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Fuzzy Cell Mapping Based Methodology for Dealing with Complex Fuzzy Systems*

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Abstract: After the introduction of the concepts of the fuzzy cell and fuzzy cell space, the fuzzy cell mapping based methodology for dealing with complex systems is outlined with the emphasis on the modelling, identification and control. The proposed paradigm opens a new way to efficiently cope with multivariable fuzzy systems and also a way to handel fuzzy systems with the existing well-developed conventional theories. Its feasibility has been proved by the real-world applications.

Key words: fuzzy systems; fuzzy cell; fuzzy cell mapping; system identification; fuzzy control

1 Introduction

It is well known that the modelling, control and analysis of complex multivariable fuzzy systems are still the "bottleneck" problem in the research and application of fuzzy control and fuzzy systems[1,2]. Based on the existing works, this paper tries to deal with the following problems: ① How to make the fuzzy relational equation based approach efficiently applicable to the modelling and dynamical analysis of the multivariable fuzzy systems? ② How to organically combine the fuzzy approach and the well-developed conventional approach and to give full play to the merits of both paradigms?

The theoretical frame proposed in this paper is the so-called fuzzy cell mapping based methodoloty[3,4]. The essential concepts concerned are introduced in Section 2, and the fuzzy cell-to-cell mapping (FCM-Ⅰ) and the fuzzy cell-to-space mapping (FCM-Ⅱ) based aproaches are outlined respectively in Section 3 and Section 4. In Section 5, a numerical example is used to show the feasibility of the proposed approaches. The paper ends with some future topics.

2 Some Basic Concepts

Definition 2.1[3,4] A fuzzy cell XC_j in the space $X = X_1 \times X_2 \times \cdots \times X_n \ni x = \{x_1, x_2, \cdots, x_n\}$ is a normal and convex fuzzy subspace in X characterized by its induced membership function (MBF):

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$$\mu_{XC_j}(x) = \text{agg}\,\{\mu_{X_1'}(x_1), \mu_{X_2'}(x_2), \cdots, \mu_{X_n'}(x_n)\} : X \to [0,1]$$

where agg is an aggregating operator usually taken as min or prod. , $\mu_{X_i'}(x_i)$ is the MBF of the XC_j' constituent fuzzy subset in the universe $X_i \ni x_i$, The fuzzy cells $XC_j, j = 1, \cdots, N$ form the fuzzy cell space $XC = \{XC_1, XC_2, \cdots, XC_N\}$ corresponding to X, and the possibility measure vector $CX(x) = \{CX_1(x), CX_2(x), \cdots, CX_N(x)\} = \{\mu_{XC1}(x), \cdots, \mu_{XCN}(x)\}$ is called the fuzzy cell vector or equally the possibility distribution of x on XC.

Definition 2. 2[5,6] Let the space X be partitioned into N fuzzy cells $XC_i, i = 1, \cdots, N$. Y be partitioned into M fuzzy cells $YC_j, j = 1, \cdots, M$, then the collection of the fuzzy cell-to-cell mappings $R_{ij}: XC_j \to YC_j, i = 1, \cdots, N, j = 1, , M$ is called the fuzzy cell-to-cell mapping (FCM- I) from the cell space XC to the cell space YC; and the collection of the fuzzy cell-to-space mappings $R_j: XC_j \to Y, j = 1, \cdots, N$ is caled the fuzzy cell-to-space mapping (FCM- II) from the cell space XC to the space Y.

3 Fuzzy Cell-to-Cell Mapping Based Approach

3. 1 The FCM- I Model of MIMO Fuzzy Systems

Let the input space $X \subset R^n$ and the output space $Y \subset R^m$ of a MIMO free system be partitioned respectively into the input cell sapce $XC = \{XC_1, XC_2, \cdots, XC_N\}$ and the output cell space $YC = \{YC_1, YC_2, \cdots, YC_M\}$, then the model constructed on the basis of the fuzzy cell-to-cell mapping form XC to YC is the FCM- I model of the system in the following form[4,6]:

$$CY(y) = CX(x) \circ R = CX(x) \circ [R_{ij}]_{N \times M} \tag{3. 1}$$

where $CY(y)$ and $CX(x)$ are respectively the M-dimensional output cell vector of the m-dimensional output vector y, and the N-dimensional input cell vector of the n-dimensional input cell vector x, "$\circ$" is the compositional inference operator ususlly taken as Zadeh's max-min($\vee$ - $\wedge$) operator[7]. R, with its generic entry R_{ij} being mapping intensity from the cell XC_i to the cell YC_j, is called the fuzzy cell mapping relational matrix.

It is not difficult to generalize the model (3. 1)to a forced MIMO system, and the resulted FCM- I model is as:

$$CY(y) = [CU(u) \times CX(x)] \circ R = [CU(u) \times CX(x)] \circ [R_{ijk}]_{L \times N \times M} \tag{3. 2}$$

where $CU(u)$ is the fuzzy cell in the control space U. The entry $R_{ijk}: UC_i \times XC_j \to YC_k$ is the mapping intensity from the control cell UC_i and the state cell XC_j to the output cell YC_k.

It is worth noting that with FCM- I , any free MIMO system can always be charactreized by a two-dimensional fuzzy relational model and any forced MIMO system by a three-dimensional fuzzy relational model, and this will obviously reduce the complexity of using fuzzy relational equations in dealing with complex high dimensional MIMO systems.

3. 2 Identification of FCM- I Systems

When modelled with FCM- I , the identification of FCM- I modelled system is to determine the matrix $\boldsymbol{R}$ according to the system's I/O data. The general procedure is firstly to get the K fuzzy cell vector associations $\{CX(x^k), CY(y^k)\}$ from the K I/O sample pairs $(x^k, y^k), k = 1, 2, \cdots, K$, and then to determine the cell mapping relatinal matrix $\boldsymbol{R}$ from the K cell vector associations. What follow are several specific approaches.

3.2.1 Logical implication (LI) Approach

With LI approach, every association $\{CX(x^k), CY(y^k)\}$ is regarded as a logical implication:

$$R^k = CX(x^k) \rightarrow CY(y^k) = CX(x^k) \odot CY(y^k) \tag{3.3}$$

where $\odot$ is the implication operator most commonly taken as min($\wedge$) or prod. ($\cdot$)[1] and here taken as min$=\wedge$. Then (3.3) becomes:

$$R^k = [R^k_{ij}]_{N\times M} = [CX_i(x^k) \wedge CY_j(y^k)]_{N\times M}, \quad k = 1, \cdots, K \tag{3.4}$$

and the aggregation of the K logical implication gives out the final fuzzy cell-to-cell mapping relation:

$$R^k = \bigcup_{k=1,\cdots,K} R^k = [R_{ij}]_{N\times N} = [\vee_{k=1,\cdots,K} R^k_{ij}]_{N\times M}. \tag{3.5}$$

3.2.2 Equation resolution (ER) Approach

For data pairs (x^k, y^k), there should be the following equation holding:

$$CY(y^k) = CX(x^k) \circ R, \quad k = 1, 2, \cdots, K \tag{3.6}$$

then the greatest solution is[8]:

$$\hat{R}^k = CX(x^k) \circledast CY(y^k) = [R^k_{ij}]_{N\times M} \tag{3.7}$$

and

$$\hat{R}^k_{ij} = CX_i(x^k) \circledast CY_j(y^k) \tag{3.8}$$

with the operator $\circledast$ defined as: $a \circledast b = b$, if $a > b$; 1, if $a \leqslant b$, for $a, b \in [0,1]$. Then the greatest solution to (3.8) is:

$$\hat{R} = \bigcap_{k=1,K} \hat{R}^k = [\hat{R}_{ij}]_{N\times M} = [\wedge_{k=1,K} \hat{R}^k_{ij}]_{N\times M} \tag{3.9}$$

which is the fuzzy cell-to-cell mapping relational matrix we are searching for.

3.2.3 Optimization based (OB) approach

For every equation in (3.6), the greatest solution is given in (3.7) and denoted here as R^k. A lower solution is obtained as[8]:

$$\hat{R}^k = CX(x^k) \textcircled{\beta} CY(y^k) = [R^k_{ij}]_{N\times M}, \tag{3.10}$$

$$\hat{R}^k_{ij} = CX_i(x^k) \textcircled{\beta} CY_j(y^k) \tag{3.11}$$

with the operator $\textcircled{\beta}$ defined as: $a \textcircled{\beta} b = 0$, if $a > b$; b, if $a < b$, for $a, b \in [0,1]$. Then a lower solution to (3.6) is:

$$\check{R} = \bigcup_{k=1,K} \check{R}_k = [\check{R}_{ij}]_{N\times M} = [\vee_{k=1,K} \check{R}^k_{ij}]_{N\times M}. \tag{3.12}$$

It is easy to prove that[5]:

$$R = \lambda\hat{R} + (1-\lambda)\check{R} = R(\lambda), \quad \lambda \in [0,1] \tag{3.13}$$

is the general solution to the equation system (3.6).

So through tuning the parameter in (3. 13) to its optimal value with respect to some specifically defined performance index, the unique optimal marix is thus obtained. It is interesting to note that this approach overcomes the drawback of the existing approaches[1] caused by the non-uniqueness of the solution, and makes it possible to model and indetify the fuzzy systems on the basis of optimization.

3. 2. 4 Approximate Approach (AP)

For a MIMO system with too many variables, to reach the goal of meeting the applicational requirement with a FCM- I model as simple aspossible, some measures can be taken. The key idea here is to use the significant input variables and the significant input and output fuzzy cells in identifying the model of the object systems. These significant variables and the significant fuzzy cells can be determined from the sampled data pairs[9].

· The approximate FCM- I model of a MIMO system with n input variables and m output variables can be identified through the following procedure:

Step 1 Determine the significant variable vector $x = (x_1, x_2, \cdots, x_{n'})$. It is obvious that $n' \leqslant n$. $X \ni x$ is called the significant input variable space.

Step 2 Partition the space X and the output space Y into respectively N and M fuzzy cells.

Step 3 Determine the significant input fuzzy cells and the significant output fuzzy cells, respectively as $\{XC_1, XC_2, XC_{N'}\}$ and $\{YC_1, YC_2, \cdots, YC_{M'}\}$. It is obvious that $N' \leqslant N$ and $M' \leqslant M$.

Step 4 Determine the fuzzy cell mapping relational matrix R with the significant fuzzy cells and thus obtaine the approximate FCM- I model. R is obviously simpler than those obtained directly through other approaches.

Up to now, the FCM- I based approaches have been outlined, and the feasibility of the FCM- I based paradigm is shown in the numerical example given in Section 5.

4 Fuzzy Cell-to-Space Mapping (FCM- Ⅱ) Based Method

4. 1 The FCM- Ⅱ Model of MISO Fuzzy Systems

The basic idea behind the FCM- Ⅱ based method is that[3,9]: any complex system is in reality the aggregation of some explicit or implicit parallelly functioning simpler subsystems, these subsystems dominate respectively in different parts of the system variable space; and the controller for a complex system can also be considered as the aggregation of some explicit or implicit parallelly functioning simpler subcontroller. The FCM- Ⅱ based modelling is firstly to eastablish the mapping relation from every input fuzzy cell XC_i to the whole output space Y, i. e. , a fuzzy cell-to-space mapping $R_i : XC_i \rightarrow Y$, which is a submodel reflecting the local I/O relation of the system informally expressed as:

$$R_i: \quad \text{IF} \quad x(t) \quad \text{IS} \quad XC_i \quad \text{THEN} \quad y(t) = f_i(t, x(t), \theta_i), \quad i = 1, 2, \cdots, N \tag{4. 1}$$

where $f_i(\cdot)$ may be in any suitable form parameterized with θ_i, and then to aggregate all the N submodels to obtain the model of the whole system $R = \bigcup_{i=1,N} R_i$ specifically expressed as:

$$y(t) = \sum\nolimits_{i=1,N} p_i(x(t)) \cdot f_i(t, x(t), \theta_i) \tag{4.2}$$

where

$$p_i(x(t)) = XC_i(x(t)) / \sum\nolimits_{j=1,N} XC_j(x(t)) \tag{4.3}$$

is the relative mapping intensity of $x(t)$ from the cell XC_i to the space Y and measures the influence and contribution of the subsystem $f_i(\cdot)$ to the behavior of the whole sysem. It's obvious that the T-S model[10] is a special case of (4.2) when $f_i(\cdot)$ takes the linear form.

From (4.3) it is easy to prove that relative intensities $p_i(x(t))$ have the following properties:

$$\begin{aligned} &XC_i(x(t)) \leqslant XC_j(x(t)) \Leftrightarrow p_i(x(t)) \leqslant p_j(x(t)), \quad i \neq j, \\ &0 \leqslant p_i(x(t)) \leqslant 1, \quad \wedge \sum\nolimits_{i=1,N} p_i(x(t)) = 1 \end{aligned} \tag{4.4}$$

which implies that for every input $x(t)$, $\{p_i(x(t)), i = 1, 2, \cdots, N\}$ forms a probability distribution generated by its fuzzy cell distribution, and the output of (4.2) is the mathematical expect of the outputs of all the submodels over the probability distribution.

It is also reasonable to conclude that any distribution satisfying (4.4) can be used to construct the FCM-Ⅱ model of the MISO systems. As a try, the generalized relative mapping intensity could be defined as:

$$p_i(x(t)) = \frac{d_i(x(t))}{\sum\nolimits_{j=1,N} d_j(x(t))} \tag{4.5}$$

and

$$d_i(x(t)) = \| x(t) - a^i \|^{\gamma}, \quad i = 1, 2, \cdots, N \tag{4.6}$$

where $\| \cdot \|$ is a norm measuring the distance between $x(t)$ and the kernel of the fuzzy cell XC_i, γ is a negative parameter. Because of the fact that $p_i(x(t))$ is general enough and has nothing to do with the definition of the MBFs, the FCM-Ⅱ model with $p_i(x(t))$ defined in this way will be much more flexible to fit the different applications without the overdependency on the situation-specific heuristics. So the FCM-Ⅱ model defined this way is called the generalized FCM-Ⅱ (GFCM-Ⅱ) model which might cover all the existing system model due to the generality of $f_i(\cdot)$ and the continuous variation of γ. The readers interested in the identification of FCM-Ⅱ modelled systems are referred to [3, 9, 10].

4.2 FCM-Ⅱ Based Control

When constructed in FCM-Ⅱ, the resulted FCM-Ⅱ based controller is in the following form:

$$u(t) = \sum\nolimits_{j=1,N} p_j(x(t)) \cdot f_j(t, x(t), \theta_j) \tag{4.7}$$

with $p_j(x(t))$ defined in (4.3), and the GFCM-Ⅱ controller is that with $p_j(x(t))$ defined in (4.5). Some interesting structural properties of the GFCM-Ⅱ Controller are given in the following theorem.

Theorem 1 For GFCM-Ⅱ controller (4.7), the following propositions are true:

1) When $|\gamma| \to 0$, the GFCM- Ⅱ controller becomes a constant structure controller,

2) when $0 < |\gamma| < \infty$, the GFCM- Ⅱ controller is a non-linear continuously variable structure controller consisting of N parallel constant structure subcontrollers with non-linear switching function;

3) When $|\gamma| \to \infty$, the GFCM- Ⅱ controller is a conventional variable structure controller with boolean switching function.

From the theorem, it is easy to conclude that the GFCM- Ⅱ controller covers all the existing controller of different types with $f_j(\cdot)$ and γ specifically defined due to the generality of $f_j(\cdot)$. Although the systematic design procedure of GFCM- Ⅱ controller is still not available, the fully parameterized analytical structure of FCM- Ⅱ and GFCM- Ⅱ controller will certainly facilitate not only the synthesis and analysis, but also the adaptation of fuzzy controller as well, through the synthetic usage of both the existing conventional paradigm and the fuzzy approaches.

5 Illustrative Examples

The well-known furnace data of Box and Jenkins[11] is used to justify the methods proposed in this paper, and the comparative study is also made. The results are shown in Table 1 and Table 2, which indicate clearly the superiority of what proposed here to the existing fuzzy approaches.

Table 1 FCM- I system identification results

Aproaches	LI	ER	OB $\lambda_{opt} = 0.731$	AP
performance Index $J = \sum_{k=1}^{K}(y^k - \hat{y}^k)^2/K$	0.453	0.527	0.312	0.725

Table 2 Comparison of fuzzy models

Models	Tong's[5]	Pedryzc's[1]	Box's[11]	FCM- I $\lambda_{opt} = 0.731$	FCM- Ⅱ [9]
performance Index $J = \sum_{k=1}^{K}(y^k - \hat{y}^k)^2/K$	0.469	0.478	0.202	0.312	0.174

6 Concluding Remarks

The fuzzy cell mapping based methodology is outlined with the emphasis on the modelling, identification and controller structure concerning with multivariable fuzzy systems. The paradigm proposed makes it possible to use not only the fuzzy relational approaches but also the existing conventional aproaches and the fuzzy approaches in dealing with complex MIMO fuzzy systems. As a developing paradigm, there are many topics to be tackled to make the fuzzy cell mapping based methodology more applicable, what follows are, not limited to, some of them:

1) The general design scheme to the FCM- Ⅱ based feedback controller,

2) The effective complex system analysis through fuzzy cell-to-cell mapping;

3) The combination of the soft-computing based approach with the ideas revealed in this paper, etc.

References

[1] Pedrycz, W.. Fuzzy Control and Fuzzy Systems. Res. Stud. Press Ltd. ,1992

[2] Gupta, M. et al.. Multivariable Structure of Fuzzy Control Systems. IEEE Trans, 1986, SMC-16:638—656

[3] Jia, L-M and Zhang, X-D. Fuzzy Cell Mapping Approach to Complex System Modelling and Control. Proc. of IEEE SICICI'92, 1992, 2:1380—1385

[4] Jia, L-M and Zhang, X-D. Fuzzy Cell Mapping Based Approach To the Multivariable Fuzzy System Modelling. Control & Decision, 1993, 8(4):271—277

[5] Jia, L-M and Zhang X-D. Fuzzy Sytem Identification Through Fuzzy Cell-to-Cell Mapping. Proc. of IFAC'93, 1993, 5:895—899

[6] Jai, L-M, Zhang, X-D and Xie, Z-T. Outline of the Fuzzy Cell Mapping Based Paradigm For Dealing with Fuzzy Multivariable Systems. in Industrial Application of Soft Computing (P. Ralston et al. , eds.), IFSIC Press, 1994, 218—231

[7] Zadeh, L. A.. Towards a Theory of Fuzzy Systems. in Aspects of Networks and System Theory (R. E. Kalman, et al eds.), 1971, 469—492

[8] Di Nola, W. et al.. Fuzzy Relation Equations and Their Applications to Knowledge Engineering. Kluwer Academic Press, 1989

[9] Jia, L-M and Zhang, X-D. Fuzzy Cell Mapping Approach To the Modelling and Control of Complex Process with Applications. Proc of China CDC'92, North-Eastem Polytech Univ. Press, 1992, 269—275

[10] Takagi, T. and Sugeno, M.. Fuzzy Identification of Systems and Its Application to Modelling and Control. IEEE Trans. 1985, SMC-15:116—132

[11] Box, G. E. P. and Jenkins, G. M.. Time Series, Analysis, Forcasting and Control. Holden Day, 1970

处理复杂模糊系统的模糊穴映射方法

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摘要:在对模糊穴和模糊穴空间等基本概念作一简单介绍之后,本文着重从模糊多变量系统的建模,辨识和控制等方面出发概述了基于模糊穴映射的理论和方法.本文的方法为有效地处理多变量模糊系统和利用已有的传统方法来解模糊系统问题开辟了一条新的途径.本文建立的方法的有效性已被实际的应用所证明.

关键词:模糊系统;模糊穴;模糊穴映射;系统辨识;模糊控制

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