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Quadratic Stability of a Class of Uncertain Large-Scale Systems with Symmetrically Interconnected Subsystems

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Abstract: This paper investigates the quadratic stability problem of a class of uncertain large-scale systems composed of several similar subsystems interconnected with an external system in a symmetrical fashion. A sufficient condition for the quadratic stability of such a system is given in terms of two modified subsystems with lower-order and corresponding H_∞-norms, and it is shown that the condition is also necessary for the quadratic stability of the overbounding system corresponding to the system.

Key words: large-scale systems; stability analysis; model uncertainties

1 Introduction

A number of large-scale interconnected systems found in the real world enjoy the particular feature of having similar units and symmetrical interconnections. Systems with these characteristics are encountered in electric power systems, industrial manipulators, computer networks, etc[1~4]. Recently, there has been a great interest on large-scale systems with similar subsystems. In [4~6], the stability and decentralized control of the large-scale systems without uncertainty are investigated by utilizing structural properties of these systems, and important characteristics are observed. This paper is concerned with the quadratic stability problem of a class of uncertain large-scale systems with symmetically interconnected subsystems.

2 System Description

The uncertain system Σ under consideration consists of N similar subsystems interconnected with an external system, the overall system Σ is described by the composite equations:

$$\dot{x}(t) = (A + \Delta A)\, x(t). \tag{2.1}$$

where $x(t) = x = [x_0^{\mathrm{T}} \quad x_1^{\mathrm{T}} \quad \cdots \quad x_N^{\mathrm{T}}]^{\mathrm{T}}, x_0 = x_0(t) \in \mathrm{R}_0^n, x_i = x_i(t) \in \mathrm{R}^n \quad (i = 1, \cdots, N)$,

$$A = [A_{ij}], \quad (i, j = 0, 1, \cdots, N). \tag{2.2a}$$

with $A_{00} = A_0$, $A_{0i} = L_0$, $A_{i0} = M_0$, $A_{ii} = A_1 \quad (i = 1, \cdots, N)$, and $A_{ij} = H_1 \quad (i, j = 1, \cdots, N; i \neq j)$.

$$\Delta A = [\Delta A_{ij}(t)], \quad (i, j = 0, 1, \cdots, N). \tag{2.2b}$$

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with $\quad \Delta A_{00}(t) = \sum_{j=1}^{k_0} A_{0j}q_{0j}(t),\quad \Delta A_{oi}(t) = \sum_{j=1}^{k_1} L_{0j}q_{ij}(t),\quad \Delta A_{i0}(t) = \sum_{j=1}^{k_2} M_{0j}q_{0ij}(t),$

$$\Delta A_{ii}(t) = \sum_{j=1}^{k_3} A_{1j}r_{ij}(t),\quad (i = 1,\cdots,N),$$

and $\quad \Delta A_{ij}(t) = \sum_{j=1}^{k_4} H_{1j}r_{1ij}(t),\quad (i,j = 1,\cdots,N; i \neq j).$

Suppose that

1) the uncertain parameters $q_{ij}(t)$, $q_{0ij}(t)$, $r_{ij}(t)$, and $r_{sij}(t)$ are Lebesgue measureable functions such that

$$|q_{ij}(t)| \leqslant 1,\quad |q_{sij}(t)| \leqslant 1,\quad |r_{ij}(t)| \leqslant 1,\quad |r_{sij}(t)| \leqslant 1. \tag{2. 3}$$

2) the matrices $A_{0j}, L_{0j}, M_{0j}, A_{1j}$, and H_{1j} are rank 1, and

$$A_{0j} = a_{1j}a_{2j}^{\mathrm{T}},\quad L_{0j} = a_{3j}a_{4j}^{\mathrm{T}},\quad M_{0j} = a_{5j}a_{6j}^{\mathrm{T}},\quad A_{1j} = a_{7j}a_{8j}^{\mathrm{T}},\quad H_{1j} = h_{1j}h_{2j}^{\mathrm{T}}. \tag{2. 4}$$

This paper is concerned with the following notion of stability for the system Σ.

Definition 2. 1[7] Consider an uncertain linear system

$$\dot{x}(t) = [A^0 + \Delta A^0(q(t))]x(t) \tag{2. 5}$$

where $x(t) \in \mathrm{R}^p$ is the state, $q(t)$ is a vector of uncertain parameters, which is restricted to a prescribed bounding set Ω where Ω is a compact set. The system (2. 5) is said to be quadratically stable if there exists a $p \times p$ positive definite symmetric matrix P and a constant $\alpha > 0$ such that, for any admissible uncertainty $q(t)$, the Lyapunov derivative for the Lyapunov function $V(x) = x^{\mathrm{T}}Px$ satisfies

$$L(x,t) = \dot{V} = 2x^{\mathrm{T}}p[A^0 + \Delta A^0(q(t))]x \leqslant -\alpha \| x \|^2,$$

for all pairs $(x,t) \in \mathrm{R}^p \times \mathrm{R}$. In the following, suppose that $\Delta A^0(q(t)) = D^0F^0(t)E^0$, where the uncertainty $F^0(t)$ satisfies $F^0(t)^{\mathrm{T}}F^0(t) \leqslant 1$.

Lemma 2. 2[8] The system (2. 5) is quadratically stable if and only if it satisfies the following conditions:

i) A^0 is a stability matrix;

ii) $\| E^0(sI - A^0)^{-1}D^0 \|_\infty < 1$.

The above result could of course be applied directly to the system Σ of (2. 1) by using the notion of overbounding in [9]. But, this involves computing H_∞-norm of a matrix with high-order, it may be very difficult to do this. The next section will present a simple method of checking the quadratic stability of the system Σ.

The following notations will be used in the sequel.

$$a_1 = [a_{11} \quad \cdots \quad a_{1k_0}],\quad a_2 = [a_{21} \quad \cdots \quad a_{2k_0}], \tag{2. 6a}$$

$$a_3 = [a_{31} \quad \cdots \quad a_{3k_1}],\quad a_4 = [a_{41} \quad \cdots \quad a_{4k_1}], \tag{2. 6b}$$

$$a_5 = [a_{51} \quad \cdots \quad a_{5k_2}],\quad a_6 = [a_{61} \quad \cdots \quad a_{6k_2}], \tag{2. 6c}$$

$$a_7 = [a_{71} \quad \cdots \quad a_{7k_3}],\quad a_8 = [a_{81} \quad \cdots \quad a_{8k_3}], \tag{2. 6d}$$

$$h_1 = [h_{11} \quad \cdots \quad h_{1k_4}],\quad h_2 = [h_{21} \quad \cdots \quad h_{2k_4}], \tag{2. 6e}$$

$$A_m = A_1 + H_1, \quad A_{p0} = A_1 - (N-1)H_1, \tag{2.7a}$$

$$A_p = \begin{bmatrix} A_0 & NL_0 \\ M_0 & A_1 - (N-1)H_1 \end{bmatrix}, \tag{2.7b}$$

$$D_p = \begin{bmatrix} a_1 & N^{1/2}a_3 & 0 & 0 & 0 \\ 0 & 0 & N^{-1/2}a_5 & N^{-1/2}a_7 & [(N-1)/N)]^{1/2}h_1, \end{bmatrix}, \tag{2.7c}$$

$$E_p^{\mathrm{T}} = \begin{bmatrix} a_2 & N^{1/2}a_6 & 0 & 0 & 0 \\ 0 & 0 & N^{1/2}a_4 & N^{1/2}a_8 & [N(N-1)]^{1/2}h_2, \end{bmatrix} \tag{2.7d}$$

$$D_m = [a_5 \quad a_7 \quad (N-1)^{1/2}h_1], \quad E_m^{\mathrm{T}} = [a_4 \quad a_8 \quad (N-1)^{1/2}h_2], \tag{2.7e}$$

$$E_m^{\mathrm{T}} = [a_4 \quad a_8 \quad (N-1)^{1/2}h_2]. \tag{2.7f}$$

3 Main Results

Theorem 3.1 The system Σ of (2.1) is quadratically stable if it satisfies the following conditions:

3.1 i) A_p and A_m are stability matrices;

3.1 ii) $\| E_p(sI - A_p)^{-1}D_p \|_\infty \leqslant 1$ and $\| E_m(sI - A_m)^{-1}D_m \|_\infty < 1$.

Theorem 3.2 If the system Σ contains no the external system (i.e., $n_0 = 0$), then the system Σ of (2.1) is quadratically stable if it satisfies the following conditions:

3.2 i) A_{p0} and A_m are stability matrices;

3.2 ii) $\| E_m(sI - A_{p0})^{-1}D_m \|_\infty < 1$ and $\| E_m(sI - A_m)^{-1}D_m \|_\infty < 1$.

In order to investigate the degree of conservativeness of the above results, consider the uncertain system Σ_u defined as follows:

$$\dot{x}(t) = (A + \Delta\bar{A})x(t), \tag{3.1}$$

where A is given by (2.2a),

$$\Delta\bar{A} = DF(t)E \tag{3.2}$$

with the uncertainty $F(t)$ satisfying

$$F(t)^{\mathrm{T}}F(t) \leqslant I, \tag{3.3a}$$

$$D = \mathrm{diag}[d_0 \quad d_1 \quad \cdots \quad d_N] \tag{3.3b}$$

where

$$d_0 = [a_1 \quad a_2 \quad \cdots \quad a_3], \quad d_i = [a_5 \quad \underbrace{h_1 \quad \cdots \quad h_1}_{i-1} \quad a_7 \quad h_1 \quad \cdots \quad h_1], \quad (i = 1, \cdots, N),$$

$$E^{\mathrm{T}} = [e_0 \quad e_1 \quad \cdots \quad e_N] \tag{3.3c}$$

where

$$e_0 = \mathrm{diag}[a_2 \quad a_4 \quad \cdots \quad a_4], \quad e_i = \mathrm{diag}[a_6 \quad \underbrace{h_2 \quad \cdots \quad h_2}_{i-1} \quad a_8 \quad h_2 \quad \cdots \quad h_2],$$

$$(i = 1, \cdots, N).$$

By the notion of overbounding in [9], it is easy to see that the system Σ_u overbounds the system Σ. For the quadratic stability of Σ_u, we have

Theorem 3.3 The system Σ_u of (3.1) is quadratically stable if and only if it satisfies the conditions 3.1i) and 3.1ii).

Theorem 3. 4 If the system Σ contains no external system (i. e. , $n_0 = 0$), then the system Σ_u of (3. 1) is quadratically stable if and only if it satifies the conditions 3. 2i) and 3. 2ii).

Remark 3. 5 Since the system Σ of (2. 1) is overbounded by the system Σ_u of (3. 1), Theorem 3. 1 and Theorem 3. 2 are immediate from Theorem 3. 3 and Theorem 3. 4. Theorem 3. 1 and Theorem 3. 2 present sufficient conditions for the quadratic stability of the system Σ in terms of two matrices with lower-order and corresponding H_∞-norm, which simplifies considerably the stability analysis of the overall system. Theorem 3. 3 and Theorem 3. 4 show that the conditions are also necessary for the quadratic stability of the overbounding system Σ_u corresponding to the system Σ. Comparing with applying directly Lemma 2. 2 to the system Σ, no extra conservativeness is introduced by using Theorem 3. 1 and Theorem 3. 2. For the degree of conservativeness introduced by overbounding the system Σ with the system Σ_u of (3. 1), the reader is referred to reference [9].

The following preliminaries will be used in the proof of Theorem 3. 3.

Lemma 3. 5 The system (2. 5) is qudaratically stable if and only if one of the following two conditions hold:

3. 5i) There exists a positive definite symmetric matrix P, such that

$$(A^0)^T P + PA^0 + PD^0(D^0)^T P + (E^0)^T E^0 < 0;$$

3. 5ii) A^0 is a stability matrix, and $\| E^0(sI - A^0)^{-1}D^0 \|_\infty < 1$.

Lemma 3. 6 If the system (2. 5) is quadratically stable, then there exists a constant $\varepsilon > 0$ and a uniquely positive definite symmetric matrix P, such that

$$(A^0)^T P + P(A^0)^T + PD^0(D^0)^T P + (E^0)TE^0 + \varepsilon I = 0.$$

and $A^0 + D^0(D^0)^T P$ is asymptotically stable.

Lemmas 3. 5 and 3. 6 are consequences of the results in [8], the proofs are omitted.

Consider the matrix $T(n_0, n, q) \in R^{(n_0+q_n)\times(n_0+q_n)}$ given by

$$T(n_0,n,1) = \mathrm{diag}[I_{n_0} \quad I_n], \quad T(n_0,n,q) = \begin{bmatrix} I_{n_0} & 0 & 0 \\ 0 & I_n & T_1 \\ 0 & T_2 & T_3 \end{bmatrix}, \quad (q > 1), \tag{3. 4}$$

where $T_1 = [-I_n \quad -I_n \quad \cdots \quad -I_n]$, $T_2^T = [I_n \quad I_n \quad \cdots \quad I_n]$, $T_3 = \mathrm{diag}[I_n \quad I_n \quad \cdots \quad I_n]$, and I_k denotes a $k \times k$ identity matrix. Let the matrix $T \in R^{(n_0+N_n)\times(n_0+N_n)}$ defined as follows:

$$T = T(0)T(1)\cdots T(N-1) \tag{3. 5}$$

where $T(i) = \mathrm{diag}[T(n_0,n,N-i) \quad I_n \quad \cdots \quad I_n]$ $(i = 0,1,\cdots \quad N-1)$, $T(n_0,n,N-i)$ is given by (3. 4).

By computing directly, we have

Lemma 3. 7 Let A be given by (2. 2a), and $J = \mathrm{diag}[J_0 \quad J_1 \quad \cdots \quad J_1] \in R^{(n_0+N_n)\times(n_0+N_n)}$ with $J_0 \in R^{n_0\times n_0}$ and $J_1 \in R^{n\times n}$. Then the following equalities hold:

3. 7i) $T^{-1}AT = \mathrm{diag}[A_p \quad A_m \quad \cdots \quad A_m]$;

3.7ii) $T^{T}AT = \mathrm{diag}[A_q \quad N(N-1)A_m \quad \cdots \quad 6A_m \quad 2A_m]$, where

$$A_q = \begin{bmatrix} A_0 & NL_0 \\ NM_0 & N[A_1 + (N-1)H_1] \end{bmatrix};$$

3.7iii) $T^{T}JT = \mathrm{diag}[J_0 \quad NJ_1 \quad N(N-1)J_1 \quad \cdots \quad 6J_1 \quad 2J_1]$;

3.7iv) $T^{-1}J(T^{-1})^{T} = \mathrm{diag}[J_0 \quad 1/NJ_1 \quad 1/[N(N-1)] \quad J_1 \quad \cdots \quad 1/6J_1 \quad 1/2J_1]$.

Proof of Theorem 3.3

Sufficiency. Suppose that the conditions 3.1i) and 3.1ii) hold. By Lemma 3.5, there exist positive definite symmetric matrices P_p and P_m such that

$$Y_p(A_p,P_p) = A_p^{T}P_p + P_pA_p + P_pD_pD_p^{T}P_p + E_p^{T}E_p < 0, \tag{3.6a}$$

$$Y_m(A_m,P_m) = A_m^{T}P_m + P_mA_m + P_mD_mD_m^{T}P_m + E_m^{T}E_m < 0. \tag{3.6b}$$

Let $P_p = \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix} (P_{11} \in \mathrm{R}^{n\times n})$. Consider the matrix $P \in \mathrm{R}^{(n_0+N_n)\times(n_0+N_n)}$ defined as follows:

$$P = [g_{ij}], \quad (i,j = 0,1,\cdots,N) \tag{3.7}$$

where $g_{00} = P_{00}, g_{0i} = 1/NP_{01}, g_{i0} = 1/NP_{10}, g_{ii} = [P_{11} + N(N-1)P_m]/N^2$ and $g_{ij} = (P_{11} - NP_m)/N^2$, $(i,j = 1,\cdots,N; i \neq j)$. Let

$$Y(A,P) = A^{T}P + PA + PDD^{T}P + E^{T}E. \tag{3.8}$$

By computing directly, we have

$$DD^{T} = \mathrm{diag}[a_1a_1^{T} + Na_3a_3^{T} \quad D_mD_m^{T} \quad \cdots \quad D_mD_m^{T}], \tag{3.9a}$$

$$E^{T}E = \mathrm{diag}[a_2a_2^{T} + Na_6a_6^{T} \quad E_m^{T}E_m \quad \cdots \quad E_m^{T}E_m], \tag{3.9b}$$

$$D_pD_p^{T} = \mathrm{diag}[a_1a_1^{T} + Na_3a_3^{T} \quad 1/ND_mD_m^{T}], \tag{3.9c}$$

$$E_p^{T}E_p = \mathrm{diag}[a_2a_2^{T} + Na_6a_6^{T} \quad NE_m^{T}E_m]. \tag{3.9d}$$

From Lemma 3.7, equalities (3.7) and (3.9), it implies

$$T^{T}Y(AP)T = \mathrm{diag}[Y_p(A_p,P_p) \quad N(N-1)Y_m(A_m,P_m) \quad \cdots \quad 2Y_m(A_m,P_m)] < 0.$$

Since the matrix T is nonsingular, $Y(A,P) < 0$. By Lemma 3.5, the system Σ_u is quadratically stable.

Necessity. Suppose that the system Σ_u is quadratically stable. By lemma 3.6, there exists a constant $\varepsilon > 0$ and a uniquely positive definite symmetric matrix P such that $Y(A,P) + \varepsilon I = 0$ and $A + DD^{T}P$ is asymptotically stable. Similar to the above argument, it is easy to show that the conditions 3.1i) and 3.1ii) hold. Thus, the proof of Theorem 3.3 is completed. Q.E.D.

The proof of Theorem 3.4 is similar, and is omitted.

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一类具有对称内联子系统的不确定大系统的二次稳定性

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摘要: 本文研究了一类由若干个相似子系统和一个外部系统通过对称地内联组成的不确定大系统的二次稳定性问题.用二个低阶子系统以及相应的 H_∞ 范数的术语给出了这样一个系统是二次稳定的充分条件,并且证明了这个条件对于这个系统的过界系统的二次稳定性也是必要的.

关键词: 大系统; 稳定性分析; 模型不确定性

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