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An optimization Design Method of Fuzzy Logic Controller

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Abstract: In this paper we come up with a new design method of optimization for fuzzy logic controller. The procedures to directly optimize the control rules in the defuzzification process are provided. The simulation results with stable and unstable models are given in curves and discussed.

Key words: fuzzy logic; optimization; dynamic system; fuzzy control, system design.

1 Introduction

Since fuzzy logic was introduced by Zadeh in 1965, fuzzy control has found a wide range of applications[1,2]. Generally, fuzzy systems work well when we can use experience or prior knowledge to articulate the fuzzy logic. Especially when there exist uncertainty or imprecision and nonlinearities or other modeling complexities, fuzzy control appears to be an effective tool. But since the design of fuzzy control systems is based on trial and observation procedures, the performance of the fuzzy systems depends on human experience. Since these uncertainties are not known a priori, the design procedures for fuzzy logic control systems are individual. Resently, some researchers are interested in design methods using an optimization algorithm to determine the membership function[3].

In this paper we propose a design method of optimization of fuzzy controller. Instead of the optmization of membership function, the control action is directly optimized in the defuzzification process.

This method consists of two stages: the fuzzy logic inference using conventional procedures and the optimization of the defuzzification which further refine inference rule base.

The computer simulation results show the control effects of the system with optimized fuzzy controller are better than that with the conventional fuzzy logic controller.

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2 Description of Fuzzy Control[1,2]

The basic fuzzy controller can be described with three processes: the fuzzification, the control decision and the defuzzification. A fuzzy system S is a transformation $S: I^n \to I^p$. Consider two fuzzy sets A and B are fuzzy subsets of E and U respectively, where $E \subseteq R$ and $U \subseteq R$ denote input universe of discourse and output universe of discourse. If A consists of n linguistic variables $\{x_i\}$ and B consists of m linguistic variables $\{y_i\}$, expressed as:

$$A = \{x_1, x_2, \cdots, x_n\}, \tag{2.1}$$

$$B = \{y_1, y_2, \cdots, y_m\}. \tag{2.2}$$

for any of input variables $e \in E \subseteq R$, constructing the antecedent fuzzy vector $V_e = [\mu_{e1}, \mu_{e2}, \cdots, \mu_{en}]^T$, we can infer, basing on the rule base, a consequent fuzzy vector:

$$W_u = [\mu_{u1}, \mu_{u2}, \cdots, \mu_{um}]^T \tag{2.3}$$

where $\mu_{ei}: e \to [0,1]$, is the ith membership value associated with the ith linguistic variable $x_i \in A$ and indicates the degree with which the crisp variable e belong to x_i. And the similar meaning holds with μ_{ui}.

The rule base is a FAM-bank matrix or a composition relation matrix up to $n \times m$ dimension which expresses the relationship between fuzzy variables A and B. such that:

$$V_e \circ M = W_u \tag{2.4}$$

where "$\circ$" the composition operator and M is the composition relation matrix.

Once the output fuzzy variable W_u is detemined the membership values corresponding to output linguistic variable are known. We can use fuzzy centroid method to obtain the crisp output value. Thus the crisp output is calculated by

$$u = \frac{\sum_{i=1}^{m} \mu_{ui} \cdot u_i}{\sum_{i=1}^{m} \mu_{ui}} \tag{2.5}$$

where $\mu_i \in U, \forall\, i \in \{I, m\}$, is a characterized value associated with linguistic variables in the output universe of discourse.

3 Optimization of Fuzzy Controller[3,4]

Assume the rule base has been set up and the membership function of the concered system has been selected by some experiences or some mothods presented such as in [5]. The crisp output of fuzzy controller is produced by fuzzy centroid method. Then we can describe the action of the controller by the following procedures.

For a sequence of observed or estimated input crisp variables $e_1, e_2, \cdots, e_K$, in general case of multi-input and single output, the ith inference rule of the fuzzy controller is defined as:

R^i: IF e_1 is E_1^i and e_2 is E_2^i and $\cdots$ and e_K is E_K^i, THEN u_i is U^i

$$\text{AND} \quad \mu_i = \mu_{U^i}(u_i) = \min\{\mu_{E_1^i}(e_1), \mu_{E_2^i}(e_2), \cdots, \mu_{E_k^i}(e_K)\}$$

where $R^i (i = 1, 2, \cdots, N)$ denodes the ith inference rule of N inference rules, U^i is a fuzzy subset in the appropriate output universe of discourse. We refine the control action u to achieve more precise control result by reconstructing it as:

$$u = \frac{\sum_{i=1}^{N} \omega_i \cdot u_i'}{\sum_{i=1}^{N} \omega_i \cdot \mu_{ui}} \tag{3.1}$$

where $\omega_i (i = 1, 2, \cdots, N)$ is the weighting factor selected to satisfy the index minimization in an extermination problem and $u_i' (= \mu_{ui} u_i)$ denotes output variable or control action derived by the ith rule. The optimization problem may be stated as:

Find $\omega_i (i = 1, 2, \cdots, N) \in \mathrm{R}.$

And
$$u = \frac{\sum_{i=1}^{N} \omega_i \cdot u_i'}{\sum_{i=1}^{N} \omega_i \cdot \mu_i}. \tag{3.2}$$

So that
$$J^* = \min_{\omega} J[u(\cdot)]. \tag{3.3}$$

Subject to
$$\dot{X} = f(X, u, t), \tag{3.4}$$
$$g_1(X, t) = 0, \tag{3.5}$$
$$g_2(X, t) \leqslant 0, \tag{3.6}$$
$$X_0 = C, \quad S(X_f) \leqslant 0 \tag{3.7}$$

where R denotes the real world space, $J[u(\cdot)]$ is an index function specified basing on the performance required by the system. By properly selecting u, i. e., the sequence of $\omega_i (i = 1, 2, \cdots, N)$, $J[u(\cdot)]$ will reach the minimum value J^*, $g_1(X_f, t_f) = 0$ and $g_2(X_f, t_s) \leqslant 0$ represent the trajectory constraints of the equality and inequality type, $X_0 = C$ and $S(X_f) \leqslant 0$ are the boundary conditions for initial and final states, and $\dot{X} = f(X, u, t)$ is the state space representation of the system.

Because the inference rule is fuzzy and the relation between control action u_i and input variables is not based on mathematical model representation, the conventional method cannot be used to solve the optimization problem. As discussed in [6] and [3], a systematic trial and error approach method will simplify the solving procedures.

4 Optimization Algorithm

The optimization procedure is to find a sequence of weighting factors so that the performance index is minimum under the constraints and boundary condition. Consider equation (3.1), if $\omega_i = 1$, for $i = 1, 2, \cdots, N$, the problem is reduced to the normal case. In order to achieve the optimization control action u, we let $\omega_i = 1 + \Delta\omega_i$ and equation (3.1) is evolved as:

$$u = \frac{\sum_{i=1}^{N} u_i' + \sum_{i=1}^{N} \Delta\omega_i u_i'}{\sum_{i=1}^{N} \mu_{ui} + \sum_{i=1}^{N} \Delta\omega_i \mu_{ui}} \tag{4.1}$$

where $\Delta\omega_i \circ u_i'$ and $\Delta\omega_i \cdot \mu_{ui}, \forall\ i \in \{1, N\}$, implies the increment of the control action and the corresponding membership degree derived from the ith inference rule. We may calculated and compare the values of the performance index by modifying $\Delta\omega_i$ and find a proper vector, $\omega^* = [\omega_1^*, \omega_2^*, \cdots, \omega_N^*]^T$, which make performance index J minimum.

5 Simulation and Results

5.1 Preliminary Design of Fuzzy Controller

In the fuzzy inference process we select both error and the change of error as input variables and choose triangular membership functions for the linguist variables as stated in [5]. There are only four of complete control rules fired in one calculation process and the control action is given similar to equation (2.5) with $N = 4$.

$$u = \frac{\sum_{k=1}^{4} \mu_k \cdot u_k}{\sum_{k=1}^{4} \mu_k} \tag{5.1}$$

where $u_k = \min\{\mu_{E_k}(e), \mu_{\dot{E}_k}(\dot{e})\}, (k = 1,2,3,4)$, is the minimal membership value of input variables in the kth rule.

5.2 Optimization Design

The performance index selected for the optimization procedure is

$$J = -\int_0^1 (e^2 + \dot{e}^2)\mathrm{d}t. \tag{5.2}$$

In this case control action u is not appeared in the index but it can be subject to any region by limiting its universe of discourse.

Consider the discrete form of the index

$$\min J = \sum_{k=0}^{N-1} (e^2(t_{k+1}) + \dot{e}^2(t_{k+1}) \cdot \Delta t_k \tag{5.3}$$

the kth stage subproblems:

$$\min J^k = (e^2(t_{k+1}) + \dot{e}^2(t_{k+1})) \cdot \Delta t_k. \tag{5.4}$$

5.3 Simulation Results

In the following examples we first show the simulation results of the second order systems controlled with the conventional fuzzy logic and the optimized fuzzy logic respectively. Then further inspect the impact on system responses owing to the variation of parameters which makes the plant vary from stable to unstable.

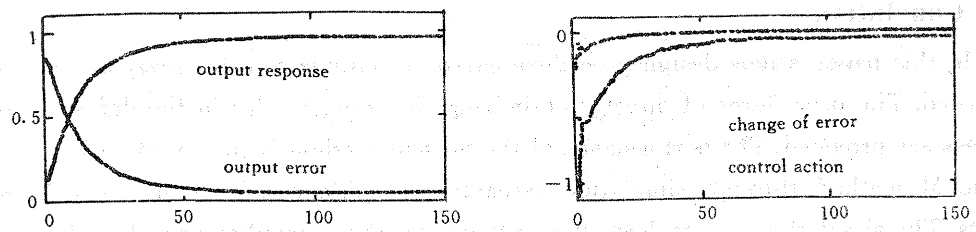
Example 1 Consider a general plant described by the following second order state equations in discrete form:

$$x_1(t_{k+1}) = (-3x_1(t_k) + x_2(t_k)) \cdot \Delta t_k + x_1(t_k), \tag{5.5}$$

$$x_2(t_{k+1}) = (-2x_1(t_k) - 0.1x_2(t_k) + u(t_k)) \cdot \Delta t_k + x_2(t_k). \quad (5.6)$$

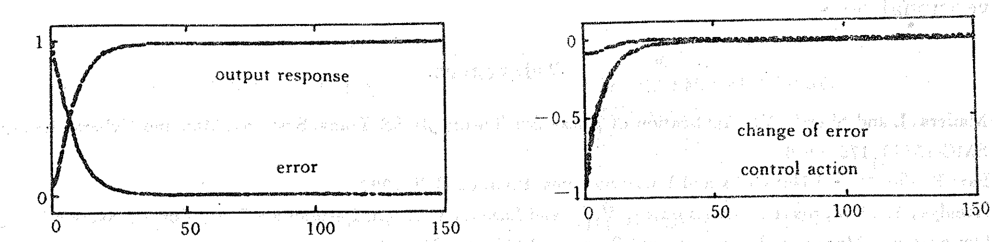
In the simulation the sample interval of 0.01s is chosen and the control action u is added into the system as an increment. The response curves are shown in Fig. 1.

In Fig. 1, the curves are corresponding to the result caused by the conventional fuzzy logic controller and Fig. 2 are corresponding to the result caused by the optimized fuzzy logic controller. The simulation curves show that the error and the change of error are reduced in the response process and the time of output response is shorter by using optimized fuzzy logic controller than by using the conventional one.



(a) Output and error curve (b) Change of error and control

Fig. 1 Process responses of example 1 with conventional fuzzy controller



(a) Output and error curve (b) Change of error and control

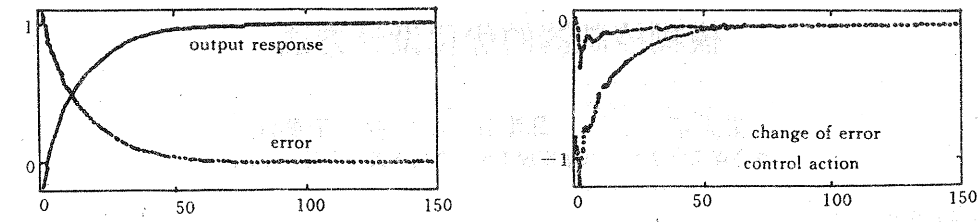
Fig. 2 Process responses of example 1 with optimized fuzzy controller

Example 2 In this simulation the modle, evolved from example 1, are described by the following state equatioins in the discrete form:

$$x_1(t_{k+1}) = -(3x_1(t_k) + x_2(t_k)) \cdot \Delta t_k + x_1(t_k), \quad (5.7)$$

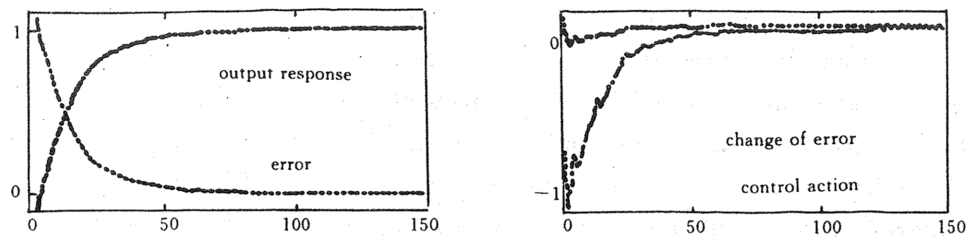
$$x_2(t_{k+1}) = (-2x_1(t_k) - 0.1x_2(t_k) + u(t_k)) \cdot t_k + x_1(t_k). \quad (5.8)$$

We may notice that the coefficient of x_2 in equation (5.7) is changed to negative from positive in equation (5.5) which results in the system becoming open loop unstable. The response curves are shown in Fig. 3 and Fig. 4. We can see the same results hold when the plant models become unstable.



(a) Output and error curve (b) Change of error and control

Fig. 3 Process responses of example 2 with Conventional fuzzy controller



(a) Output and error curve (b) Change of error and control

Fig. 4 Process responses of example 2 with optimized fuzzy controller

6 Conclution

In this paper, a new design procedure based on optimization for fuzzy controller is proposed. The procedures of directly optimizing the control rules in the defuzzification process are provided. The performance of the present method is compared with the conventional method through simulation research to stable and unstable second order plants. The simulation results demonstrate that since the squared error and its derivative is taken as the optimized index the responses are more favorable than that of conventional ones.

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模糊控制器的优化设计方法

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摘要：本文提出了一种模糊控制器的优化设计方法，讨论了直接优化去模糊过程的控制规则，并对基于稳定和不稳定模型进行了仿真研究.

关键词：模糊逻辑；最优化；动力系统；模糊控制；系统设计

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