>q$

In this case, $(p-q)$ augmented input variables will be introduced, such as $u_{q+1}(t)$, $u_{q+2}(t),\cdots,u_p(t)$. Let $\bar{U}(t)=[u_1(t),\cdots,u_q(t),\cdots,u_p(t)]^{\mathrm{T}}$, we have

$$\begin{aligned}Y_0(t)&=D_0^{-1}[N_0,0][I+\mu_1\mathrm{diag}[\Delta_1,0]]\bar{U}(t)+\mu_2W[\Delta_2,0]\bar{U}(t)\\&\triangleq D_0^{-1}\bar{N}_0[I+\mu_1\bar{\Delta}_1]\bar{U}(t)+\mu_2W\bar{\Delta}_2\bar{U}(t),\end{aligned}\tag{3}$$

Now the dimension of the input of system (3) is the same as that of the output, and the transfer function matrix from $[u_{q+1}(t), \cdots, u_p(t)]^T$ to $Y_0(t)$ is a zero matrix. It is obvious that both the system (2) and (3) satisfy the assumptions A1)～A3).

Interactor matrices play an important role in parametrizing MIMO plants for the studies of MRAC[15,16]. Literature [17] proposed the concept of modified right interactor (MRI) matrix for plant parametrization. In paper [11], the MRI matrix of G_0 is assumed to be known. In addition, the method given in the literature [11] is valid only when the dimensions of input and output are equal to each other. In our design, less priori informations about the modeled part of the plant (1) are needed. For example, we do not need the knowledge of the MRI matrix of G_0.

Definition 2.1 For MIMO plant (1), in assumption A1), let $B_0(\alpha)=(b_{ij}^{(0)})$. If $|b_{ii}^{(0)}|>\sum_{j\neq i}^{q}|b_{ij}^{(0)}|$ for every $\alpha\in\Omega$ $(i=1,2,\cdots,p)$, then $B_0(\alpha)$ is said to be uniformly strictly row diagonally dominant matrix. $B_0(\alpha)$is called the gain matrix of the plant (1).

In the subsequent discussion, the following notations will be used.

1) $|\cdot|$ Absolute value of a scalar or a scalar function.

2) $\|\cdot\|$ Euclidean norm of a vector, or the norm of a stable rational operator

3) $[x]_{ij}$ Element of matrix X in the ith row and jth column.

4) $[h]_i$ The ith component of vector h.

5) $(b_{ij})_{p\times q}$ The $p\times q$ order matrix.

6) $\lambda(N)$ Root of polynomial $N(s)$ or the root of eigenpolynomial.

3 Design Scheme of VSC for MIMO System

In this section, we will introduce a strictly positive real rational matrix function $W_m(s)$. The main contribution of this section is that a VSC scheme is given with consideration of unmodeled dynamics. Moreover, our results are still valid when G_0 is a non-minimum phase system. In the following discussion, the designs will be given for two different cases: i) B_0 is a uniformly strictly row diagonally dominant. ii) B_0 is a positive or negative definite matrix.

i) B_0 is a Uniformly Strictly Row Diagonally Dominant

For the simplicity, in the sequal, only the case where $p<q$ is discussed. In this case, we assume that the sign of elements $b_{ii}^{(0)}$ of B_0 are known $(i=1,2,\cdots,p)$. The principle of the control scheme is shown in Fig. 1.

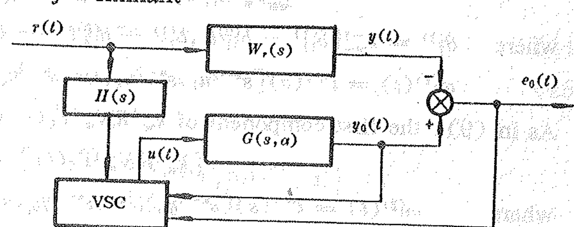

Fig. 1 The control scheme

Now $H(s)=W_m^{-1}(s)[W_r(s)^T, 0]^T$, where $W_m(s)=k_mN_m(s)/D_m(s)$ is a strictly positive real rational matrix function in s, and k_m is a positive constant. $D_m(s)$ and $N_m(s)$ are given by

$$D_m(s)=\mathrm{diag}[d_1(s), d_2(s), \cdots, d_q(s)], \tag{4}$$

$$N_m(s)=l(s)I, \tag{5}$$

where $d_1(s)$, $d_2(s)$, $\cdots$, $d_q(s)$ are monic stable polynomial whose degree are n; $l(s)$ is a monic stable polynomial whose degree is $(n-1)$; I is a $q\times q$ unit matrix. Denote $D_m(s)=Is^n+D_1s^{n-2}+\cdots+D_n$, and let $R(s)=D_m(s)-\overline{D}_0(s,\alpha)$, then $R(s)$ is a matrix polynomial with degree less than n. Define

$$\bar{r}(t)=H(s)r(t),\quad \overline{Y}_m(t)=[I_p,0]^{\mathrm{T}}Y_m(t),$$

then we have $\overline{Y}_m(t)=W_m(s)\bar{r}(t)$. The augmented tracking error is given by

$$\bar{e}(t)=\overline{Y}_0(t)-\overline{Y}_m(t). \tag{6}$$

It is easy to see that $e_0(t)=[I_p,0]\bar{e}(t)$. From $D_m=\overline{D}_0+R$, we have

$$\overline{Y}_0(t)=D_0^{-1}\overline{N}_0U(t)+D_m^{-1}R\overline{Y}_0(t)+\mu_1D_m^{-1}\overline{N}_0\Delta_1u(t)+\mu_2D_m^{-1}\overline{D}_0\overline{W}\Delta_2U(t). \tag{7}$$

Since $N_m(s)=l(s)I$, it is commutative when any rational matrix mutiplies it from left or right. Hence, Eq. (6) can be rewritten as follows:

$$\begin{aligned}\bar{e}_0(t)=&\ k_mD_m^{-1}N_m[k_m^{-1}\overline{N}_0N_m^{-1}U(t)+k_m^{-1}RN_m^{-1}\overline{Y}_0(t)-\bar{r}+\mu_1k_m^{-1}N_m^{-1}\overline{N}_0\Delta_1U(t)\\&+\mu_2k_m^{-1}N_m^{-1}\overline{D}_0\overline{W}\Delta_2U(t)].\end{aligned} \tag{8}$$

Let
$$\begin{aligned}&l(s)=s^{n-1}+l_1s^{n-2}+l_2s^{n-3}+\cdots+l_{n-1}\triangleq s^{n-1}+l^*(s),\\&\overline{N}_0(s,\alpha)=\overline{B}_0s^{n-1}+\overline{B}_1s^{n-1}+\cdots+B_{n-1},\\&\overline{B}_i=[b_{hj}^{(i)}]_{q\times q},\quad(i=1,2,\cdots,q),\end{aligned}$$

then $\overline{B}_0=[B_0^{\mathrm{T}},\ [0,\ I_1]^{\mathrm{T}}]^{\mathrm{T}}$. Hence, we obtain

$$\begin{aligned}[k_m^{-1}\overline{N}_0N_m^{-1}U(t)]_1=&\,k_m^{-1}b_{11}^{(0)}u_1+k_m^{-1}b_{12}^{(0)}u_2+\cdots+k_m^{-1}b_{1q}^{(0)}u_q\\&+k_m^{-1}b_{11}^{(0)}l^{-1}(s)(-l^*(s))u_1+\cdots+k_m^{-1}b_{1q}^{(0)}l^{-1}(s)(-l^*(s))u_q\\&+k_m^{-1}l^{-1}(s)\Big(\sum_{j=1}^{q}b_{1j}^{(1)}s^{n-2}u_j\Big)+k_m^{-1}l^{-1}(s)\Big(\sum_{j=1}^{q}b_{1j}^{(2)}s^{n-3}u_j\Big)\\&+\cdots+k_m^{-1}l^{-1}(s)\Big(\sum_{j=1}^{q}b_{1j}^{(n-1)}u_j\Big)\\=&\,k_m^{-1}b_{11}^{(0)}u_1+\cdots+k_m^{-1}b_{1q}^{(0)}u_q+k_m^{-1}\sum_{j=1}^{q}(b_{1j}^{(1)}-b_{1j}^{(0)}l_1)l^{-1}(s)s^{n-2}u_j\\&+\cdots+k_m^{-1}\sum_{j=1}^{q}(b_{1j}^{(n-1)}-b_{1j}^{(0)}l_{n-1})l^{-1}(s)u_j\\\triangleq&\,k_m^{-1}b_{11}^{(0)}u_1+\cdots+k_m^{-1}b_{1q}^{(0)}u_q+\theta_1^{(1)\mathrm{T}}\omega_1^{(1)}(t),\end{aligned} \tag{9}$$

where $\theta_1^{(1)}=k_m^{-1}[b_{11}^{(1)}-b_{11}^{(0)}l_1,b_{12}^{(1)}-b_{12}^{(0)}l_1,\cdots,b_{1q}^{(1)}-b_{1q}^{(0)}l_{n-1},\cdots,b_{1q}^{(n-1)}-b_{1q}^{(0)}l_{n-1}]^{\mathrm{T}}$,

$\omega_1^{(1)}(t)=l^{-1}(s)[s^{n-2}u_1,s^{n-2}u_2,\cdots,s^{n-2}u_q,s^{n-3}u_1,\cdots,s^{n-3}u_q,\cdots,u_1,\cdots,u_q]^{\mathrm{T}}$.

As in (9), the first component of $k_m^{-1}RN_m^{-1}\overline{Y}_0(t)$ is denoted by

$$[k_m^{-1}RN_m^{-1}\overline{Y}_0(t)]_1=\theta_1^{(2)\mathrm{T}}\omega_1^{(2)}(t),$$

where $\omega_1^{(2)}(t)=l^{-1}(s)[s^{n-1}y_{01},\cdots,s^{n-1}y_{0q},s^{n-2}y_{01},\cdots,s^{n-2}y_{0q},\cdots,y_{01},\cdots,y_{0q}]^{\mathrm{T}}$.

In Eq. (8), let $\bar{r}_1$ denotes the first component of $\bar{r}(t)$, and let

$$\begin{aligned}&[\mu_1k_m^{-1}N_m^{-1}\overline{N}_0\Delta_1U(t)]_1\triangleq\mu_1k_m^{-1}l^{-1}(s)(\Delta_1^{(11)}u_1+\cdots+\Delta_1^{(1q)}u_q),\\&[\mu_2k_m^{-1}N_m^{-1}\overline{D}_0\overline{W}\Delta_2U(t)]_1\triangleq\mu_2k_m^{-1}l^{-1}(s)(\Delta_2^{(11)}u_1+\cdots+\Delta_2^{(1q)}u_q),\end{aligned}$$

where $\Delta_1^{(1j)}=[\overline{N}_0\Delta_1]_{1j}$, $\Delta_2^{(1j)}=[\overline{D}_0\overline{W}\Delta_2]_{1j}$, then it is obvious that the $l^{-1}(s)\Delta_1^{(1j)}$ and $l^{-1}(s)\Delta_2^{(1j)}$ are causal operators. Define

$$\theta_1^{\mathrm{T}}\triangleq[\theta_1^{(1)\mathrm{T}},\theta_1^{(2)},1]^{\mathrm{T}},\quad\omega_1^{\mathrm{T}}\triangleq[\omega_1^{(1)\mathrm{T}},\omega_1^{(2)},\bar{r}_1]^{\mathrm{T}},$$

then in Eq. (8), the first component of $\bar{e}_0(t)$ can be written by

$$\begin{aligned} e_{01} &= k_m^{-1}d_1^{-1}(s)l(s)[k_m^{-1}b_{11}^{(0)}u_1 + \cdots + k_m^{-1}b_{1q}^{(0)}u_q + \theta_1^{\mathrm{T}}\omega_1 - \bar{r}_1 \\ &\quad + \mu_1 k_m^{-1}l^{-1}(s)(\Delta_1^{(11)}u_1 + \Delta_1^{(1q)}u_q) + \mu_2 k_m^{-1}l^{-1}(s)(\Delta_2^{(11)}u_1 + \cdots + \Delta_2^{(1q)}u_q)] \\ &\triangleq k_m^{-1}d_1^{-1}(s)l(s)u_1^*, \end{aligned} \tag{10}$$

In general, for $1 \leqslant i \leqslant p$, we have

$$\begin{aligned} e_{0i} &= k_m^{-1}d_i^{-1}(s)l(s)[k_m^{-1}b_{ii}^{(0)}u_i + k_m^{-1}b_{ii}^{(0)}u + \cdots + k_m^{-1}b_{i,i-1}^{(0)}u_{i-1} + k_m^{-1}b_{i,i+1}^{(0)}u_{i+1} \\ &\quad + \cdots + k_m^{-1}b_{iq}^{(0)}u_q + \mu_1 k_m^{-1}l^{-1}(s)(\Delta_1^{(i1)}u_1 + \cdots + \Delta_1^{(iq)}u_q) - \bar{r}_i \\ &\quad + \mu_2 k_m^{-1}l^{-1}(s)(\Delta_2^{(i1)}u_1 + \cdots + \Delta_2^{(iq)}u_q) + \theta_i^{\mathrm{T}}\omega_i] \\ &\triangleq k_m^{-1}d_1^{-1}(s)l(s)u_1^*, \end{aligned} \tag{11}$$

where

$$\begin{aligned} u_i^* &\triangleq (k_m^{-1}b_{ii}^{(0)}u_i + k_m^{-1}b_{ii}^{(0)}u_1 + \cdots + k_m^{-1}b_{i,i-1}^{(0)}u_{i-1} + k_m^{-1}b_{i,i+1}^{(0)}u_{i+1} + \cdots + k_m^{-1}b_{iq}^{(0)}u_q \\ &\quad + \mu_1 k_m^{-1}l^{-1}(s)(\Delta_1^{(i1)}u_1 + \Delta_1^{(iq)}u_q) + \mu_2 k_m^{-1}l^{-1}(s)(\Delta_2^{(i1)}u_1 + \cdots + \Delta_2^{(iq)}u_q) + \theta_i^{\mathrm{T}}\omega_i - \bar{r}_i, \end{aligned}$$

$\omega_i^{(1)} = \omega_1^{(1)}$, $\omega_i^{(2)} = \omega_1^{(2)}$, $\omega_i = [\omega_i^{(1)}, \omega_i^{(2)}, \bar{r}_i]^{\mathrm{T}}$. And when $p < i \leqslant q$, $b_{ii}^{(0)} = 1$ and $b_{ij}^{(0)} = 0$. The state-space description of (11) can now be written as follows:

$$\begin{cases} \dot{z}_i = A_i^* z_i + b_i^* u_i^*, \\ e_{0i} = c_i^{*\mathrm{T}} z_i. \end{cases} \tag{12}$$

let $A \triangleq \mathrm{diag}(A_1^*, A_2^*, \cdots, A_q^*)$, $B \triangleq \mathrm{diag}(b_1^*, b_2^*, \cdots, b_q^*)$, and $C \triangleq \mathrm{diag}(c_1^*, c_2^*, \cdots, c_q^*)$, then the minimum state-space realization of the augmented tracking error model (8) is given by

$$\begin{cases} \dot{z} = Az + Bu^*, \\ \bar{e}_0 = C^{\mathrm{T}} z, \end{cases} \tag{13}$$

where $u^*(t) = [u_1^*(t), u_2^*(t), \cdots, u_q^*(t)]^{\mathrm{T}}$, $z(t) = [z_1(t), z_2(t), \cdots, z_q(t)]^{\mathrm{T}}$. Since $l(s)d_i^{-1}(s)$ is a strictly positive real transfer function, there exist positive definite matrix G_i and Q_i such that

$$A_i^{*\mathrm{T}} G_i + G_i A_i^* = -Q_i, \quad G_i b_i^* = c_i^*.$$

Let $G = \mathrm{diag}(G_1, G_2, \cdots, G_q)$, $Q = \mathrm{diag}(Q_1, Q_2, \cdots, Q_q)$, then we have

$$A^{\mathrm{T}} G + GA = -Q, \quad GB = C.$$

For Eq. (13), taking liapunov function

$$V(z) = (1/2) z^{\mathrm{T}} G z, \tag{14}$$

we obtain

$$\dot{V}(z)|_{(13)} = -z^{\mathrm{T}} Q z + \bar{e}_0^{\mathrm{T}} u^*. \tag{15}$$

The variable structure control law is given by

$$\begin{aligned} u_i(t) &= -\mathrm{sgn}(e_{0i}(t)b_{ii}^{(0)})\Big(\sum_{j=1}^{(n-1)q} k_j^{(1)}|\omega_i^{(1j)}| + \sum_{j=1}^{nq} k_j^{(2)}|\omega_i^{(2j)}| + k_0|r_i|\Big) \\ &\quad - k\,\mathrm{sgn}(e_{0i}(t)b_{ii}^{(0)}) - \mathrm{sgn}(e_{0i}(t)b_{ii}^{(0)})(M_{11}m_1(t) + M_{12}m_2(t) + M_{13}), \end{aligned} \tag{16}$$

where k, $k_j^{(1)}$, $k_j^{(2)}$, M_{11}, M_{12}, and M_{12} are some appropriate positive constants; $\omega_i^{(1j)} = [\omega_i^{(1)}]_j$, $\omega_i^{(2j)} = [\omega_i^{(2)}]_j$, $m_1(t)$ and $m_2(t)$ are determined by the following equation:

$$\dot{m}_h(t) = -\beta_{0h} m_h(t) + \beta_{1h}|u_1(t)| + \beta_{2h}|u_2(t)| + \cdots + \beta_{qh}|u_q(t)| + \beta_h, \tag{17}$$

where $\beta_{0h} < \min\{a_h, \lambda_0\}$, $\lambda_0 = \min|\mathrm{Re}\lambda(l(s))|$; β_h and β_{jh} are some positive constants ($h = 1, 2$; $j = 1, 2, \cdots, q$). Let

$$v_i^{(1)}(t) \triangleq |l^{-1}(s)\sum_{j=1}^{q}\Delta_1^{(ij)}u_j)|, \quad v_i^{(2)}(t) \triangleq |l^{-1}(s)\sum_{j=1}^{q}\Delta_2^{(ij)}u_j)|.$$

From the assumption A2) in Section 2, when the parameters β_h and β_{jh} are appropriately chosen, there exist $\mu_1^*>0$ and $\mu_2^*>0$, for $\mu_1\in(0,\mu_1^*)$ and $\mu_2\in(0,\mu_2^*)$, such that

$$\mu_1 k_m^{-1}v_i^{(1)}(t) \leqslant \overline{M}_{11}m_1(t)+\overline{M}_{13}+\mu_1\bar{\delta}_1(|u_1|+|u_2|+\cdots+|u_q|),$$

$$\mu_2 k_m^{-1}v_i^{(2)}(t) \leqslant \overline{M}_{12}m_2(t)+\overline{M}_{13}+\mu_2\bar{\delta}_2(|u_1|+|u_2|+\cdots+|u_q|).$$

Since $\omega_1^{(1)}=\omega_2^{(1)}=\cdots=\omega_q^{(1)}$, and $\omega_1^{(2)}=\omega_2^{(2)}=\cdots=\omega_q^{(2)}$, we obtain

$$|u_j| \leqslant |u_i|+k_0|\bar{r}_i|+k_0|\bar{r}_j|.$$

According to the above analysis, if the parameters in the control law (16) and (17) are chosen appropriately, we can prove that

$$e_{0i}u_i^* < -\rho_i|e_{0i}|, \tag{18}$$

and thus

$$\bar{e}_0^{\mathrm{T}}u^* < -\rho_1|e_{01}|-,\cdots,-\rho_q|e_{0q}|, \tag{19}$$

where $\rho_i's$ are positive constants. Therefore, the following result is obtained:

$$\dot{V}(z)|_{(13)} \leqslant -z^{\mathrm{T}}Qz-\sum_{j=1}^{q}\rho_j|e_{0i}|. \tag{20}$$

From (20) and (14) it is evident that $\|z(t)\|$ decreases at least exponentially. Therefore, from (13) and (6), $\|\bar{e}_0(t)\|$ and $\|e_0(t)\|$ also tends to zero exponentially. Moreover, the exponential stability of $\|z(t)\|$ and $\|e_0(t)\|$ are independent of the external excitation $r(t)$. Owing to the strict positive realness of $d_i^{-1}(s)l(s)$ we have $c_i^{*\mathrm{T}}b_i^*>0$. Then from (18) we have

$$(1/2)\frac{\mathrm{d}}{\mathrm{d}t}(e_{0i}^2(t)) = e_{0i}(t)(c_i^{*\mathrm{T}}A_iz_i+c_i^{*\mathrm{T}}b_i^*u_i^*)$$

$$\leqslant |e_{0i}(t)|\,\|c_i^*A_i\|\,\|z_i(t)\|+c_i^{*\mathrm{T}}b_i^*e_{0i}(t)u_i^* \leqslant (\varepsilon(t)-\rho_ic_i^{*\mathrm{T}}b_i^*)|e_{0i}|, \tag{21}$$

where $\varepsilon(t)$ tends to zero exponentially. Hence, there exists $\tau>0$ such that

$$(1/2)\frac{\mathrm{d}}{\mathrm{d}t}(e_{0i}^2(t)) < -\rho_i^*|e_{0i}(t)|, \tag{22}$$

for all $t>\tau(i=1,2,\cdots,q)$. Therefore, from (22) we can see that the sliding surface $\bar{e}_0=C^{\mathrm{T}}z=0$ is guaranteed to be reached in $t>\tau$.

When the sliding mode is achieved, the equivalent control can be calculated by setting $\bar{e}_0(t)=0$ in (13). However, in practice, the equivalent control can not measured on-line due to the structured and unstructured uncertainties of plant (1). In implementation, the average control obtained by using a low-pass filter (such as, for example, $u_{iav}(t)=(1/(\tau_is+1))u_i(t)$ with sufficiently small time constant τ_i may be used instead of the equivalent control on the switcing surface.

From $e_0(t)=Y_0(t)-Y_m(t)$, and the uniform boundedness of $Y_m(t)$, we obtain

$$\|Y_0(t)\| \leqslant \|e_0(t)\|+\|Y_m(t)\| \leqslant M. \tag{23}$$

In conclusion, the global stability of VSC system with control signal $u(t)$ defined by (16) and (17) can be proved by the Liapunov function method.

The above results obtained for MIMO systems now can be summarized in the following theorem:

Theroem 3. 1 For the MIMO system (1), if the dimension of input q is greater than that of output and the assumptions A1)～A3) are satisfied, and the gain matrix B_0 of the plant is strictly row diagonally dominant and the sign of the elements $b_{ii}^{(0)}$ of B_0 is known, then there exist $\mu_1^*>0$ and $\mu_2^*>0$, for $\mu_1\in(0, \mu_1^*)$ and $\mu_2\in(0, \mu_2^*)$, such that the closed-loop system is globally asymptotically stable and the sliding mode control is achieved when the VSC law $U(t)$ with appropriately selected parameters is given by (16) and (17) and the strictly positive real model $W_m(s)$ is taken appropriately.

ii) B_0 is a Positive Definite Matrix ($p=q$) (or Negative Definite Matrix)

In (1), let $\overline{D}_0(s)=B_0^{-1}D_0(s)$, $N_0(s)=B_0^{-1}N_0(s)$, then (1) can be rewritten in the following form:

$$Y_0(t) = \overline{D}_0^{-1}(s)\overline{N}_0(s)(I+\mu_1\Delta_1)\overline{U}(t) + W\mu_2\Delta_2 U(t), \tag{24}$$

Let $D_m(s)=d(s)I$, $N_m(s)=l(s)I$, and $R(s)=B_0^{-1}D_m(s)-\overline{D}_0(s)$, then (6) can be given by

$$e_0(t) = k_m(B_0^{-1}D_m)^{-1}N_m[k_m^{-1}\overline{N}_0N_m^{-1}U(t) + k_m^{-1}RN_m^{-1}Y_0(t) - \bar{r} + \mu_1k_m^{-1}N_m^{-1}\overline{N}_0\Delta_1U(t) + \mu_2k_m^{-1}N_m^{-1}\overline{D}_0W\Delta_2U(t)], \tag{25}$$

where $d(s)^{-1}l(s)$ is a strictly positive real transfer function. Since B_0^{-1} is a positive definite matrix, $k_m(B_0^{-1}D_m(s))N_m(s)$ is a strictly positive real tranfer matrix. The following discussion is similar with the case i) since $\overline{N}_0(s)=B_0^{-1}N_0(s)$. We omit the manipulation and give the following theorem.

Theroem 3. 2 For the MIMO system (1), if the dimension of input is the same as that of output and the assumptions A1)～A3) are satisfied, and the gain matrix B_0 of the plant is a positive or negative definite matrix, then there exist $\mu_1^*>0$ and $\mu_2^*>0$, for $\mu_1\in(0, \mu_1^*)$ and $\mu_2\in(0, \mu_2^*)$, such that the closed-loop system is globally asymptotically stable and the sliding mode control is achieved when the VSC law $U(t)$ with appropriately selected parameters is given by (16) and (17) and the strictly positive real model $W_m(s)$ is taken appropriately.

4 Simulation Results

In this section the performance of the proposed VSC scheme will be illustrated by simulation example.

Example An MIMO system with B_0 to be strictly row diagonally dominant is considered.

In the plant (1), let

$$D_0(s) = \begin{bmatrix} s^2+1.9s-0.2 & s+1 \\ 0 & s^2+0.8s-0.2 \end{bmatrix}, \quad N_0(s) = \begin{bmatrix} s+1 & 0.1s+1 \\ 0.2s+2 & s+3 \end{bmatrix},$$

$$\Delta_1(s) = \begin{bmatrix} 1/(s+\alpha_1) & 1/(s+9) \\ 1/(s+11) & 1/(s+\alpha_1) \end{bmatrix}, \quad \Delta_2(s) = \begin{bmatrix} 1/(s+\alpha_2) & 1/(s+7) \\ 1/(s+12) & 1/(s+\alpha_2) \end{bmatrix},$$

$W(s)=\mathrm{diag}[1/(s+3), 2/(s+5)]$, $\mu_1=\mu_2=0.2$, $\alpha_1=\alpha_2=20$. We select the comparison model as $W_m(s)=(s+3)/(s^2+3s+2))I$. The external input signal is taken as $r(t)=[(1/2)\sin(5t), (1/5)\cos(2t)]^T$. In the control law (16) and (17), the constant parameters are chosen to be $k_1^{(1)}=4$, $k_2^{(1)}=5$, $k_1^{(2)}=k_2^{(2)}=3.2$, $k_3^{(2)}=k_4^{(2)}=3$, $k=4$, $M_{11}=M_{12}=2.5$, $M_{13}=2$, $k_0=3$. We take $m_i(t)=m(t)$ $(i=1,2)$, here $m(t)$ is given by the following equation:

$$\dot{m}(t) = -2.9m(t) + 3|u(t)| + 5.$$

Fig. 2, Fig. 3, Fig. 4 and Fig. 5 show the tracking error and the variable structrue control signals.

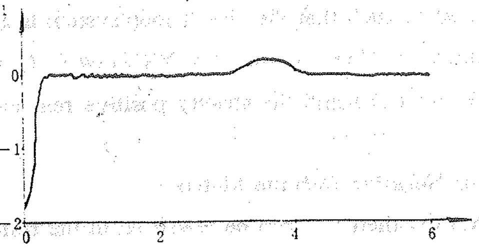

Fig. 2 The tracking error $e_{01}(t)$

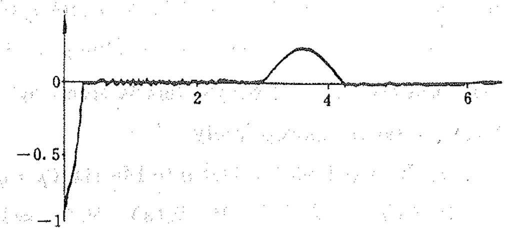

Fig. 3 The tracking error $e_{02}(t)$

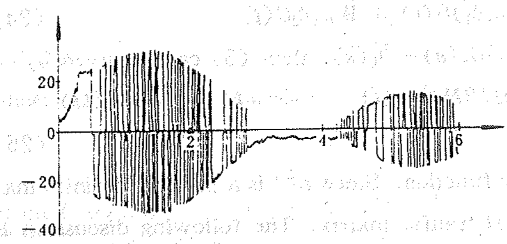

Fig. 4 The control signal $u_1(t)$

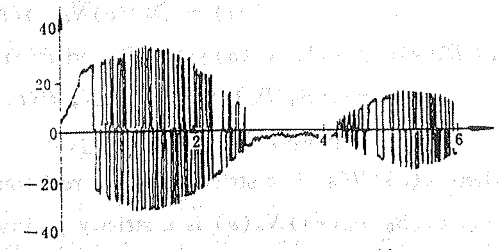

Fig. 5 The control signal $u_2(t)$

5 Conclusions

This paper presents a method for designing a variable structure controller by using only the input and output measurements of the plant for MIMO systems. Both structured and unstructrued uncertainties are considered. The modeled part of the system may contain unstable zeros and unstable poles. Therefore, the systems to be considered are very general. When the gain matrix B_0 is strictly row diagonally dominant or positive definite our design can make the whole system globally asymptotically stable. The defect of our proposed method is that the chattering phenomenon will exist, which is inevitable in sliding mode control. How to reduce or eliminate this chattering is now under study. We conclude that if the adaptive control is taken in the sufficiently small areas of the sliding surface the chattering can be reduced.

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动态不确定多输入多输出系统的变结构鲁棒控制

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摘要: 本文仅利用输入输出数据给出了一种动态不确定多输入多输出系统的变结构控制器设计机制. 控制系统的设计中引入了一个严格正实的模型. 应用适当逻辑切换的状态变量滤波器实现变结构控制. 证明了所提方案能保证全局输出有界稳定并能实现滑模控制和渐近跟踪参考模型的输出. 该设计方案对结构不确定性具有一定的鲁棒性. 仿真结果表明了所提方案的有效性.

关键词: 变结构控制; 鲁棒控制; 动态不确定系统

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