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Delay-Dependent Stability for Linear Uncertain Time-Delay Systems

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Abstract: In this paper, a sufficient condition for the stability of linear uncertain systems with a time-varying delay is presented, Razumikhin's technique and a vector inequality are employed to derive the condition. The obtained criterion can include information on the size of delay, and therefore, belongs to delay-dependent criteria. Also, an illustrative example is given to show that the obtained ctiterion is better than the existing one in the literature.

Key words: delay-dependent stability; time-delay systems; linear uncertain systems

1 Introduction

Time delay is commonly encountered in various engineering systems, such as chemical processes, long transmission lines in pneumatic, hydraulic, and rolling mill systems. In general, the existence of time delay makes the stability analysis of time-delay systems much more complicated, and many researchers have been seeking various convenient methods to check the stability over the past decade. The existing stability criteria for linear time-delay systems can be classified into two categories according to their dependence upon the size of delays. One category which do not include information on delay is called delay-independent criteria[1~12], another carrying information on the delays is referred to as delay-dependent criteria[13~16]. However, abandonment of information on the delays in the former catalogue generally causes conservativeness of the criteria especially when the delay is comparatively small[14].

In this paper, a delay-dependent stability criterion for linear uncertain time-delay systems is established by employing Razumikhin's technique and a vector inequality. An example is given to show the application of the obtained criterion and to compare the results with those given in [15,16]. The organization of this paper in the following is: Section 2 develops the main theorem, Section 3 provides an illustrative example, and the last section ends with the conclusion.

Notation x^{T} and M^{T} denote the transpose of a vector $x \in \mathrm{R}^n$ and a matrix $M \in \mathrm{R}^{n \times n}$, respectively. $\lambda_{\max}(M)$ and $\lambda_{\min}(M)$ denote the maximum and minimum eigenvalue of M, respectively. $\|x\| = (\sum_{i=1}^{n} |x_i|^2)^{1/2}$, $\|M\| = [\lambda_{\max}(M^{\mathrm{T}}M)]^{1/2}$, and $\mu(M) = \lambda_{\max}[(M + M^{\mathrm{T}})/2]$.

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2 Main Results

Let us consider the linear uncertain time-delay system described by the following differential-difference equation of the form

$$\dot{x}(t) = (A + \Delta A)x(t) + (A_1 + \Delta A_1)x(t - \tau),$$
$$x(t) = \varphi(t), \quad t \in [-r, 0] \tag{1}$$

where $x \in \mathrm{R}^n$ is the state, $A \in \mathrm{R}^{n\times n}$ and $A_1 = \mathrm{R}^{n\times n}$ are constant matrices, $\tau = \tau(t) \geqslant 0$ denotes a time-varying delay with a upper bound $r > 0$ which will be derived in this paper, $\varphi(t)$ denotes a continuous vector-valued initial function, ΔA and ΔA_1 are linear parameter uncertainties with bounds as follows:

$$\| \Delta A \| \leqslant \| \alpha \|, \quad \| \Delta A_1 \| \leqslant \alpha_1, \tag{2}$$

where α and α_1 which are greater than zero are given.

Now let us consider the initial time to be zero and let $x(t), t \geqslant 0$, be the solution of system (1) through $(0, \varphi)$. Since $x(t)$ is continuously differentiable for $t \geqslant 0$, one can write[17]

$$x(t-\tau) = x(t) - \int_{t-\tau}^{t} \dot{x}(s)\mathrm{d}s$$
$$= x(t) - \int_{t-\tau}^{t} [(A + \Delta A)x(s) + (A_1 + \Delta A_1)x(s - \tau)]\mathrm{d}s \tag{3}$$

for $t \geqslant \tau$. If we return to (1) using this expression for $x(t-\tau)$, we construct the following auxiliary problem

$$\dot{x} = (A + \Delta A + A_1 + \Delta A_1)x(t)$$
$$- (A_1 + \Delta A_1)]\int_{t-\tau}^{t} [(A + \Delta A)x(s) + (A_1 + \Delta A_1)x(s - \tau)]\mathrm{d}s,$$
$$x(t) = \Psi(t), \qquad t \in [-2r, 0], \tag{4}$$

where $\Psi(t)$ is arbitrary continuous function on $[-2r, 0]$. If the zero solution of (4) is asymptotically stable, then the zero solution of (1) is asymptotically stable since (1) is a special case of (4)[17].

Theorem 1 Suppose that $A + A_1$ is asymptotically stable and $\mu(A + A_1) < 0$. Then system (1) is asymptotically stable if

$$\tau(t) < r = \frac{-\mu(A + A_1) - (\alpha + \alpha_1)}{\sqrt{2\beta}}, \tag{5}$$

where

$$\beta = \| A_1(AA^{\mathrm{T}} + A_1A_1^{\mathrm{T}})A_1^{\mathrm{T}} \| + \| AA^{\mathrm{T}} + A_1A_1^{\mathrm{T}} \| (2\alpha_1 \| A_1 \| + \alpha_1^2)$$
$$+ (2\alpha \| A \| + 2\alpha_1 \| A_1 \| + \alpha^2 + \alpha_1^2)(\| A_1 \| + \alpha_1)^2. \tag{6}$$

Proof Let

$$V(x) = x^{\mathrm{T}}x, \quad x \in \mathrm{R}^n. \tag{7}$$

By deriving the derivate of V along the solution of (4), we obtain

$$\dot{V} = 2x^{\mathrm{T}}(t)\dot{x}(t)$$

$$=x^{\mathrm{T}}(t)[(A+\Delta A+A_1+\Delta A_1)+(A+\Delta A+A_1+\Delta A_1)^{\mathrm{T}}]x(t)$$

$$-2x^{\mathrm{T}}(t)(A_1+\Delta A_1)\int_{t-\tau}^{t}[(A+\Delta A)x(s)+(A_1+\Delta A_1)x(s-\tau)]\mathrm{d}s$$

$$\leqslant x^{\mathrm{T}}(t)[(A+A_1)+(A+A_1)^{\mathrm{T}}]x(t)+\|x^{\mathrm{T}}(t)[(\Delta A+\Delta A_1)+(\Delta A+\Delta A_1)^{\mathrm{T}}]x(t)\|$$

$$+\int_{t-\tau}^{t}2x^{\mathrm{T}}(t)(A_1+\Delta A_1)(A+\Delta A)x(s)\mathrm{d}s$$

$$+\int_{t-\tau}^{t}2x^{\mathrm{T}}(t)(A_1+\Delta A_1)(A_1+\Delta A_1)x(s-\tau)\mathrm{d}s. \tag{8}$$

In (8), we have

$$\begin{aligned}&\|x^{\mathrm{T}}(t)[\Delta A+\Delta A_1)+(\Delta A+\Delta A_1)^{\mathrm{T}}]x(t)\|\\&=\|2x^{\mathrm{T}}(t)(\Delta A+\Delta A_1)x(t)\|\\&\leqslant 2(\|\Delta A\|+\|\Delta A_1\|)\|x(t)\|^2\\&\leqslant 2(a+a_1)\|x(t)\|^2.\end{aligned} \tag{9}$$

Furthermore, by using the following inequality

$$2u^{\mathrm{T}}\nu\leqslant\frac{1}{\varepsilon}u^{\mathrm{T}}u+\varepsilon\nu^{\mathrm{T}}\nu,\quad u,\nu\in\mathrm{R}^n. \tag{10}$$

where $\varepsilon>0$ is any real constant, we can obtain

$$\int_{t-\tau}^{t}2x^{\mathrm{T}}(t)(A_1+\Delta A_1)(A+\Delta A)x(s)\mathrm{d}s$$

$$\leqslant\int_{t-\tau}^{t}[\frac{1}{\varepsilon}x^{\mathrm{T}}(t)(A_1+\Delta A_1)(A+\Delta A)(A+\Delta A)^{\mathrm{T}}(A_1+\Delta A_1)^{\mathrm{T}}x(t)+\varepsilon x^{\mathrm{T}}(s)x(s)]\mathrm{d}s$$

$$\leqslant\frac{\tau}{\varepsilon}x^{\mathrm{T}}(t)(A_1+\Delta A_1)(A+\Delta A)(A+\Delta A)^{\mathrm{T}}(A_1+\Delta A_1)^{\mathrm{T}}x(t)+\varepsilon\tau\|x(\xi_1)\|^2, \tag{11}$$

and, similarly,

$$\int_{t-\tau}^{t}2x^{\mathrm{T}}(t)(A_1+\Delta A_1)(A_1+\Delta A_1)x(s-\tau)\mathrm{d}s$$

$$\leqslant\frac{\tau}{\varepsilon}x^{\mathrm{T}}(t)(A_1+\Delta A_1)(A_1+\Delta A_1)(A_1+\Delta A_1)^{\mathrm{T}}(A_1$$

$$+\Delta A_1)^{\mathrm{T}}x(t)+\varepsilon\tau\|x(\xi_2)\|^2. \tag{12}$$

In (11) and (12), $\xi_1\in[t-\tau,t]$ and $\xi_2\in[t-2\tau,t-\tau]$, respectively. Substituting (9), (11) and (12) into (8), we obtain

$$\dot V\leqslant 2\mu(A+A_1)\|x(t)\|^2+2(a+a_1)\|x(t)\|^2+2\varepsilon\tau\|x(\xi)\|^2$$

$$+\frac{\tau}{\varepsilon}\|(A_1+\Delta A_1)(A+\Delta A)(A+\Delta A)^{\mathrm{T}}(A_1+\Delta A_1)^{\mathrm{T}}$$

$$+(A_1+\Delta A_1)(A_1+\Delta A_1)(A_1+\Delta A_1)^{\mathrm{T}}(A_1+\Delta A_1)^{\mathrm{T}}\|\,\|x(t)\|^2$$

$$<-\{-2\mu(A+A_1-2(\alpha+\alpha_1)-2\varepsilon rq-\frac{r}{\varepsilon}\beta\}\|x(t)\|^2, \tag{13}$$

where $\xi\in[t-2\tau,t]$, $q>1$ is a constant, and β is defined by (6). In (13), Razumikhin's technique[17] has been used, that is

$$\|x(t+\theta)\|<q\|x(t)\|,\quad\theta\in[t-2\tau,t],\quad q>1. \tag{14}$$

Let

$$\varepsilon = \frac{-\mu(A + A_1) - (\alpha + \alpha_1)}{2r}. \tag{15}$$

We have

$$\dot{V} < -\rho \| x(t) \|^2, \tag{16}$$

where

$$\rho = -\mu(A + A_1) - (\alpha + \alpha_1) - \frac{2r^2\beta}{-\mu(A + A_1) - (\alpha + \alpha_1)} \tag{17}$$

If the condition (5) of the theorem is satisfied, then $\rho \geqslant 0$ and $\dot{V} < 0$. This proves the theorem[17]. Q. E. D.

Remark 1 If using the method in [15], we obtain the following bound for τ[16]:

$$\tau(t) < \frac{\lambda_{\min}(Q) - 2\|P\|(\alpha + \alpha_1)}{2\|P\|(\|P\| \|P^{-1}\|)^{1/2}\beta}, \tag{18}$$

where P is the solution of Lyapunov equation $P(A + A_1) + (A + A_1)^T P = -Q$ with $Q > 0$. We will compare (18) with (5) by the same example in [15,16] in the following section and show that (5) is better than (18) for the considered example.

3 An Illustrative Example

Example 1 Let us consider the following linear uncertain time-delay system[15,16]

$$\dot{x}(t) = (A + \Delta A)x(t) + (A_1 + \Delta A_1)x(t - \tau(t)),$$

where

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, \quad \Delta A = \begin{bmatrix} 0.3\cos t & 0 \\ 0 & 0.2\sin t \end{bmatrix},$$

$$A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \quad \Delta A_1 = \begin{bmatrix} 0.2\cos t & 0 \\ 0 & 0.3\sin t \end{bmatrix},$$

and $\|\Delta A\| = 0.3$, $\|\Delta A_1\| = 0.3$. We have

$$(A + A_1) + (A + A_1)^T = \begin{bmatrix} -6 & -1 \\ 1 & -4 \end{bmatrix}.$$

Therefore, $\mu(A + A_1) = -1.7929$, $\alpha = \alpha_1 = 0.3$, and $\beta = 28.3918$. From (5), we obtain $\tau(t) < 0.1583$. A stable simulation result for case $\tau(t) \equiv 0.16$ can be found in [15].

Discussion Form (18), we obtain the bounds [16]: $\tau(t) < 0.1575$ for case $P = I$ and $\tau(t) < 0.1198$ for case $Q = I$, respectively. Both of them are small than the obtained result.

4 Conclusion

A stability criterion for the linear uncertain systems with a time-varying delay is derived by employing Razumikhin's technique togather with a vector inequality. The obtained result can include information on the delay τ. The stability criterion is delay dependent and less conservative than the existing delay dependent criterion. The provided example shows that it is convenient to use the obtained criterion for checking the stability of linear uncertain time-delay systems.

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线性不确定性时滞系统的滞后相关稳定性

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摘要:本文采用拉什密辛技巧和一个矢量不等式建立了一个具有变时滞的线性不确定性系统的稳定性充分条件.所得的稳定性判据包含了时滞的信息,属于滞后相关稳定性判据.一个说明例子证明了所给

的判据好于文献中已有的判据.

关键词：滞后相关稳定性；时滞系统；线性不确定性系统

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