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The Transient Solution of the Repairable Queueing System $M^X/G(M/H)/1$*

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Abstract: This paper is concerned with the Repairable Queueing Systems (RQS), the $M^X/G(M/H)/1$ is typical bulk-arrival RQS. The notatin (M/H) represents that the server lifetime is exponentially distributed, while its repair time has a general continuous distribution. By using the method of vector Markov process, its transient solution is easily obtained. Especially, the reliability indices of the server are only dependent on the idle probability of the RQS, or equivalently, on the busy period and busy cycle.

Key words: queueing theory; reliability theory; repairable queueing system; vector Markov process (method)

1 Model Definition

The model $M^X/G(M/H)/1$ is parameterized by

1) The bulk-arrival process is a Poisson process of the rate λ, and the probabilities of the batch size are $P(\gamma=m)=a_m$, $m=1,2,\cdots$, with the finite mean a;

2) The service time distribution, $G(t)$, with the finite mean μ^{-1} and the service rate $\mu(t)=g(t)/\overline{G}(t)$ where $\overline{G}(t)=1-G(t)$;

3) The life distribution of the server is an exponential, with the failure rate α;

4) The repair time distribution of the server, $H(t)$, with the finite mean β^{-1} and the repair rate $\beta(t)=h(t)/\overline{H}(t)$.

The lifetime and the repair time of the server and the service time and the arrival process of the customers are all assumed to be mutually independent.

2 Vector Markov Process

System states are defined as follows:

1) The state (0) represents that there is no customer in the system;

2) The state (n,x) $n=1,2,\cdots,0\leqslant x<\infty$, represents that there are n customers in the system where a customer is being served and the elapsed service time is x, but others are waiting for their service;

3) The state (n,x,y), $n=1,2,\cdots,0\leqslant x,y<\infty$, represents that there are n customers in

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the system and x's value keeps till the state changed, and y is the elapsed repair time of the failed server.

The transitions among the system states are shown as Fig. 1.

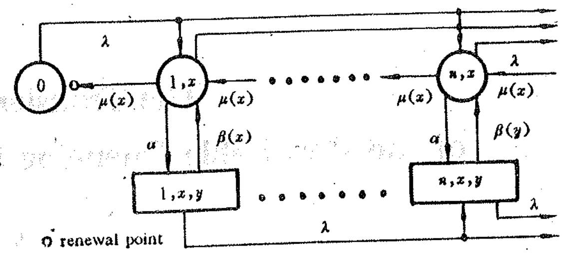

Fig. 1 State-transition-rate diagram for $M^X/G(M/H)/1$

Let

$$p_0(t) = P[S(t) = (0)],$$
$$p_n(t,x)\mathrm{d}x = P[S(t) = (n), x \leqslant X(t) < x + \mathrm{d}x],$$
$$q_n(t,x,y)\mathrm{d}y = P[S(t) = (n), X(t) = x, y \leqslant Y(t) < Y + \mathrm{d}y]$$

where $n=1,2,\cdots$. Thus the evolution of the vector Markov process $[S(t), X(t), Y(t)]$ can be determined by

1) The set of differential-integral equations

$$\left[\frac{\mathrm{d}}{\mathrm{d}t} + \lambda\right] p_0(t) = \int_0^\infty p_1(t,x)\mu(x)\mathrm{d}x, \tag{1}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda + \alpha + \mu(x)\right] p_n(t,x) = \lambda\sum_{j=1}^{n} a_j p_{n-j}(t,x) + \int_0^\infty q_n(t,x,y)\beta(y)\mathrm{d}y, \tag{2}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \lambda + \beta(y)\right] q_n(t,x,y) = \lambda\sum_{j=1}^{n} a_j q_{n-j}(t,x,y) \tag{3}$$

where $p_0(t,x)=0$, $q_0(t,x,y)=0$, $n=1,2,\cdots$.

2) The boundary conditions

$$p_n(t,0) = \lambda a_n p_0(t)u(t) + u(t)\int_0^\infty p_{n+1}(t,x)\mu(x)\mathrm{d}x + \delta_{ni}\delta(t) \tag{4}$$

where $u(t)=\begin{cases}1, & t>0,\\ 0, & t=0,\end{cases}$ and $\delta(x)$ is the Dirac delta function.

$$q_n(t,x,0) = \alpha p_n(t,x), \quad n = 1,2,\cdots. \tag{5}$$

3) The initial conditions

$$p_0(0) = \delta_{0i}, \quad p_n(0,x) = \delta_{ni}\delta(x), \quad n = 1,2\cdots, \text{ where } \delta_{ij} = \begin{cases}1, & i = j,\\ 0, & i \neq j.\end{cases}$$

3 Solving the Set of Equations

Let

$$A(z) = \sum_{n=1}^{\infty} a_n z^n, \quad P(t,x,z) = \sum_{n=1}^{\infty} p_n(t,x)z^n, \quad Q(t,x,y,z) = \sum_{n=1}^{\infty} q_n(t,x,y)z^n,$$

$$f^*(s) = \int_0^\infty e^{-st}f(t)\mathrm{d}t, \quad f(s) = \int_0^\infty e^{-st}\mathrm{d}F(t).$$

By taking Laplace transform and z transform for equations (3) and (5), we get

$$Q^*(s,x,y,z) = \alpha P^*(s,x,z)e^{-\gamma y}\overline{H}(y) \tag{6}$$

where

$$\gamma = s + \lambda - \lambda A(z).$$

Similarly, from equation (2), we have

$$P^*(s,x,z) = P^*(s,o,z)e^{-\nu x}\overline{G}(x) \tag{7}$$

where $$\nu = \gamma + a - ah^*(\gamma).$$

In order to determine $P^*(s,o,z)$, from equations (1) and (4), we obtained

$$P^*(s,o,z) = \frac{z\{z^i - (s+\lambda-\lambda A(z)]p_0^*(s)\}}{z - g^*(\nu)}. \tag{8}$$

By Rouche's theorem, $z - g^*(\nu)$ has exactly one zero $\tilde{b}(s)$ inside unit circle $|z|=1$, and $\tilde{b}(s)$ is also a zero of the numerator in (8). Therefore, we get

$$p_0^*(s) = \tilde{b}^i(s)[s+\lambda-\lambda A(\tilde{b}(s))]^{-1} \tag{9}$$

and

$$P^*(s,z) = \int_0^\infty P^*(s,x,z)\mathrm{d}x = z\overline{G}^*(\nu)[z^i - \nu p_0^*(s)]/[z - g^*(\nu)], \tag{10}$$

$$Q^*(s,z) = \int_0^\infty\int_0^\infty Q^*(s,x,y,z)\mathrm{d}x\mathrm{d}y = a\overline{H}^*(\gamma)P^*(s,z), \tag{11}$$

$$P^*(s) = \lim_{z\to 1-} P^*(s,z) = [1 - sp_0^*(s)]/[s + a - ah^*(s)], \tag{12}$$

$$Q^*(s) = \lim_{z\to 1-} Q^*(s,z) = a\overline{H}^*(s)P^*(s). \tag{13}$$

4 Some Characters of the System

Theorem 1 The $M^X/G(M/H)/1$ queue corresponds to the $M^X/\tilde{G}/1$ queue where

$$\tilde{g}^*(s) = g^*[s + a - ah^*(s)], \tag{14}$$

Proof By Ref. [1].

Theorem 2 The system is stable if and only if

$$\rho = \frac{\lambda\alpha}{\mu}\left(1 + \frac{a}{\beta}\right) < 1. \tag{15}$$

Proof By Ref. [2].

Theorem 3 The Laplace transform of the system renewal (i. e. the system becomes empty) density

$$m^*(s) = (s+\lambda)p_0^*(s) - \delta_{0i}.$$

Proof By using the general formula of Ref. [3], and noting Fig. 1 and equation (1), we have

$$m^*(s) = \int_0^\infty p_1^*(s,x)\mu(x)\mathrm{d}x = (s+\lambda)p_0^*(s) - \delta_{0i}.$$

Corollary 1 The Laplace-Stieltjes transform of the renewal time distribution of the system

$$c(s) = \lambda A[\tilde{b}(s)]/(s+\lambda).$$

Proof In the general, the system becomes empty forms a delay renewal process. When $i=0$ in the initial conditions, it forms a renewal process. In this case,

$$p_0^*(s) = [s+\lambda-\lambda A(\tilde{b}(s))]^{-1}.$$

Hence, the conclusion comes immediately from $c(s) = m^*(s)/[1+m^*(s)]$.

5 The Quantitative Indices of the Queue

Theorem 4 The L and Z transforms of the queue length distribution at arbitrary time is

$$L^*(s,z) = p_0^*(s) + p^*(s,z) + Q^*(s,z).$$

Proof Obvious.

Corollary 2 When $\rho<1$, the stationary distribution of the queue length exists and its z transform

$$L(z)=\lim_{s\to 0+} sL^*(s,z)=\frac{(z-1)(1-\rho)g^*(\kappa)}{z-g^*(\kappa)}$$

where
$$\kappa=\lambda-\lambda A(z)+\alpha-\alpha h^*[\lambda-\lambda A(z)].$$

Corollary 3 When $\rho<1$, the mean queue length of the system

$$\bar{L}=\lim_{z\to 1-} L'(z)=\rho+\lim_{z\to 1-}[g^*(\kappa)]''/2(1-\rho)$$

where

$$\lim_{z\to 1-}[g^*(\kappa)]''=\lambda^2a^2\left(1+\frac{\alpha}{\beta}\right)^2\sigma_G^2+A''(1)\rho/a+\lambda^2a^2\alpha(\sigma_H^2+1/\beta^2)/\mu+\rho^2.$$

Theorem 5 The LS transform of the busy period distribution is

$$d(s)=A[\tilde{b}(s)]$$

and when $\rho\leqslant 1$, it is a proper distribution.

Proof Let $D_n(n=1,2,\cdots)$ be the busy period random variable started with n customers. Denote by $d_n(s)$ $(n=1,2,\cdots)$ the LS transform of D_n. The busy period D is defined to be the time from server's beginning service until the queue empty. It is obvious that

$$D=\sum_{n=1}^{\infty}a_nD_n,\quad d_n(s)=[d_1(s)]^n.$$

Thus the LS transform of D is given by

$$d(s)=\sum_{n=1}^{\infty}a_nd_n(s)=A[d_1(s)].$$

Next, D_1 may be regarded as the busy period of the $M/\tilde{G}/1$ queue. It is well known that $d_1(s)=\tilde{b}(s)$. Hence

$$d(s)=A[\tilde{b}(s)].$$

As to that $P\{D\leqslant t\}$ is a proper distribution, it directly follows from the following fact, i. e. when $\rho\leqslant 1$,

$$\lim_{s\to 0+}\tilde{b}(s)=1.$$

Corollary 4 When $\rho<1$, the mean busy period

$$\bar{D}=\lim_{s\to 0+}-(A[\tilde{b}(s)])'=\frac{a}{\mu}\left(1+\frac{\alpha}{\beta}\right)\left[1-\frac{\lambda a}{\mu}\left(1+\frac{\alpha}{\beta}\right)\right]^{-1}.$$

Theorem 6 When $\rho<1$, the LS transform of the idle cycle (i. e the system begins from the idle period and again returns to the idle period through the busy period) is

$$c(s)=\frac{\lambda}{s+\lambda}A[\tilde{b}(s)].$$

Proof Let C and I be the idle cycle and the idle period of the RQS $M^X/G(M/H)/1$, respectively. We have $C=I+D$ and I and D are mutually independent. Since the distributions of I and D are $1-\exp(-\lambda t)$ and $A[\tilde{b}(s)]$, respectively, taking LST of $C=I+D$, we have

$$c(s)=\frac{\lambda}{s+\lambda}A[\tilde{b}(s)].$$

This result consists with one of Corollary 1, hence the LST of the busy cycle also is $c(s)$.

Theorem 7 The L and LS transforms of the virtual waiting time distribution is

$$w^*(s,\theta)=\frac{[\tilde{g}^*(\theta)]^i-\theta p_0^*(s)}{s+\lambda-\lambda A[\tilde{g}^*(\theta)]-\theta}\cdot\frac{1-A[\tilde{g}^*(\theta)]}{a[1-\tilde{g}^*(\theta)]}.$$

Proof Let $W(t)$ represent the virtual waiting time for an arbitrary (test) customer in an arriving batch. Then $W(t)=W_1(t)+W_2$, where $W_1(t)$ is the waiting time of the first customer in the test customer's batch, and W_2 is the waiting time for the service of the batch-mates who are served before the test customer under consideration.

Denote by $W_1(t,x)$ the distribution of $W_1(t)$, i.e. $W_1(t,x)=P[W_1(t)\leqslant x]$. Since the $M^X/G(M/H)/1$ queue corresponds to the $M^X/\tilde{G}/1$ queue, for the $M^X/\tilde{G}/1$ queue, by letting $\alpha=0$ in Section 3, we have

$$\tilde{P}^*(s,x,z)=\tilde{P}^*(x,0,z)e^{-[s+\lambda-\lambda A(z)]x}\bar{\tilde{G}}(x),$$

$$\tilde{P}^*(s,0,z)=\frac{z\{z^i-[s+\lambda-\lambda A(z)]\tilde{p}_0^*(s)\}}{z-\tilde{g}^*[s+\lambda-\lambda A(z)]}.$$

Also, because $\tilde{g}^*[s+\lambda-\lambda A(z)]=g^*(\gamma)=g^*[\gamma+\alpha-\alpha h^*(\gamma)]=g^*(\nu)$ it implies $\tilde{p}_0^*(s)=p_0^*(s)$. Thus

$$W_1(t,x)=\tilde{p}_0(t)u(x)+\sum_{n=1}^{\infty}\int_0^{\infty}\tilde{p}_n(t,u)\,\overbrace{\tilde{g}_0*\tilde{g}*\cdots*\tilde{g}}^{n-1}(x)\mathrm{d}u$$

where $*$ represent the convolution and $\tilde{g}_0(x)=\tilde{g}(u+x)/\bar{\tilde{G}}(u)$. By taking L and LS transforms, we get

$$\begin{aligned}
w_1^*(s,\theta)&=\tilde{p}_0^*(s)+\sum_{n=1}^{\infty}\int_0^{\infty}\tilde{p}_n(s,u)[\tilde{g}^*(\theta)]^{n-1}\Big[\int_0^{\infty}e^{-\theta x}\tilde{g}(u+x)/\bar{\tilde{G}}(u)\mathrm{d}x\Big]\mathrm{d}u\\
&=\tilde{p}_0^*(s)+\frac{1}{\tilde{g}^*(\theta)}\tilde{P}^*(s,0,\tilde{g}^*(\theta))\int_0^{\infty}e^{-\tilde{\gamma}u}\int_0^{\infty}e^{-\theta x}\tilde{g}(u+x)\mathrm{d}x\mathrm{d}u\\
&=\tilde{p}_0^*(s)+\frac{[\tilde{g}^{*i}(\theta)-\tilde{\gamma}p_0^*(s)]}{\tilde{g}^*(\theta)-\tilde{g}^*(\tilde{\gamma})}\int_0^{\infty}e^{-(\tilde{\gamma}-\theta)u}\int_u^{\infty}e^{-\theta y}\tilde{g}(y)\mathrm{d}y\mathrm{d}u\\
&=\tilde{p}_0^*(s)+\frac{[\tilde{g}^{*i}(\theta)-\tilde{\gamma}p_0^*(s)]}{\tilde{g}^*(\theta)-\tilde{g}^*(\tilde{\gamma})}\int_0^{\infty}e^{-\theta y}\tilde{g}(y)\int_0^{y}e^{-(\tilde{\gamma}-\theta)u}\mathrm{d}u\mathrm{d}y\\
&=\tilde{p}_0^*(s)+\frac{[\tilde{g}^{*i}(\theta)-\tilde{\gamma}p_0^*(s)]}{\tilde{\gamma}-\theta}=\frac{(\tilde{g}^*(\theta))^i-\theta p_0^*(s)}{\tilde{\gamma}-\theta}
\end{aligned}$$

where $$\tilde{\gamma}=s+\lambda-\lambda A[\tilde{g}^*(\theta)].$$

Let $r_n(n=1,2\cdots)$ be the probability of a test customer being in the n-th position of an arriving batch. Using a result in the renewal theory, Burke[4] has showed that it is given by

$$r_n=\frac{1}{a}\sum_{i=n}^{\infty}a_i.$$

Thus, the LS transform of W_2 is given by

$$w_2(\theta)=\sum_{n=1}^{\infty}r_n[\tilde{g}^*(\theta)]^{n-1}=\sum_{n=1}^{\infty}\frac{1}{a}\sum_{i=n}^{\infty}a_i[\tilde{g}^*(\theta)]^{n-1}$$

$$=\frac{1}{a}\sum_{n=1}^{\infty}[1-\sum_{i=1}^{n-1}a_i][\tilde{g}^*(\theta)]^{n-1}$$

$$=\frac{1}{a}\frac{1}{1-\tilde{g}^*(\theta)}-\frac{1}{a}\sum_{n=2}^{\infty}\sum_{i=1}^{n-1}a_i[\tilde{g}^*(\theta)]^{n-1}$$

$$=\frac{1}{a}\frac{1}{1-\tilde{g}^*(\theta)}-\frac{1}{a}\sum_{i=1}^{\infty}\sum_{n=1}^{\infty}a_i[\tilde{g}^*(\theta)]^{n+i-1}$$

$$=\frac{1}{a}\frac{1}{1-\tilde{g}^*(\theta)}-\frac{1}{a}\frac{A[\tilde{g}^*(\theta)]}{1-\tilde{g}^*(\theta)}$$

$$=\frac{1}{a}\frac{1-A[\tilde{g}^*(\theta)]}{1-\tilde{g}^*(\theta)}.$$

Since $W_1(t)$ and W_2 are independent, the L and LS transforms of $W(t)$ is given by

$$w^*(s,\theta)=w_1^*(s,\theta)w_2(\theta)$$

$$=\frac{[\tilde{g}^*(\theta)]^i-\theta p_0^*(s)}{s+\lambda-\lambda A[\tilde{g}^*(\theta)]-\theta}\cdot\frac{1-A[\tilde{g}^*(\theta)]}{a[1-\tilde{g}^*(\theta)]}.$$

Corollary 5 When $\rho<1$, the stationary distribution of the waiting time exists and its LS transform

$$w(\theta)=\lim_{s\to 0+}sw^*(s,\theta)$$

$$=\frac{(1-\rho)\theta}{\theta-\lambda+\lambda A[\tilde{g}^*(\theta)]}\cdot\frac{1-A[\tilde{g}^*(\theta)]}{a[1-\tilde{g}^*(\theta)]}.$$

Corollary 6 When $\rho<1$, the mean waiting time

$$\overline{W}=\lim_{\theta\to 0+}-[w(\theta)]'$$

$$=\frac{[\lambda a(1+\alpha/\beta)]^2\sigma_G^2+\lambda^2a^2\alpha(\sigma_H^2+1/\beta^2)/\mu+\rho^2+\rho A''(1)/a]}{2\lambda a(1-\rho)}.$$

Proof By direct calculation.

6 The Reliability Indices of the Server

Theorem 8 The Laplace transforms of the server availability and unavailability

$$A^*(s)=[1+\alpha\overline{H}^*(s)sp_0^*(s)]/[s+\alpha-\alpha h^*(s)],$$

$$U^*(s)=[1-sp_0^*(s)]\alpha\overline{H}^*(s)]/[s+\alpha-\alpha h^*(s)].$$

Proof By the definition of availability, $A(t)=P\{\text{the server is good at time } t\}=p_0(t)+P(t)$. Thus the results are clear.

Corollary 7 When $\rho<1$, the limiting availability of the server

$$A=1-\frac{\lambda\alpha}{\mu\beta}.$$

Proof Since $\lim_{t\to\infty}p_0(t)$ exists, the conclusion follows from $A=\lim_{s\to 0+}sA^*(s)$.

Theorem 9 The Laplace transform of the server failure frequency

$$W_f^*(s)=[1-sp_0^*(s)]\alpha/[s+\alpha-\alpha h^*(s)].$$

Proof Using the general formula of Ref. [3], we have

$$W_f^*(s)=\sum_{n=1}^{\infty}\int_0^{\infty}p_n^*(s,x)\alpha dx$$

$$=[1 - sp_0^*(s)]a/[s + a - ah^*(s)].$$

Corollary 8 When $\rho<1$, the limiting failure frequency of the server

$$W_f = \frac{\lambda a\alpha}{\mu}.$$

Proof It is similar to the proof of Corollary 7.

Corollary 9 When $\rho<1$,

$$1 - sp_0^*(s) = 1 - d_i(s) + \frac{1-d(s)}{1-c(s)}c_i(s), \quad \text{for } i = 0,1,2,\cdots$$

where

$$d_0(s) = 1, \quad c_i(s) = \frac{\lambda}{s+\lambda}d_i(s).$$

Proof

$$sp_0^*(s) = \frac{s\tilde{b}^i(s)}{s+\lambda-\lambda A[\tilde{b}^i(s)]}$$

$$= \frac{s}{s+\lambda}d_i(s)/[1-c(s)]$$

$$= \frac{d_i(s)-c_i(s)}{1-c(s)},$$

$$1 - sp_0^*(s) = 1 - \frac{d_i(s)-c_i(s)}{1-c(s)}$$

$$= 1 - d_i(s) + d_i(s) - \frac{d_i(s)-c_i(s)}{1-c(s)}$$

$$= 1 - d_i(s) + \frac{c_i(s)-c(s)d_i(s)}{1-c(s)}$$

$$= 1 - d_i(s) + \frac{1-d(s)}{1-c(s)}c_i(s).$$

The server reliability $R(t)$ is defined as the probability that a new server does not fail by t. Hence, it is the complementary distribution of the server first failure time. In order to find out the first failure time distribution of the server, we regard the state (n,x,y) (the server fails) as an absorbing state. In this case, the state probabilities of new process must satisfy the following: the set of equations

$$\left(\frac{d}{dt}+\lambda\right)r_0(t) = \int_0^\infty r_1(t,x)\mu(x)dx,$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda+\alpha+\mu(x)\right]r_n(t,x) = \lambda\sum_{j=1}^{n}a_j r_{n-j}(t,x), \quad n = 1,2,\cdots,$$

the boundary conditions

$$r_n(t,0) = \lambda a_n r_0(t)u(t) + u(t)\int_0^\infty r_{n+1}(t,x)\mu(x)dx + \delta_{ni}\delta(t), \quad n = 1,2,\cdots,$$

the initial conditions

$$r_0(0) = \delta_{0i}, \quad r_n(0,x) = \delta_{ni}\delta(x), \quad n = 1,2,\cdots.$$

By solving above set of equations, we obtain.

Theorem 10 The Laplace transform of the server reliability

$$R^*(s)=r_0^*(s)+\sum_{n=1}^{\infty}\int_0^{\infty}r_n^*(s,x)\mathrm{d}x$$
$$=[1+\alpha r_0^*(s)]/(s+\alpha)$$
$$=\frac{1}{s+\alpha}\left[1+\frac{\alpha b^i(s+\alpha)}{s+\lambda-\lambda A(b(s+\alpha))}\right]$$

and the mean time to the first failure (MTTFF) of the server

$$\text{MTTFF}=\lim_{s\to 0+}R^*(s)$$
$$=1/\alpha+b^i(\alpha)/[\lambda-\lambda A(b(\alpha))]$$

where $b(s)$ is the root with minimal absolute value in z of the equation

$$z=g^*[s+\lambda-\lambda A(z)].$$

Corollary 10 The LS transform of the first failure time distribution of the server

$$f(s)=[1-sr_0^*(s)]\alpha/(s+\alpha).$$

Proof By direct calculation.

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可修排队系统 $M^X/G(M/H)/1$ 的瞬态解

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摘要：本文研究一个典型的批到达可修排队系统 $M^X/G(M/H)/1$. 记号(M/H)表服务台寿命服从指数分布，而其修理时间为一连续型分布. 利用向量马氏过程方法，我们得到了它的瞬态解. 特别是发现了服务台的可靠性指标仅依赖于可修排队系统的空闲概率，或等价地仅依赖于它的忙期和忙循环.

关键词：排队论；可靠性理论；可修排队系统；向量马氏过程(方法)

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