第 11 卷第 6 期
1994 年 12 月

控制理论与应用
CONTROL THEORY AND APPLICATIONS

Vol. 11, No. 6
Dec., 1994

H$_\infty$ Robust Sub-Optimal Control with Robust Regulation Performance*

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Abstract: In this paper, an H$_\infty$ robust sub-optimal contoller synthesis problem with robust regulation performance is investigated for a plant with structured uncertainty. It is shown that the problem can be reduced to an H$_\infty$ standard problem with a scaling parameter. A design example applying the presented approach to the D. D. manipulator is given.

Key words: H$_\infty$ robust control; robust regulation; structured uncertainty.

1 Introduction

Consider a system given by Fig. 1. The plant with structured uncertainty $P_\Sigma(s)$ is given by

$$\dot{x} = (A + \Delta A)x + B_1 w + (B_2 + \Delta B)u, \tag{1}$$

$$z = C_1 x + D_{12} u, \tag{2}$$

$$y_m = C_2 x + D_{21} w + D_{22} u, \tag{3}$$

where $x \in R^n$ is state vector, $u \in R^m$ control input, y_r reference input, $y_m \in R^q$ measured output, $w \in R^p (w \in L_2[0, +\infty))$ disturbance input and $z \in R^r$ controlled output. A, B_1, B_2, C_1, C_2, D_{12}, D_{21} and D_{22} are known matrices with appropriate dimensions. It is assumed that uncertainty ΔA, ΔB are described with unknown matrix $\Sigma \in R^{h \times k}$ and known matrices $E \in R^{n \times h}$, $F_a \in R^{k \times n}$, $F_b \in R^{k \times m}$ as follows,

$$[\Delta A \ \Delta B] = E \Sigma [F_a \ F_b]. \tag{4}$$

Unknown matrix Σ belongs to a given bounded set $\Omega = \{\Sigma \mid \bar{\sigma}(\Sigma) \leqslant 1\}$.

We consider the following controller synthesis problem: design a compensator $K(s)$ which satisfies the following requirements:

1) Robust Stability: The closed loop system of plant P_Σ with the controller $K(s)$ is internally stable for all $\Sigma \in \Omega$.

2) H$_\infty$ robust sub-optimality: for a given $\gamma > 0$,

$$\| T_{zw}(s) \|_\infty < \gamma, \quad \forall \Sigma \in \Omega \tag{5}$$

where T_{zw} denotes closed loop transfer function from w to z.

3) Robust regulation performance: for a given reference signal y_r with $w = 0$, regulated error becomes

* Manuscript received May 23, 1992, revised Apr. 26, 1994.

$$\lim_{t\to\infty} e = 0, \quad \forall\, \Sigma \in \Omega \tag{6}$$

where $e := y_m - y_r$. The reference signal y_r satisfies a differential equation

$$y_r^{(p)} + a_p y_r^{(p-1)} + \cdots + a_2 y_r' + a_1 y_r = 0 \tag{7}$$

and the characteristic roots of (7) $\lambda_i (i=1,\cdots,p)$ are all in the closed right half complex plane with unknown initial condition.

There are many literatures in which one or two of the three objectives described above are considered. The design problem to satisfy 1) can be reduced to an H_∞ standard problem since the objective is equivalent to an H_∞ norm bound constraint[1]. The objective 3) can be achieved by incorporating an internal model into appropriate channels of controller if plants exist in some neighborhood of nominal plant in the graph topology[2]. The controller synthesis problem with objectives 1) and 3) is equivalent to the problem of robust regulation with an H_∞ constraint, which is solved by Nagpal and his coworkers in paper [3]. As well known, the objective 2) with a fixed Σ is the H_∞ performance problem. Hence, multi-objective synthesis problem with 1) and 2) is nothing but an H_∞ robust performance problem. The problem has been investigated using Riccati inequality methods[4,5] or the μ-synthesis method.

In this paper, we design a controller $K(s)$ to meet the requirements 1)～3) simultaneously. It is clear that one possible approach to solve this problem is to find a robust stabilizing controller which has an internal model and satisfies the H_∞ robust performance requirement. For this end, it is important to clarify that under what condition the solution of the H_∞ robust performance problem can include the internal model. The remaining part of the controller out of the internal model should be designed to satisfy the H_∞ robust performance requirement. It is shown that this problem can be reduced to a H_∞ standard problem for an extended plant with a scaling parameter.

2 Preliminary Results

First, we consider a strictly proper compensator $K(s)$ give by

$$K(s) = \left[\begin{array}{c:c} A_c & B_c \\ \hdashline C_c & 0 \end{array}\right]. \tag{8}$$

The following theorem established by the well-known internal model principle gives a sufficient and necessary condition for $K(s)$ satisfying performance requirements 1) and 3) which is a slight modification of the results given by Devison[2].

Let M be a $p\times p$ matrix given by

$$M = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -a_1 & -a_2 & -a_3 & \cdots & -a_p \end{bmatrix}. \tag{9}$$

Theorem 1 Assume that

$$\operatorname{rank}\begin{bmatrix} A+\Delta A-\lambda_i I & B_2+\Delta B \\ C_2 & D_{21} \end{bmatrix} = n+q, \quad i=1,2,\cdots,p, \tag{10}$$

hold for all $\Sigma \in \Omega$. Then, $K(s)$ is a solution of controller synthesis problem with objectives 1) and 3) if and only if

$$K(s) = \{Cm + \tilde{C}(sI - \tilde{A})^{-1}\tilde{B}_1\}(sI - A_m)^{-1}B_m + \tilde{C}(sI - A)^{-1}\tilde{B}_2. \tag{11}$$

where $A_m = T$ block diag$\{M, M, \cdots, M\}T^{-1}$, $B_m = T\, B_n$ with the property that {block diag$\{M, M, \cdots, M\}$, $B_n\}$ is controllable and T is arbitrary nonsingular matrix. The matrices $\tilde{A}$, $\tilde{B}_1$, $\tilde{B}_2$, $\tilde{C}$ and C_m should be determined such that

$$\tilde{K}(s) = \left[\begin{array}{c|cc} \tilde{A} & \tilde{B}_1 & \tilde{B}_2 \\ \hline \tilde{C} & C_m & 0 \end{array}\right] \tag{12}$$

is a robust stabilizing controller for the augmented plant

$$P_{\text{aug}} = \left[\begin{array}{cc|c} A + \Delta A & 0 & B_2 + \Delta B \\ B_m C_2 & A_m & B_m D_{22} \\ \hline 0 & I_q & 0 \\ C_2 & 0 & D_{22} \end{array}\right]. \tag{13}$$

If the full state of plant P_Σ is measurable, i. e. $C_2 = I$, $D_{21} = D_{22} = 0$, then use of the internal model principle gives the next result similar to Theorem 1.

Theorem 2 Assume $C_2 = I$, $D_{21} = D_{22} = 0$ and (10) hold for all $\Sigma \in \Omega$. Then, a solution of controller synthesis problem with objectives 1) and 3) is given by

$$K(s) = C_m(sI - A_m)^{-1}B_m + \tilde{D} \tag{14}$$

where matrices C_m and $\tilde{D}$ should be determined such that

$$\tilde{K}(s) = [C_m \quad \tilde{D}] \tag{15}$$

is a robust stabilizing controller for the augmented plant P_{aug} given in (13).

In controller (11) and (14), $\tilde{K}(s)$ and $\tilde{K}$ are free parameters which are constrained only to be a robust stabilizing controller for the plant P_{aug}. Using these freedom we can find a controller $K(s)$ such that the closed loop system satisfies 1)~3).

Let a state space realization of closed loop system T_{zw} be given by

$$\dot{x}_c = (A_c + \Delta A_c)x_c + B_c w, \tag{16}$$

$$z = C_c x_c. \tag{17}$$

It is easy to show that the system satisfies 1) and 2), if there exists a $P > 0$ such that

$$(A_c + \Delta A_c)^T P + P(A_c + \Delta A_c) + \gamma^{-2} P B_c B_c^T P + C_c^T C_c < 0, \quad \forall\ \Delta A_c. \tag{18}$$

If there exists P>0 satisfying (18), then we call the system is H$_\infty$ robust sub-optimal.

3 Compensator Design

Instituting the controller (11) or (14) into the plant (1), (2), the system given in Fig. 1 can be equivalently described as Fig. 2(a) with extended plant P_{temp} given by

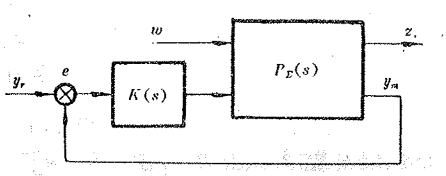
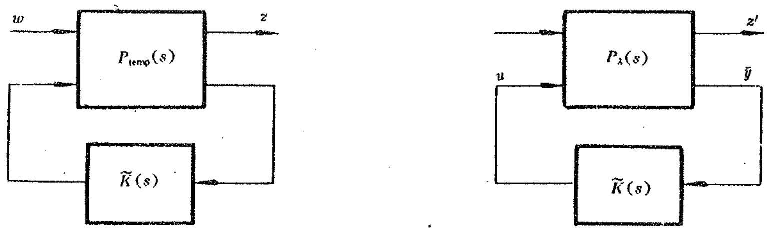


Fig. 1 The closed loop system

$$P_{temp} = \left[\begin{array}{cc:cc} A+\Delta A & 0 & B_1 & B_2+\Delta B \\ B_mC_2 & A_m & B_mD_{21} & B_mD_{22} \\ \hdashline C_1 & 0 & 0 & D_{12} \\ \begin{bmatrix} 0 \\ C_2 \end{bmatrix} & \begin{bmatrix} I_q \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ D_{21} \end{bmatrix} & \begin{bmatrix} 0 \\ D_{22} \end{bmatrix} \end{array}\right]. \tag{19}$$

It is clear the $\widetilde{K}(s)$ robust stabilizes P_{aug} if and only if the $\widetilde{K}(s)$ robust stabilizes P_{temp}. Hence, we can obtain a compensator satisfying 1)~3) by finding such a controller $\widetilde{K}(s)$ that the closed loop system in Fig. 2(a) is H_∞ robust sub-optimal.



(a) (b)

Fig. 2 Equivalent system

Theorem 3 Given $\gamma>0$. The system given in Fig. 2(a) is H_∞ robust sub-optimal if and only if there exists a scale $\lambda>0$ such that the system given in Fig. 2(b) is internally stable and satisfies $\| T_{z'w'}(s) \|_\infty<\gamma$ where scaled plant $P_\lambda(s)$ is defined by

$$P_\lambda = \left[\begin{array}{c:cc} \begin{matrix} A & 0 \\ B_mC_2 & A_m \end{matrix} & \begin{bmatrix} B_1 & \gamma\lambda E \\ B_mD_{21} & 0 \end{bmatrix} & \begin{bmatrix} B_2 \\ B_mD_{22} \end{bmatrix} \\ \hdashline \begin{bmatrix} C_1 & 0 \\ \lambda^{-1}F_a & 0 \end{bmatrix} & 0 & \begin{bmatrix} D_{12} \\ \lambda^{-1}F_b \end{bmatrix} \\ \begin{bmatrix} 0 & I_q \\ C_2 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ D_{21} & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ D_{22} \end{bmatrix} \end{array}\right]. \tag{20}$$

Proof We only show the case with feedback contorller $\widetilde{K}(s)$ given by (11). The case with controller given by (14) can be shown similarly. A state space realization of the closed loop system in Fig. 2(a) is given by

$$\dot{\hat{x}}_c = (\hat{A}+\Delta\hat{A})\hat{x} + \hat{B}w, \tag{21}$$

$$z = \hat{C}\hat{x} \tag{22}$$

where

$$\hat{A} = \begin{bmatrix} A & B_2C_m & B_2\widetilde{C} \\ B_m\overline{C}_2 & A_m + B_mD_{22}C_m & B_mD_{22}\widetilde{C} \\ \widetilde{B}_2C_2 & \widetilde{B}_1 + \widetilde{B}_2D_{22}C_m & \widetilde{B}_2D_{22}\widetilde{C} \end{bmatrix},$$

$$\hat{B} = \begin{bmatrix} B_1 \\ B_mD_{21} \\ \widetilde{B}_2D_{21} \end{bmatrix},$$

$$\hat{C} = [C_1 \quad D_{12}C_m \quad D_{12}\widetilde{C}]$$

and $\Delta\hat{A}=\hat{E}\Sigma\hat{F}$ with $\hat{E}=[E^{\mathrm{T}} \quad 0 \quad 0]^{\mathrm{T}}$, $\hat{F}=[F_a \quad F_bC_m \quad F_b\widetilde{C}]$.

From Theorem 2.4 in [6], $\widetilde{K}(s)$ is such a controller that the closed loop system in Fig. 2

(a) is H$_\infty$ robust sub-optimal, if and only if there exists a scale $\lambda>0$ such that algebraic Riccati inequality.

$$\hat{A}^T P + P\hat{A} + \gamma^{-2}P\hat{B}\hat{B}^T P + \lambda^2 P\hat{E}\hat{E}^T P + \hat{C}^T\hat{C} + \lambda^{-2}\hat{F}^T\hat{F} < 0 \qquad (23)$$

has a positive definite solution P.

On the other hand, the closed loop system shown in Fig. 2(b) is described by

$$\dot{\zeta} = \hat{A}\zeta + [\hat{B}\ \gamma\lambda\hat{E}]w', \qquad (24)$$

$$z' = \begin{bmatrix} \hat{C} \\ \lambda^{-1}\hat{F} \end{bmatrix}\zeta. \qquad (25)$$

From Lemma 2.2 in [7], Riccati inequality (23) have positive definite solution P if and only if the system (24), (25) internally stable and satisfies $\| T_{z'w'} \|_\infty<\gamma$.

Theorem 3 shows that our design problem can be reduced to the H$_\infty$ standard problem with a scaled plant P_λ. Application of Sampei's method[8] gives the following results which characterize compensator $K(s)$ in (11) with a desirable $\widetilde{K}(s)$.

Theorem 4 Assume the (A,B_2) is stabilizable, (C_2,A) is detectable and

$$\operatorname{rank}\begin{bmatrix} A-\lambda_i I & B_2 \\ C_2 & D_{22} \end{bmatrix} = n+q, \quad i=1,2,\cdots,q. \qquad (26)$$

Then, there exists an internally stabilizing controller $\widetilde{K}(s)$ for plant $P_\lambda(s)$ such that $\| T_{z'w'} \|_\infty<\gamma$ for given $\lambda>0$ and $\gamma>0$ if and only if there exists a $C_m\in R^{m\times q}$ such that the following conditions hold

1) There exists a matrix $F\in R^{m\times(n+q)}$ such that Riccati inequality

$$A_F^T P + PA_F + \gamma^{-2}PB_F B_F^T P + C_F^T C_F < 0 \qquad (27)$$

has positive definite solution P.

2) There exists a matrix $K\in R^{(n+m)\times 2q}$ such that Riccati inequality

$$A_K^T Q + QA_K + QB_K B_K^T Q + \gamma^{-2}C_K^T C_K < 0 \qquad (28)$$

has positive definite solution Q.

3) $Q>\gamma^{-2}P$,

where

$$A_F = \begin{bmatrix} A & B_2C_m \\ B_mC_2 & A_m+B_mD_{22}C_m \end{bmatrix} + \begin{bmatrix} B_2 \\ B_mD_{22} \end{bmatrix} F,$$

$$B_F = \begin{bmatrix} B_1 & \gamma\lambda E \\ B_mD_{21} & 0 \end{bmatrix}, \quad C_F = \begin{bmatrix} C_1 & D_{12}C_m \\ \lambda^{-1}F_a & \lambda^{-1}F_bC_m \end{bmatrix} + \begin{bmatrix} D_{12} \\ \lambda^{-1}F_a \end{bmatrix} F,$$

$$A_K = \begin{bmatrix} A & 0 \\ B_mC_2 & A_m+B_mD_{22}C_2 \end{bmatrix} + K\begin{bmatrix} 0 & 0 \\ D_{21} & 0 \end{bmatrix},$$

$$B_K = \begin{bmatrix} B_1 & \gamma\lambda E \\ B_mD_{21} & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ D_{21} & 0 \end{bmatrix}, \quad C_K = \begin{bmatrix} C_1 & D_{12}C_m \\ \lambda^{-1}F_a & \lambda^{-1}F_bC_m \end{bmatrix}.$$

Proof Outline of proof is as follows. Let $\hat{y}=\bar{y}-\begin{bmatrix} 0 \\ D_{22} \end{bmatrix}u$. Then, the existence of internally stabilizing controller $\widetilde{K}(s)$ satisfying $\| T_{z'w'} \|_\infty<\gamma$ in Fig. 2(b) is equivalent to the existence of feedback controller $\widetilde{K}(s)$ $(u=\widetilde{K}(s)\hat{y})$ such that the closed loop system is internally stable and $\| T_{\tilde{z}w} \|_\infty<\gamma$. The condition 1)～3) follows by use of Theorem 2 in [8].

Theorem 5 Assume that the plant with uncertainty satisfies the assumption in Theorem 4 and condition:

$$\operatorname{rank}\begin{bmatrix} A+\Delta A-\lambda_i I & B_2+\Delta B \\ C_2 & D_{22} \end{bmatrix}=n+q, \quad \forall\, \Sigma\in\Omega, \quad i=1,2,\cdots,p. \tag{29}$$

If there exists a matrix $C_m\in \mathrm{R}^{m\times q}$ and a scale $\lambda>0$ satisfying the conditions 1)～3) in Theorem 4, then such a desired controller (11) that the closed loop system in Fig. 1 satisfies performance requirements 1)～3) is given by

$$K(s)=\left[\begin{array}{cc:c} A_m & 0 & [B_m \quad 0] \\ \widetilde{B}_1 & \widetilde{A}_1 & \widetilde{B}_2 \\ \hdashline C_m & \widetilde{C} & 0 \end{array}\right], \tag{30}$$

where

$$\widetilde{A}=A_F+(Q-\gamma^{-2}P)^{-1}\{QK\begin{bmatrix} 0 & I \\ C_2 & 0\end{bmatrix}+QK_2D_{22}F-\gamma^{-2}M\},$$

$$\widetilde{B}_2=-(Q-\gamma^{-2}P)^{-1}QK_2, \quad \widetilde{B}_1=-(Q-\gamma^{-2}P)^{-1}Q(K_1-K_2D_{22}C_m), \quad \widetilde{C}=F,$$

$$M=H+F^{\mathrm{T}}\{[B^{\mathrm{T}} \quad D_{22}^{\mathrm{T}}B_m^{\mathrm{T}}]P+[D_{12}^{\mathrm{T}}C_1+\lambda^{-2}F_b^{\mathrm{T}}F_a \quad 0]\}+F^{\mathrm{T}}(C_1^{\mathrm{T}}C_1+\lambda^{-2}F_b^{\mathrm{T}}F_b)F$$
$$-Q\left\{K\begin{bmatrix} 0 & 0 \\ D_{21}B_c^{\mathrm{T}} & D_{21}D_{21}^{\mathrm{T}}B_m^{\mathrm{T}}\end{bmatrix}+\begin{bmatrix} B_1B_1^{\mathrm{T}}+\gamma^2\lambda^2EE^{\mathrm{T}} & B_1D_{21}^{\mathrm{T}}B_m^{\mathrm{T}} \\ B_mD_{21}B_1^{\mathrm{T}} & B_mD_{21}D_{21}^{\mathrm{T}}B_m^{\mathrm{T}}\end{bmatrix}\right\},$$

$$H=-\{A_F^{\mathrm{T}}P+PA_F+\gamma^{-2}PB_FB_F^{\mathrm{T}}P+C_F^{\mathrm{T}}C_F\}, \quad K=[K_1 \quad K_2].$$

Similarly, we can obtain a desired compensator for the case $y_m=x$ as follwos.

Thoerem 6 Let $C_2=I$, $D_{21}=D_{22}=0$. Assume that (A,B_2) is stabilizable and D_{12} is of full column rank. If there exists a scale $\lambda>0$ such that

$$\hat{A}^{\mathrm{T}}P+P\hat{A}+P\{\gamma^{-2}\hat{B}_1\hat{B}_1^{\mathrm{T}}-\hat{B}_2(\hat{D}_{12}^{\mathrm{T}}\hat{D}_{12})^{-1}\hat{B}_2^{\mathrm{T}}\}P+\hat{C}_1^{\mathrm{T}}\{I-\hat{D}_{12}\hat{D}_{12}^{\mathrm{T}}\}^{-1}\hat{C}_1<0 \tag{31}$$

has positive definite solution P, then a desired compensator such that the closed loop system in Fig. 1 satisfies 1)～3) is given by

$$K(s)=C_m(sI-A_m)^{-1}B_m+\widetilde{D} \tag{32}$$

with

$$[C_m \quad \widetilde{D}]=-\begin{bmatrix} 0 & I \\ I & 0\end{bmatrix}\{\hat{D}_{12}^{\mathrm{T}}\hat{D}_{12}\}^{-1}\{\hat{B}_2^{\mathrm{T}}P+\hat{D}_{12}^{\mathrm{T}}\hat{C}_1\}, \tag{33}$$

where

$$\hat{A}=\begin{bmatrix} A & 0 \\ B_m & A_m\end{bmatrix}-\hat{B}_2(\hat{D}_{12}^{\mathrm{T}}\hat{D}_{12})^{-1}\hat{D}_{12}^{\mathrm{T}}\hat{C}_1, \quad \hat{B}_2=\begin{bmatrix} B_2 \\ 0\end{bmatrix},$$

$$\hat{B}_1=\begin{bmatrix} B_1 & \gamma\lambda E \\ 0 & 0\end{bmatrix}, \quad \hat{C}_1=\begin{bmatrix} C_1 & 0 \\ \lambda^{-1}F_a & 0\end{bmatrix}, \quad \hat{D}_{12}=\begin{bmatrix} D_{12} \\ \lambda^{-1}F_b\end{bmatrix}.$$

Proof It follows from Theorem 2 and Theorem 3 by showing that $\hat{K}(s)=[C_m \quad \hat{D}]$ given by (32) is such a controller that the closed loop system in Fig. 2(b) satisfies $\| T_{\tilde{z}w} \|_\infty<\gamma$ for the scaled plant $P_\lambda(s)$ given in (20).

4 Example

Consider the D. D. manipulator with two degree of freedom shown in Fig. 3, where θ_1 and θ_2 denote the angles of the joint 1 and 2, respectively. As well-known, this is a nonlinear system. In order to use linear control theory to obtain a desired performance, an effective approach is the linearization with certain nonlinear compensator using parameters of plant. However, when there exist some uncertainty in the parmaeters caused by unknown varying weight, uncertain viscosity, et al., then the linearized system can be represented by the following linear model with nonlinear parameter perturbation

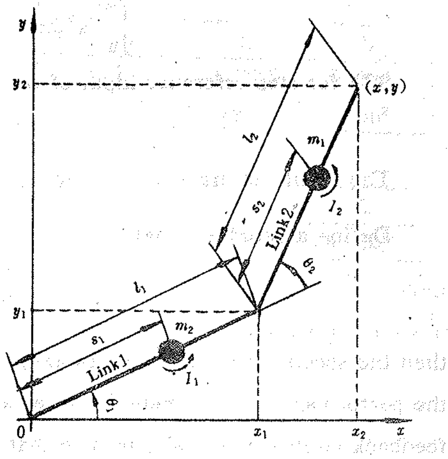


Fig. 3 Equivalent system with scaled plant P_λ

$$\ddot{\theta} = -(J_0 + \Delta J)^{-1}\Delta f\dot{\theta} + (J_0 + \Delta J)^{-1}J_0 u, \tag{34}$$

where $\theta = [\theta_1 \quad \theta_2]^T$ and u is the control input of the linearized system. The matrices J_0, ΔJ and Δf are given by

$$J_0 = \begin{bmatrix} j_{10} + j_{20} + 2j_{30}\cos\theta_2 & j_{20} + j_{30}\cos\theta_2 \\ j_{20} + j_{30}\cos\theta_2 & j_{20} \end{bmatrix}, \tag{35}$$

$$\Delta J = \begin{bmatrix} \Delta j_1 + \Delta j_2 + 2\Delta j_3\cos\theta_2 & \Delta j_2 + \Delta j_3\cos\theta_2 \\ \Delta j_2 + \Delta j_3\cos\theta_2 & \Delta j_2 \end{bmatrix}, \tag{36}$$

$$\Delta f = \begin{bmatrix} -2\Delta j_3\dot{\theta}_1\dot{\theta}_2\sin\theta_2 - \Delta j_3\dot{\theta}_2^2\sin\,\sin\theta_2 \\ \Delta j_3\dot{\theta}_1^2\sin\theta_2 \end{bmatrix}, \tag{37}$$

with $j_{10} = m_1 s_1^2 + m_2 l_1^2 + I_1$, $j_{20} = m_2 s_2^2 + I_2$ and $j_{30} = m_2 l_1 s_2$, where m_i, l_i, s_i and I_i $(i = 1, 2)$ denote the mass containing the weight, the length of link, the mass center of link and moment of inertia, respectively. Δj_i $(i = 1, 2, 3)$ denote parameter perturbations caused by unknown varying weight et al.

Define the 4×1 state vector $x = [\dot{\theta}^T \quad \theta^T]^T$. Then, (34) is expressed by

$$\dot{x} = (A + \Delta A)x + (B_2 + \Delta B)u + w, \tag{38}$$

where the w denote the 4×1 disturbance input vector, and

$$A = \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix}, \quad B = \begin{bmatrix} I \\ 0 \end{bmatrix}, \quad \Delta A = \begin{bmatrix} \Delta A_{11} & 0 \\ 0 & 0 \end{bmatrix}, \quad \Delta B = \begin{bmatrix} \Delta B_{21} \\ 0 \end{bmatrix},$$

$$\Delta A_{11} = \begin{bmatrix} \sigma_1 & \sigma_2 \\ \sigma_3 & \sigma_4 \end{bmatrix}, \quad \Delta B_{21} = \begin{bmatrix} \sigma_5 & \sigma_6 \\ \sigma_7 & \sigma_8 \end{bmatrix}.$$

Assume that the parameter perturbations satisfy bounded condition $|\sigma_i| < \delta_i$, $i = 1, \cdots, 8$ for given $\delta_i (i = 1, \cdots, 8)$ and the x is measureable.

We consider the design specifications as follows:

S1) the closed loop system is robust stable for all σ satisfying $|\sigma_i| < \delta_i$, $i = 1, \cdots, 8$.

S2) for given positive definite matrices Q and R and scale γ, the system response of disturbance satisfies

$$\int_0^\infty (x^TQx + u^TRu)\mathrm{d}t \leqslant \gamma, \quad \forall\, w \in L_2. \tag{39}$$

S3) for step reference input of the angles of joint y_r which satisfies differential equation

$$\dot{y}_r = 0, \tag{40}$$

The regulated error defined by $e := y - \begin{bmatrix} 0 \\ I \end{bmatrix} y_r$ go to 0 for all uncertainty.

Define a penalty signal by

$$z = \begin{bmatrix} Q^{1/2} \\ 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ R^{1/2} \end{bmatrix} u, \tag{41}$$

then the specification S2) implies $\|z\|_2 \leqslant \gamma \|w\|_2$, $\forall\, w \in L_2$ for all uncertainty. It is equivalent to the performance requirement 2) in section 1. Hence our design problem is equivalent to finding a feedback controller $u = K(s)e$ such that the closed loop system satisfies preformance requirement 1)～3) in section 1.

Since the uncertainties ΔA and ΔB can be described as the form (4) by defining the matrices

$$\Sigma(t) = \mathrm{diag}\left[\frac{\sigma_1}{\delta_1}, \frac{\sigma_2}{\delta_2}, \frac{\sigma_3}{\delta_3}, \frac{\sigma_4}{\delta_4}, \frac{\sigma_5}{\delta_5}, \frac{\sigma_6}{\delta_6}, \frac{\sigma_7}{\delta_7}, \frac{\sigma_8}{\delta_8}\right],$$

$$E = \left[\begin{array}{cccc:cccc} \delta_1 & \delta_2 & 0 & 0 & \delta_5 & \delta_6 & 0 & 0 \\ 0 & 0 & \delta_3 & \delta_4 & 0 & 0 & \delta_7 & \delta_8 \\ \hdashline \multicolumn{4}{c:}{0} & \multicolumn{4}{c}{0} \end{array}\right], \quad F_a = \left[\begin{array}{cc:c} 1 & 0 & \\ 0 & 1 & \\ 1 & 0 & 0 \\ 0 & 1 & \\ \hdashline \multicolumn{2}{c:}{0} & 0 \end{array}\right], \quad F_b = \left[\begin{array}{cc:c} \multicolumn{2}{c:}{0} & 0 \\ \hdashline 1 & 0 & \\ 0 & 1 & \\ 1 & 0 & 0 \\ 0 & 1 & \end{array}\right]$$

and x is measurable, the $K(s)$ can be found from Theorem 6 with $A_m = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $B_m = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

We now consider the physical values of the D. D. manipulator used for experiment as Table 1 and the bounds of uncertainty as Table 2 in which about 20% parameter perturbations are considered. When $Q_2 = 0.1I$, $R = I$ and $\gamma = 1$, the Riccati inequality in Theorem 6 has positive definite solutin

$$P = \begin{bmatrix} 5.1623 & 0 & 3.1623 & 0 \\ 0 & 5.1623 & 0 & 3.1623 \\ 3.1623 & 0 & 8.1623 & 0 \\ 0 & 3.1623 & 0 & 8.1623 \end{bmatrix} \tag{42}$$

for the scale $\lambda = 1$. Then, a desired controller is obtained by Theorem 7 as follows

$$K(s) = \begin{bmatrix} 3.2654 & 0 & 1.5811\frac{1}{s} + 3.5813 & 0 \\ 0 & 3.2654 & 0 & 1.5811\frac{1}{s} + 3.5813 \end{bmatrix}. \tag{43}$$

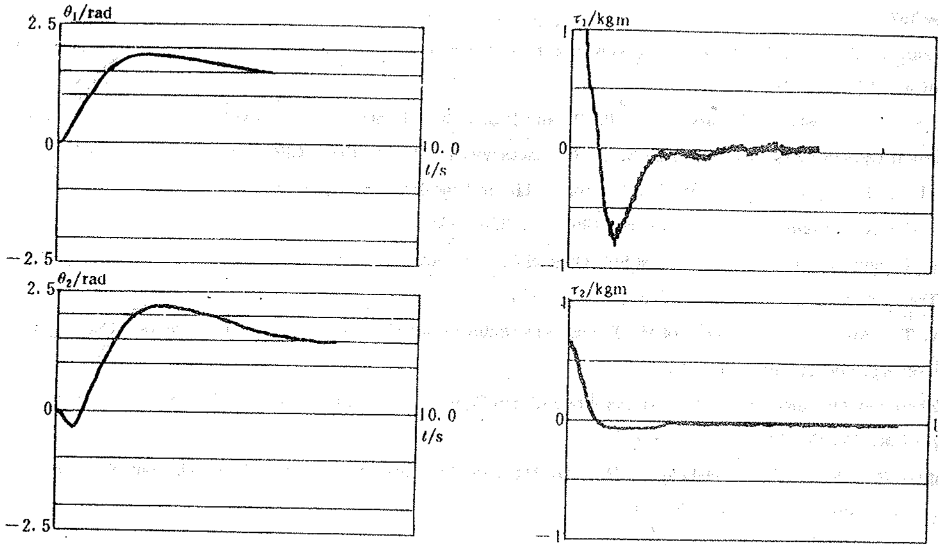
Table 1 Physical Values

m_1	m_2	l_1	l_2	s_1	s_2	I_1	I_2
5.1522 kg	1.6786 kg	0.2500 m	0.2200 m	0.1116 m	0.1012 m	0.1069 kgm^2	0.0058 kgm^2

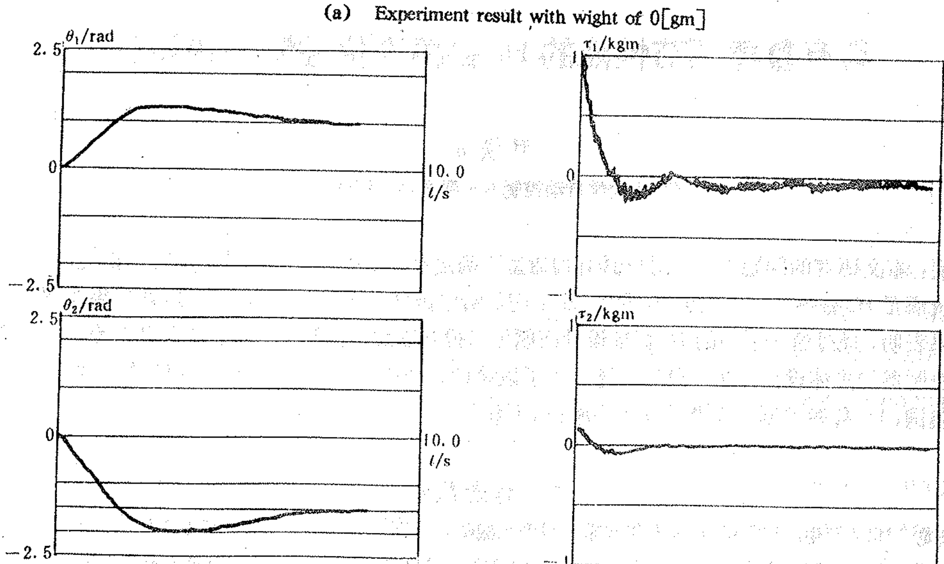
Table 2 Values of δ_i

δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
0.0678	0.0006	0.4164	0.1742	0.1187	0.0701	0.4674	0.1521

Fig. 4 shows the experiment results using the controller for different weight, where $\tau_i(i=1, 2)$ is the torque intput. It is shown that the system has relative good performance for uncertain parameter perturbation.



(a) Experiment result with wight of 0[gm]



(b) Experiment result with wight of 500[gm]

Fig. 4 Experiment result

5 Conclusion

In this paper a synthesis problem of H_∞ robust sub-optimal control with robust regulation performance is investigated for plant with structural uncertainty. It is shown that the problem can be reduced to H_∞ standard problem for a plant with a scaling parameter λ. The design example applying the presented approach to D. D. manipulator shows the validity of the solution given in this paper.

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具有鲁棒调节性能的H_∞鲁棒次优控制系统设计

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摘要：本文研究的问题如下，对于具有构造型不确定性的被控对象，设计一动态反馈控制器，使得闭环系统满足H_∞鲁棒次优性能要求．同时，对于满足给定微分方程的参考输入信号，具有鲁棒调节性能．研究结果表明，通过解一适当的H_∞标准设计问题可以得到理想的控制器．基于黎卡提不等式的正定解，给出了控制器的具体设计方法．最后，还给出了该设计方法在二自由度机械手上应用的实验结果．

关键词：H_∞鲁棒控制；鲁棒调节；不确定性系统

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申铁龙 1957年生．1982年至1986年在东北重型机械学院分别获工学学士及工学硕士学位．1989年赴日留学，1992年3月在日本东京上智大学获工学博士学位．目前研究领域为H_∞最优控制，鲁棒控制理论及应用．日本计测自动控制学会适应控制委员会委员，日本电气学会适应与学习控制委员会委员．