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A New Approach to Robust Variance-Constrained Estimation for Continuous Systems*

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Abstract: A new method to performance robust state estimator design is presented for linear continuous systems. The primary purpose of this paper is to find the set of filter gains such that the error variance for each system state is less than or equal to some prespecified values while the noise intensity is uncertain. A numerical example is provided to demonstrate the directness and simplicity of the design method.

Key words: linear systems; error covariance; robust state estimation

1 Introduction

It is quite common in state estimation problems to have performance objectives that are expressed as upper bounds on the variances of the estimation error. For example, in the problem of tracking a maneuverable target, it is desired to obtain filter gain such that the estimation value of the system state is situated in the prespecified effective region. Clearly, this performance requirement can be described as upper bounds on the estimation error variances of the states. Several indirect approaches attempt to achieve these constraints, for instance the theory of weighted least-squares estimation[1] minimizes a weighted scalar sum of the error variances of the state estimation. However, minimizing a scalar sum does not ensure that the multiple variance requirements will be satisfied.

The error covariance assignment (ECA) theory[2] was developed to provide an alternative and more straightforward methodology for designing filter gains which satisfy the above performance objectives. This methodology could provide a closed form solution for directly assigning the specified steady state estimation error covariance P, where the diagonal elements of P could be chosen as the desired constrained variance values and the off-diagonal elements of P were arbitrary. On the other hand, [3～5] studied the covariance assignment control (CAC) problem, and it was surprise that[2] no apparent duality appeared between the ECA theory and the CAC theory. Furthermore, there were very few papers dealing with the robust state estimation for uncertain systems, such as systems with noise intensity uncertainty.

Among many practical systems, the intensity W of the measure noise $w(t)$ can be accurately

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computed but the intensity V of system noise $v(t)$ is not this case. Fox example, in the problem of tracking a maneuverable target, the intensity V means the maneuverablity of the target and can not be accurately measured. Owing to the above important reason, an attempt will be made in this work to design the performance robust state estimator such that the error variance for each state always meets the specified constraint while the intensity V of the system noise varies between 0 and $\bar{V}$. Here, $\bar{V}$ is supposed to be the possible maximum value of V. It should be pointed out that the ECA theory can not be applied again in this problem because the assignability condition of [2] is closely related to the intensity V.

The present paper is organized as follows. In section 2, the problem statement is introduced. Section 3 studies the robust state estimator design for linear continuous systems. An illustrative example is provided in section 4. Finally, section 5 contains a conclusion.

2 Problem Statement

In this paper we propose the robust performance state estimation problem. Instead of a scalar cost function, the design objectives are inequality constraints on all state estimation error variances while the system noise intensity varies between 0 and $\bar{V}$. The plant considered here is the following time-invariant stabilizable and detectable system

$$\dot{x}(t) = Ax(t) + Bu(t) + v(t) \tag{1a}$$

and the measurement equation

$$y(t) = Cx(t) + w(t) \tag{1b}$$

where $x \in \mathrm{R}^n$ is the state, $u \in \mathrm{R}^m$ is the deterministic input and $y \in \mathrm{R}^p$ is the measured output. $v(t)$ and $w(t)$ are uncorrelated zero mean Gaussian white noise processes of intensity $V > 0$ and $W > 0$, respectively. The initial state $x(0)$ has mean $\bar{x}(0)$ and covariance $P(0)$. The state estimate vector $\hat{x}(t)$ is generated by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K(y(t) - C\hat{x}(t)), \tag{2}$$

whose estimation error covariance in the steady state is

$$P = \lim_{t \to \infty} P(t) = \lim_{t \to \infty} E[e(t)e^{\mathrm{T}}(t)]$$

where

$$e(t) = x(t) - \hat{x}(t).$$

Now, let $\sigma_i^2 (i = 1, 2, \cdots, n)$ denote the constraints on the variances of the estimation error. Then our goal is to find the filter gains appropriately such that the requirement

$$[P]_{ii} \leqslant \sigma_i^2 \quad (i = 1, 2, \cdots, n). \tag{3}$$

are always satisfied while intensity V varies between 0 and $\bar{V}$. Here, $[P]_{ii}$ is the ith diagonal element of P and the performance requirements $\sigma_i^2 (\mathrm{i} = 1, 2, \cdots, \mathrm{n})$ may be determined via computation or experimentation. Of course, the error variance constraints σ_i^2 should be more than or equal to the minimum error variance λ_i^2 which can be gotten via the optimal mean-square filter theory.

Using the above exposition, the problem statements may be formulated as follows. Given the variance constraints $\sigma_i^2 (i=1,2,\cdots,n)$, determine the set of filter gains K such that the estimation error variance value for each system state satisfy the specified performance requirement σ_i^2, i. e., the condition (3) will be satisfied, while V varies between 0 and $\bar{V}$.

3 Main Results and Proofs

In order to solve the previous problem, we start with the follwing well known result[2]

$$\dot{e}(t) = (A - KC)e(t) + v(t) - Kw(t), \tag{4}$$

and

$$\dot{P}(t) = (A - KC)P(t) + P(t)(A - KC)^{\mathrm{T}} + KWK^{\mathrm{T}} + V. \tag{5}$$

If $(A-KC)$ is a stable matrix, then in the steady state, we have the algebraic version of (5) as (see[2])

$$0 = (A - KC)P + P(A - KC)^{\mathrm{T}} + KWK^{\mathrm{T}} + V \tag{6}$$

where its solution satisfies $P=P^{\mathrm{T}}\geqslant 0$.

Now, we want to seek K such that the specified requirement can be satisfied. Here, we define a positive definite matrix Y satisfying

$$[Y]_{ii} \leqslant \sigma_i^2, \quad (i = 1,2,\cdots,n). \tag{7}$$

Using the matrix Y, the equation (6) may be expressed as

$$(A-KC)(P-Y)+(P-Y)(A-KC)^{\mathrm{T}}+(A-KC)Y+Y(A-KC)^{\mathrm{T}}+KWK^{\mathrm{T}}+V=0. \tag{8}$$

From the Lyapunov stability theory, we kow that if

$$(A - KC)Y + Y(A - KC)^{\mathrm{T}} + KWK^{\mathrm{T}} + V < 0 \tag{9}$$

holds, then $(P-Y)<0$ by virtue of the stability of $(A-KC)$, i. e., $\lambda(A-KC)\in C_g$, where $C_g=\{s\in\psi \mid \mathrm{Re}(s)<0, \psi \text{ is a complex plane}\}$ and $\lambda(Z)$ denotes the eigenvalues of matrix Z.

Consequently,

$$[P]_{ii} < [Y]_{ii} \quad (i = 1,2,\cdots,n). \tag{10}$$

It is clear that our purpose is to choose K such that (9) is satisfied subject to (7).

Now we consider the robustness of the filter gain K, i. e., (9) is always satisfied subject to (7) in spite of the system noise intensity V varies between 0 and $\bar{V}$. It is easy to see that if

$$(A - KC)Y + Y(A - KC)^{\mathrm{T}} + KWK^{\mathrm{T}} + \bar{V} < 0 \tag{11}$$

holds, then (9) is true where $0\leqslant V\leqslant \bar{V}$.

By defining $$K_y=YC^{\mathrm{T}}W^{-1},$$

(11) can be rewritten as

$$-(AY + YA^{\mathrm{T}} - YC^{\mathrm{T}}W^{-1}CY + \bar{V}) > (K - K_Y)W(K - K_Y)^{\mathrm{T}}. \tag{11'}$$

Assume that

$$AY + YA^{\mathrm{T}} - YC^{\mathrm{T}}W^{-1}CY + \bar{V} < 0 \tag{12}$$

holds and choose a positive definite matrix Q whose eigenvalues are appropriate such that the following inequality

$$-Q-(AY+YA^{\mathrm{T}}-YC^{\mathrm{T}}W^{-1}CY+\bar{V})\geqslant 0 \tag{13}$$

with the left side of (13) is of maximum rank p can be satisfied, then the equation

$$-Q-(AY+YA^{\mathrm{T}}-YC^{\mathrm{T}}W^{-1}CY+\bar{V})=(K-K_Y)W(K-K_Y)^{\mathrm{T}}, \tag{14}$$

or

$$(A-KC)Y+Y(A-KC)^{\mathrm{T}}+KWK^{\mathrm{T}}+\bar{V}=-Q. \tag{14'}$$

will yield (11'), (11), (9) and (10).

Now, by defining

$$LL^{\mathrm{T}}=-Q-(AY+YA^{\mathrm{T}}-YC^{\mathrm{T}}W^{-1}CY+\bar{V}) \tag{15}$$

with $L\in\mathrm{R}^{n\times p}$, equation (15) becomes

$$LL^{\mathrm{T}}=(K-K_Y)W^{1/2}W^{1/2}(K-K_Y)^{\mathrm{T}}, \tag{16}$$

and we have

$$LU=(K-K_Y)W^{1/2},$$

or

$$K=YC^{\mathrm{T}}W^{-1}+LUW^{-1/2} \tag{17}$$

where U is an arbitrary orthogonal matrix with appropriate dimension.

It should be noticed that, if the conditions (12), (13) hold and $Y>0$ is specified a priori, then all $(A-KC)$ which solve (14') are stable according to Lyapunov stability theory.

The following theorem states the main idea of the design method to robust state estimation for continous systems with error variance constraints and noise intensity uncertainty.

Theorem 1 Consider the system (1). If there exists a positive definite matrix Y which satisfies (7) and (12), the desired filter gain K which makes the estimation error covariance P meet (3) subject to $0\leqslant V\leqslant\bar{V}$ can be chosen by (17).

Remark 1 We can see that condition (7), (12) is very easy to test and the filter gain K is very easy to calculate. It is the difference from the traditional filter theory.

Remark 2 It is apparent from the above results that the possible filter gains (17) satisfying desired performance requirements is a set because the problem considered here is a multiobjective design task. The study of exploiting this freedom in Y,Q,U to meet other performance objectives still requires further development and investigation.

Remark 3 The research of constructing Y directly from the explicit conditions (7) and (12) is important but difficult. Similar problem arised in [2~5]. It still requires further study.

Next, the following well known results will provide the conditions for the existence of positive definite solution of inequality (12) and help us to solve (7) and (12).

Lemma 1 (sufficient conditions) If $W>0$ and (A,C) is controllable, there exists a symmetric positive definite solution to (12).

In fact, if (A,C) is controllable, the equation

$$AY+YA^{\mathrm{T}}-YC^{\mathrm{T}}W^{-1}CY+\bar{V}=-R$$

will have unqiue positive definite solution Y for an arbitrary positive definite matrix R.

Lemma 2[2] (Necessary conditions) Suppose that $W>0$ and $(A,\overline{V}^{1/2})$ is controllable. If there exists a positive definite solution to (12), (A,C) will be detectable.

Algorithm to design the robust state estimator:

Step 1 Determine the constraints $\sigma_i^2 (i=1,2,\cdots,n)$ on the estimation error variance and the possible variational limit $\overline{V}$ for the system noise intensity (σ_i^2 and $\overline{V}$ may be gotten by the performance requirements of practical systems).

Step 2 Solve inequality (7) and (12) with the help of Lemma 1, 2 and then obtain Y.

Step 3 Choose a proper positive definite matrix Q such that (13) holds.

Step 4 Obtain desired filter gain K from (17).

4 A Numerial Example

Consider the system in equation (1) with $n=2$ and the system parameters

$$A=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix},\quad C=[0\quad 1],\quad W=1.$$

It is assumed that the constraints on variances of error estimation are

$$\sigma_1^2\leqslant 2.8,\quad \sigma_2^2\leqslant 1.7$$

and the possible maximum value of V is

$$\overline{V}=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}.$$

Using (7) and (12), we can obtain

$$Y=\begin{bmatrix}2.8957 & 1.6125\\ 1.6125 & 1.7958\end{bmatrix}.$$

From (13), we choose

$$Q=\begin{bmatrix}0.2 & 0\\ 0 & 3.2\end{bmatrix},$$

then we have

$$LL^{\mathrm{T}}=\begin{bmatrix}2.4 & 0\\ 0 & 0\end{bmatrix},\quad L=[1.5492\quad 0].$$

Finally, we can obtain the desired filter gain from (17)

$$K=[3.1617\quad 1.7958].$$

5 Conclusions

The purpose of this paper is to design the filter gain such that the error variance for each system state is less than or equal to some prespecified value in spite of the system noise intensity varies between 0 and the possible maximum value. The present design methodology is based on Lyapunov stability theory. The main result of this paper can easily extended to discrete systems and systems driven by the colour noise. These results will appear in the near future.

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连续系统鲁棒约束方差估计的一种新方法

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摘要：本文提出一种新的性能鲁棒滤波增益的设计方法，即设计滤波增益，使得当系统噪声强度不确定时，系统每个状态的误差方差稳态值都不大于预先指定值，并举例说明这种设计方法的直接性与简单性.

关键词：线性系统；误差协方差；鲁棒状态估计

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