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Identification of Nonliear Systems in the Presence of Correlated Noise Using Hammerstein Model *

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Abstract: A bias-eliminated least-square method for the estimation of parameters in a Hammerstein model is developed for the case when the observed output data are correlated with an unknown colored noise. The presented method can obtain asymptotically unbiased estimates without any a priori knowledge of the noise.

Key words: Hammerstein model; nonlinear identification; consistent estimation; least-squares method

1 Introduction

In general, it is difficult to characterize a nonlinear system exactly unless the form of the nonlinearity is known. A particular form of Hammerstein model suggested by Narendra and Gallman[1] has been employed to describe a nonlinear system. It is evident that this model is of great significance in practical engineering. Up to now, a number of methods have been available for estimating the parameters of Hammerstein model[3]. However, the performance of these methods in the presence of mesurement noise depend on the model of the noise. Particularly in the case of low signal-to-noise ratio(SNR), the obtained parameter estimates are usually biased.

Recently, a new kind of bias-eliminated least-square method (BELS) was proposed to treat the bias problem in the identification of linear systems[6]. The distinguishing feature of this method is that it can obtain consistent estimates even if there is no any a priori knowledge on the noise. In this paper the idea of the BELS method is exploited to achieve asymptotically unbiased estimates of the parameters in a Hammerstein model when an unknown colored noise presents in the output data. A designed filter of low order is inserted to the system at the input terminal so that the parameters of the augmented system satisfy some known linear constraints, which can be used for extracting the estimation bias from the least-square (LS) estimates. With the bias being removed, the consistent estimate can be obtained.

2 Unbiased Parameter Estimation for Hammerstein Model

Consider a Hammerstein system depicted in Fig. 1.

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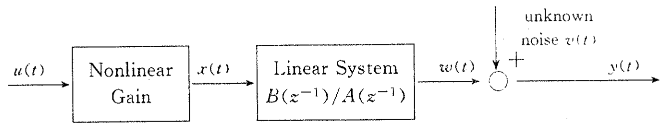

Fig. 1 Structure of Hammerstein Model

The nonlinear gain is assumed to be of the polynomial form

$$x(t) = \gamma_1 u(t) + \gamma_2 u^2(t) + \cdots + \gamma_p u^p(t). \tag{1}$$

The linear system is modeled by

$$A(z^{-1})w(t) = B(z^{-1})x(t) \tag{2}$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_n z^{-n}, \tag{3}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_m z^{-m} \tag{4}$$

and z denotes a unit-delay operator (i. e. $z^{-1}u(t) = u(t-1)$).

Combining Eqs. (1) and (2), the output $y(t)$ is related to u(t) through the following equation

$$A(z^{-1})y(t) = B(z^{-1})\sum_{i=1}^{p}\gamma_i u^i(t) + \varepsilon(t) \tag{5}$$

where

$$\varepsilon(t) = A(z^{-1})v(t). \tag{6}$$

$v(t)$ is a colored noise with unknown statistics. It is assumed to be uncorrelated with the input signal $u(t)$. Thus $\varepsilon(t)$ is independent of $u(t)$.

The problem under consideration is to estimate the coefficients a_i, b_i and γ_i from the observed data sequence $\{u(t), y(t)\}_1^N$. In particular it is desirable to obtain their consistent estimates.

By normalizing the parameter γ_1 such that $\gamma_1 = 1$, the system (5) can be rewritten in a signal-regressive form.

$$y(t) = \varphi_t^{\mathrm{T}}\theta + \varepsilon(t) \tag{7}$$

with

$$\begin{aligned}\varphi_t^{\mathrm{T}} = [&-y(t-1), \cdots, -y(t-n), u(t), \cdots, , \cdots, u(t-m), \\ &u^2(t), \cdots, u^2(t-m), \cdots, u^p(t), \cdots, u^p(t-m)],\end{aligned} \tag{8}$$

$$\theta^{\mathrm{T}} = [a_1, \cdots, a_n, b_0, \cdots, b_m, \gamma_2 b_0, \cdots, \gamma_2 b_m, \cdots, \gamma_p b_0, \cdots, \gamma_p b_m]. \tag{9}$$

The LS estimate of θ is

$$\hat{\theta}_{\mathrm{LS}}(N) = \left[\frac{1}{N}\sum_{t=1}^{N}\varphi_t\varphi_t^{\mathrm{T}}\right]^{-1}\left[\frac{1}{N}\sum_{t=1}^{N}\varphi_t y(t)\right], \tag{10}$$

and its asymptotical property is given by

$$\lim_{N\to\infty}\hat{\theta}_{\mathrm{LS}}(N) = \theta + \mathrm{R}_{\varphi\varphi}^{-1}R_{\varphi\varepsilon} \tag{11}$$

where

$$R_{\varphi\varphi} = \mathrm{E}[\varphi_t\varphi_t^{\mathrm{T}}] = \lim_{N\to\infty}\left(\frac{1}{N}\sum_{t=1}^{N}\varphi_t y(t)\right),$$

$$R_{\varphi\varepsilon}=[\mathrm{R}_{y\varepsilon}^{\mathrm{T}},0]^{\mathrm{T}}=[\gamma_{y\varepsilon}(1),\cdots,\gamma_{y\varepsilon}(n);0,\cdots,0]^{\mathrm{T}}.$$

Eq. (11) shows that if the $\gamma_{y\varepsilon}(i)$'s are known then asymptotically consistent estimate of θ can be obtained. Following the idea of [6], it is necessary to introduce n linear constraints for determining the n unkonwn $\gamma_{y\varepsilon}(i)$ $(i=1,2,,\cdots,n)$. We design a stable filter of order k.

$$F(z^{-1})=1+f_1z^{-1},\cdots,f_kz^{-k} \tag{12}$$

where the zero of $F(z^{-1})$, λ_i, are properly chosen to be real numbers such that they satisfy the condition

$$\lambda_i\neq\lambda_j,\forall\ i\neq j\quad\text{and}\quad|\lambda_i|<1,\quad i=1,2,\cdots,k. \tag{13}$$

The order k is taken to be such that

$$\frac{(k-1)p(p+1)}{2}<n\leqslant\frac{kp(p+1)}{2}. \tag{14}$$

For the convenience of discussion, we further assume that

$$n=\frac{p(p+1)}{2}. \tag{15}$$

It will be seen from the following discussions that this assumption does not affect the final results. Under such assumption the filter $F(z^{-1})$ has only one zero denoted by λ.

The filter $F^{-1}(z^{-1})$ is connected to the system at the input terminal. The augmented system thus obtained is expressed by the model

$$A(z^{-1})y(t)=\bar{B}_1(z^{-1})\bar{u}(t)+\sum_{i=2}^{p}\gamma_i\bar{B}_i(z^{-1})\bar{u}^i(t)+\varepsilon(t) \tag{16}$$

where

$$\begin{aligned}\bar{B}_i(z^{-1})&=B(z^{-1})(1+f_1z^{-1})^i\\&=\bar{b}_{i,0}+\bar{b}_{i,1}z^{-1}+\cdots+\bar{b}_{i,m+i}z^{-(m-i)},\quad i=1,2,\cdots,p.\end{aligned} \tag{17}$$

Denote

$$\bar{\theta}=[a_1,\cdots,a_n,\bar{b}_{1,0},\cdots,\bar{b}_{1,m+1},\cdots,\bar{b}_{p,0},\cdots,\bar{b}_{p,m+p}]^{\mathrm{T}}, \tag{18}$$

$$\begin{aligned}\bar{\varphi}_i=[&-y(t-1),\cdots,-y(t-n),\bar{u}(t),\cdots,\bar{u}(t-m-1),\cdots,\\&\bar{u}^p(t),\cdots,\bar{u}^p(t-m-p)]^{\mathrm{T}}.\end{aligned} \tag{19}$$

Eq. (16) can be rewritten as

$$y(t)=\bar{\varphi}_i^{\mathrm{T}}\bar{\theta}+\varepsilon(t). \tag{20}$$

In a similar way, the LS estimate of the parameters of the augmented system (16) can be obtained as follows:

$$\hat{\bar{\theta}}_{\mathrm{LS}}(N)=\Big[\frac{1}{N}\sum_{t=1}^{N}\bar{\varphi}_i\bar{\varphi}_i^{\mathrm{T}}\Big]^{-1}\Big[\frac{1}{N}\sum_{t=1}^{N}\bar{\varphi}_iy(t)\Big] \tag{21}$$

and asymptotical analysis gives

$$\lim_{N\to\infty}\hat{\bar{\theta}}_{\mathrm{LS}}(N)=\bar{\theta}+R_{\bar{\varphi}\bar{\varphi}}^{-1}R_{\bar{\varphi}\varepsilon} \tag{22}$$

where

$$R_{\bar{\varphi}\bar{\varphi}}=\mathrm{E}[\bar{\varphi}_i\bar{\varphi}_i^{\mathrm{T}}]=\lim_{N\to\infty}\Big[\frac{1}{N}\sum_{t=1}^{N}\bar{\varphi}_i\bar{\varphi}_i^{\mathrm{T}}\Big],$$

$$R_{\bar{\varphi}\varepsilon}^{\mathrm{T}} = [R_{y\varepsilon}^{\mathrm{T}}, 0]^{\mathrm{T}} = [r_{y\varepsilon}(1), \cdots, r_{y\varepsilon}(n), 0, \cdots, 0] \in \mathrm{R}^{1\times \bar{m}},$$

$$\bar{m} = n + \frac{p(2m+p+3)}{2}.$$

Since λ is the zero of $F(z^{-1})$, it must be the zero of $\bar{B}_i(z^{-1})$ which has a multiplicity of i. Therefore it holds that

$$\begin{aligned} &\bar{b}_{i,0}\lambda^{m+i} + \bar{b}_{i,1}\lambda^{m+i-1} + \cdots + \bar{b}_{i,m+i} = 0, \\ &\bar{b}_{i,0}P_{m+i}^{1}\lambda^{m-i-1} + \bar{b}_{i,1}P_{m+i-1}^{1}\lambda^{m+i-2} + \cdots + P_1^1\bar{b}_{i,m+i-1} = 0, \\ &\bar{b}_{i,0}P_{m+i}^{i-1}\lambda^{m-1} + \bar{b}_{i,1}P_{m-i-1}^{i-1}\lambda^{m} + \cdots + \bar{b}_{i,m+1}P_{i-1}^{i-1} = 0 \end{aligned} \tag{23}$$

where

$$P_m^n \triangleq \frac{m!}{(m-n)!}. \tag{24}$$

Introduce the matrix

$$H = \left[\begin{array}{c|cccc} & H_1 & & & \\ & & H_2 & & \\ 0 & & & \ddots & \\ & & & & H_p \end{array}\right] \in \mathrm{R}^{n\times \bar{m}} \tag{25}$$

where

$$H_1 = [\lambda^{m+i}, \lambda^{m+i-1}, \cdots, \lambda, 1] \in \mathrm{R}^{1\times(m+i)}, \tag{26}$$

$$H_i = \begin{bmatrix} \lambda^{m+i} & \lambda^{m+i-1} & \cdots & \lambda^{m} & \lambda^{m-1} & \cdots & \lambda & 1 \\ P_{m+i}^1\lambda^{m+i-1} & P_{m+i-1}^1\lambda^{m+i-2} & \cdots & P_1^i\lambda^{i-1} & P_{i-2}^1\lambda^{i-2} & \cdots & P_1^1 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ P_{m+i}^{i-1}\lambda^{m+1} & P_{m+i-1}^{i-1}\lambda^{m} & \cdots & P_{i-1}^{i-1} & 0 & \cdots & 0 & 0 \end{bmatrix},$$

$$i = 2, 3, \cdots, p. \tag{27}$$

Then Eq. (23) can be expressed in a compact form

$$H\bar{\theta} = 0. \tag{28}$$

Eq. (28) provides $p(p+1)/2$ linear constraints for the parameters of the augmented system which can be used for determining the $r_{y\varepsilon}(i)$'s from the $\bar{\theta}_{\mathrm{LS}}(N)$.

Applying H on both sides of Eq. (22) yields

$$H \lim_{N\to\infty} \bar{\theta}_{\mathrm{LS}}(N) = HR_{\bar{\varphi}\bar{\varphi}}^{-1}R_{\bar{\varphi}\varepsilon} = HR_{\bar{\varphi}\bar{\varphi}}^{-1}[\mathrm{R}_{y\varepsilon}^{\mathrm{T}}, 0]^{\mathrm{T}}. \tag{29}$$

Thus

$$R_{\bar{\varphi}\varepsilon} = Q[HR_{\bar{\varphi}\bar{\varphi}}^{-1}Q]^{-1}H \lim_{N\to\infty} \hat{\bar{\theta}}_{\mathrm{LS}}(N) \tag{30}$$

where

$$Q^{\mathrm{T}} = [I_n, 0] \in \mathrm{R}^{n\times \bar{m}}. \tag{31}$$

Now the following identification algorithm can be established.

Algorithm 1

1) Design a stable filter $F^{-1}(z^{-1})$ of suitable order k, where the coefficients of $F(z^{-1})$ are known and the roots of $F(z^{-1})$ satisfy the condition Eq. (13). Connect $F(z^{-1})$ to the

input terminal to form the augmented system.

2) Estimate the parameters of the augmented system by using the standard LS method, which gives

$$\hat{\bar{\theta}}_{LS}(N) = \left[\frac{1}{N}\sum_{t=1}^{N}\bar{\varphi}_t\bar{\varphi}_t^T\right]^{-1}\left[\frac{1}{N}\sum_{t=1}^{N}\bar{\varphi}_t y(t)\right]. \tag{32}$$

3) Calculate the correlation vector $R_{\bar{\varphi}e}$ by Eq. (30)

$$R_{\bar{\varphi}e}(N) = Q\left[H\left(\frac{1}{N}\sum_{t=1}^{N}\bar{\varphi}_t\bar{\varphi}_t^T\right)^{-1}Q\right]^{-1}H\ \hat{\bar{\theta}}_{LS}(N). \tag{33}$$

4) Calculate the estimate $\hat{\bar{\theta}}_{BELS}(N)$ of the parameter $\bar{\theta}$:

$$\hat{\bar{\theta}}_{BELS}(N) = \hat{\bar{\theta}}_{LS}(N) - \left(\frac{1}{N}\sum_{t=1}^{N}\bar{\varphi}_t\bar{\varphi}_t^T\right)^{-1}R_{\bar{\varphi}e}(N). \tag{34}$$

5) Compute the estimate $\hat{\theta}_{BELS}(N)$ of the original system parameter vector θ from $\hat{\bar{\theta}}_{BELS}(N)$ (see Eq. (17)).

6) Determine the estimate of a_i, b_i, and γ_i from the estimated $\hat{\theta}_{BELS}(N)$ (see Eq. (9)).

This algorihtm can also be transformed into the following on-line recursive scheme.

Algorithm 2

1) Design a stable filter $F(z^{-1})$ and connect it to the input terminal to form the augumented system.

2) Set the recursive initial values for $\hat{\bar{\theta}}_{LS}(0)$ and $\bar{P}_0$, properly with $t = 0$.

3) Calculate the bias-eliminated estimate $\hat{\bar{\theta}}_{BELS}(t)$ by the following equations:

$$\hat{\bar{\theta}}_{LS}(t) = \hat{\bar{\theta}}_{LS}(t-1) + \bar{P}_t\bar{\varphi}_t[y(t) - \bar{\varphi}_t^T\ \hat{\bar{\theta}}_{LS}(t-1)], \tag{35}$$

$$\bar{P}_t = \bar{P}_{t-1} - \bar{P}_{t-1}\bar{\varphi}_t[1 + \bar{\varphi}_t^T\bar{P}_{t-1}\bar{\varphi}_t]^{-1}\bar{\varphi}_t^T\bar{P}_{t-1}, \tag{36}$$

$$R_{\bar{\varphi}e}(t) = Q[H\bar{P}_tQ]^{-1}H\ \hat{\bar{\theta}}_{LS}(t), \tag{37}$$

$$\hat{\bar{\theta}}_{BELS}(t) = \hat{\bar{\theta}}_{LS}(t) - \bar{P}_tR_{\bar{\varphi}e}(t). \tag{38}$$

4) Compute the estimate $\bar{\theta}_{BELS}(t)$ of the original system parameter vector θ from $\hat{\bar{\theta}}_{BELS}(t)$ (see Eq. (17)).

5) Determine the estimate of a_i, b_i and γ_i from the estimated $\hat{\theta}_{BELS}(N)$ (see Eq. (9)).

Remark 1

By the same procedure as that used in the proof of Theorem 1 in [6], it can be proved that the parameter estmates $\hat{\bar{\theta}}_{BELS}(N)$ obtained by the above algorithm is asymptotically consistent, i. e., $\lim_{N\to\infty}\bar{\theta}_{BELS}(N) = \bar{\theta}$, w. p. 1. Therefore, it follows from Eq. (17) and (9) that the estimates obtained by the steps 5 and 6 in the above Algorithm 1 are consistent with their true values.

Remark 2

If $n < p(p+1)/2$, we select the first n rows of the matrix H and form a matrix H' which also supports the Eq. (28). Therefore, asymptotically unbiased estimate of $\bar{\theta}$ can be obtained by performing the procedure presented by Eqs. (28) and (30).

3 Simulation Example

Consider a Hammerstein model given in [2] where

$$x(t) = u(t) + 0.01625u^2(t) - 0.13295u^3(t) - 0.00616u^4(t), \tag{39}$$

$$A(z^{-1}) = 1 + 0.9z^{-1} + 0.15z^{-2} + 0.02z^{-3}, \tag{40}$$

$$B(z^{-1}) = 0.42z^{-1} - 0.9z^{-2}. \tag{41}$$

Table 1 Results of simulation example

para-meters	true value	SNR=11.26		SNR=1.69	
		MSLS method	BELS method	MSLS method	BELS method
a_1	0.9000	0.9078 ± 0.0023	0.9062 ± 0.0027	0.7899 ± 0.0087	0.8893 ± 0.0031
a_2	0.1500	0.1540 ± 0.0079	0.1486 ± 0.0031	0.1265 ± 0.0069	0.1396 ± 0.0050
a_3	0.0200	0.0164 ± 0.0031	0.0190 ± 0.0034	0.0119 ± 0.0072	0.0188 ± 0.0043
b_1	0.4200	0.4277 ± 0.0012	0.4163 ± 0.0017	0.3997 ± 0.0046	0.4317 ± 0.0026
b_2	−0.9000	−0.9088 ± 0.0019	−0.8905 ± 0.0020	−1.1053 ± 0.0055	−0.9245 ± 0.0041
r_2	0.01625	0.01047 ± 0.0048	0.01429 ± 0.0038	0.01914 ± 0.0052	0.01598 ± 0.0039
r_3	−0.13295	−0.13370 ± 0.0024	−0.13391 ± 0.0016	−0.11685 ± 0.0061	−0.12857 ± 0.0032
r_4	−0.00616	−0.00526 ± 0.0019	−0.00594 ± 0.0017	−0.00719 ± 0.0068	−0.00584 ± 0.0040

The colored noise $v(t)$ is simulated by a AR model

$$v(t) = \frac{1}{1 + z^{-1} + 0.41z^{-2}} e(t), \tag{42}$$

where $e(t)$ is zero-mean white noise. To examine the performance of the presented algorithm under various SNR, the variance of the $e(t)$ is taken to be 0.09 and 0.49 respectively.

In this example the filter is designed as

$$F(z^{-1}) = 1 + 0.5z^{-1}. \tag{43}$$

We have used the algorithm ten times with the size of sampled data being 450. The mean values and the standard deviations of the estimated parameters are listed in Table 1. The results obtained in [2] are also included for comparison. It can be seen from the Table that the presented algorithm in this paper can achieve a much improved accuracy for the parameter estimation of Hammerstein model.

4 Conclusion

This paper deals with the problem of unbiased parameter estimation of a Hammerstein model . The bias in the parameter estimates resulting from the colored mesurement noise is eliminated by introducing some linear constraints of the estimated parameters. The discussion of this paper reveals that in order to get desirable identification results it is an efficient way to draw much more useful information from the sampled data by utilizing some signal processing techniques.

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应用 Hammerstein 模型辨识受相关噪声扰动的非线性系统

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摘要: 本文提出了一种用于 Hammerstein 模型参数估计的偏差补偿最小二乘法。当观测数据被未知有色噪声污染时,应用本文方法可在缺少任何有关噪声先验信息的情况下实现参数的渐近无偏估计。

关键词: Hammerstein 模型; 非线性系统辨识; 一致估计; 最小二乘法

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