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Robust Circular Pole Placement via Dynamic Output Feedback for Perturbed Linear Systems *

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Abstract: This paper discusses design of robust dynamic output feedback control laws for a class of perturbed linear systems that place the closed-loop poles in a specified circular region of complex plane. Utilizing a modified Riccati eqution, existence conditions of the robust controller are given, and the general solution of the robust controller is parameterized by the inverse solution of this modified Lyapunov equation.

Key words: perturbed linear systems; robust pole placement; dynamic output feedback

1 Introduction

Pole placement is a fundamental problem in control theory and system synthesis. Much of the pole placement literature focuses on the problem of exact pole placement in which closed-loop poles are required to lie at (or arbitrary close to) given locations[1]. However, generally the exact closed-loop pole locations are not required in practice, for example American Army Services Standard MIL-F-8785B confines that the closed-loop poles of man manued plane to a "clipped" sector region in Fig. 1[2]. In fact, if the closed-loop poles are confined in the region, then the closed-loop system can have good transient behavior and the system modes damp at desired rate. Another motivation for us to study regional pole placement is that usually modelling error or system uncertainty is inavoidable in practical engineering.

By now there are no direct design techniques which can place the closed-loop poles in the "clipped" sector. Some indirect method have been proposed by [1] and [3] and some of their references by utilizing a variety regions such as circular, elliptic, parabolic region to approximate the region in Fig. 1. For deterministic system, [1] and [3] have studied the design of control laws that place closed-loop poles in a given circular region $\Omega(q,r)$ with center at $(-q,0)$ and radius $r<q$ (Fig. 2). In this paper we utilize a modified Lyapunov equation to study the robust circular pole placement via dynamic output feedback for a class of perturbed systems. The above problem is first converted into an auxillary " Q -matrix assi-

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gnment" problem, then the necessary and sufficient conditions of its assignability are derived, and the desired robust controller is parameterized by inverse solution of the modified Lyapunov equation.

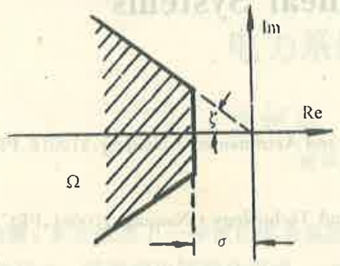

Fig. 1 "Clipped" sector region

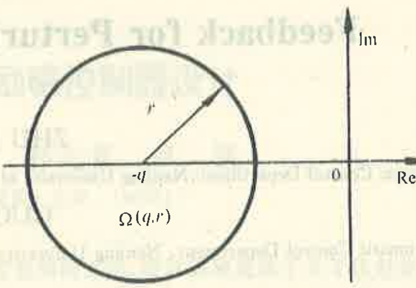

Fig. 2 Circular pole placement region

2 Preliminaries

Consider a perturbed linear continuous system

$$\begin{cases} \dot{x}_p(t) = (A_p + \Delta A_p)x_p(t) + B_p u(t), \\ y_p(t) = C_p x_p(t) \end{cases} \tag{2.1}$$

where state vector $x_p(t) \in \mathrm{R}^{n_x}$, output vector $y_p(t) \in \mathrm{R}^{n_y}$, control vector $u_p(t) \in \mathrm{R}^{n_u}$ and $A_p, \Delta A_p, B_p, C_p$ are matrices with proper dimensions. Perturbation ΔA_p describes the uncertainty of the system and it belongs to the following set:

$S \triangleq \{\Delta A_p \mid \Delta A_p = E_p \Sigma F_p, \sigma_{\max}(\Sigma) \leqslant 1$, the elements of Σ are Lebesgue measurable$\}$

where Σ is the uncertainty, and E_p and F_p are known real matrix which characterized the structure of the uncertainty [4], $\sigma_{\max}[\cdot]$ denotes the maximum singular value of $[\cdot]$.

A generalized class of dynamic output feedback controllers is described by

$$\begin{cases} \dot{z}(t) = G_{22}z(t) + G_{21}y_p(t), \\ u(t) = G_{12}z(t) + G_{11}y_p(t). \end{cases} \tag{2.2}$$

where $z(t) \in \mathrm{R}^{n_z}, G_{22} \in \mathrm{R}^{n_z \times n_z}, G_{21} \in \mathrm{R}^{n_z \times n_y}, G_{12} \in \mathrm{R}^{n_u \times n_z}$ and $G_{11} \in \mathrm{R}^{n_u \times n_y}$,

By defining

$$x(t) = \begin{bmatrix} x_p(t) \\ z(t) \end{bmatrix}, \quad y(t) = \begin{bmatrix} y_p(t) \\ z(t) \end{bmatrix},$$

$$A = \begin{bmatrix} A_p & 0 \\ 0 & 0 \end{bmatrix}, \quad \Delta A = \begin{bmatrix} \Delta A_p & 0 \\ 0 & 0 \end{bmatrix} = E\Sigma F = \begin{bmatrix} E_p \\ 0 \end{bmatrix} \Sigma [F_p \quad 0],$$

$$B = \begin{bmatrix} B_p & 0 \\ 0 & I_{n_z} \end{bmatrix}, \quad C = \begin{bmatrix} C_p & 0 \\ 0 & I_{n_z} \end{bmatrix}, \quad G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}.$$

the closed-loop system can be expressed as

$$\begin{cases} \dot{x}(t) = (A_c + \Delta A)x(t), \quad A_c = A + BGC, \\ y(t) = Cx(t). \end{cases} \tag{2.3}$$

With these preliminaries the robust circular pole placement problem via dynamic feedback is to be solved in next section.

3 Main results

Define $Q_1 \triangleq \beta_1 EE^{\mathrm{T}} + \dfrac{QF^{\mathrm{T}}FQ}{\beta_1}$, $Q_2 \triangleq QF^{\mathrm{T}}(\beta_2 I - FQF^{\mathrm{T}})^{-1}FQ$.

Theorem 3.1 Assume $\beta_1, \beta_2 > 0$ and $\beta_2 I - FQF^{\mathrm{T}} > 0$ if the following equation

$$A_c(Q+Q_2)A_c^{\mathrm{T}} + q(A_cQ + QA_c^{\mathrm{T}} + Q_1) + (q^2 - r^2)Q + \beta_2 EE^{\mathrm{T}} + W = 0 \quad (3.1)$$

exists symmetric positive definite solution Q, then closed-loop poles lie in circular region $\Omega(q,r)$ i.e. $\Lambda(A_c + \Delta A) \subset \Omega(q,r)$, where $W > 0$ is arbitrary.

Proof By lemma 3.1 in [5], from equation (3.1) we can know

$$(A_c + \Delta A)Q(A_c + \Delta A)^{\mathrm{T}} + q[(A_c + \Delta A)Q + Q(A_c + \Delta A)^{\mathrm{T}}] + (q^2 - r^2)Q + W < 0 \quad (3.2)$$

It is obvious that[3] $\Lambda(A_c + \Delta A) \subset \Omega(q,r)$.

Since $Q + Q_2 > 0$, it has unique positive definite square root $(Q+Q_2)^{1/2}$. Define

$Q_0 \triangleq (q^2 - r^2)Q + \beta_2 EE^{\mathrm{T}} + W$,

$$P \triangleq [A(Q+Q_2) + qQ](Q+Q_2)^{-1}[qQ + (Q+Q_2)A^{\mathrm{T}}] - A(Q+Q_2)A^{\mathrm{T}} - q(AQ + QA^{\mathrm{T}} + Q_1) - Q_0,$$

$$M \triangleq \begin{bmatrix} (I - BB^+)T \\ (I - C^- C)(Q+Q_2)^{-1/2} \end{bmatrix} = U_M \begin{bmatrix} \Lambda_M & 0 \\ 0 & 0 \end{bmatrix} V_M^{\mathrm{T}} \quad \text{(SVD)},$$

$$N \triangleq \begin{bmatrix} (I - BB^+)[A(Q+Q_2) + qQ)^{-1/2} \\ (I - C^- C)(Q+Q_2)^{-1}[qQ + (Q+Q_2)A^{\mathrm{T}}]T^{-1} \end{bmatrix} = U_N \begin{bmatrix} \Lambda_N & 0 \\ 0 & 0 \end{bmatrix} V_N^{\mathrm{T}} \quad \text{(SVD)},$$

$r_M \triangleq \operatorname{rank}(M)$.

where $[\cdot]^+$ denotes the Moore-Penrose inverse of matrix $[\cdot]$, and T denotes unique positive definite square root of matrix P (if P is positive definte).

Definition Assume Q is a positive definite matrix, if equation (3.1) has soluton G then Q is called Ω- assignable.

Theorem 3.2 Assume positive definite matrix Q makes $P > 0$, then Q is Ω - assignable if and only if

1) $$(I - BB^-)\{[A(Q+Q_2) + qQ](Q+Q_2)^{-1} \cdot [qQ + (Q+Q_2)A^{\mathrm{T}}] - P\}(I - BB^+) = 0, \quad (3.3a)$$

2) $$(I - C^+C)\{(Q+Q_2)^{-1} - (Q+Q_2)^{-1}[qQ + (Q+Q_2)A^{\mathrm{T}}]P^{-1} \cdot [A(Q+Q_2) + qQ](Q+Q_2)^{-1}\}(I - C^+C) = 0. \quad (3.3b)$$

Proof Substitute $A_c = A + BGC$ into the equation (3.1) then

$$\begin{aligned}
0 &= A(Q+Q_2)A^{\mathrm{T}} + BGC(Q+Q_2)A^{\mathrm{T}} + A(Q+Q_2)(BGC)^{\mathrm{T}} + BGC(Q+Q_2)(BGC)^{\mathrm{T}} \\
&\quad + qBGCQ + qQ(BGC)^{\mathrm{T}} + q(AQ + QA^{\mathrm{T}} + Q_1) + Q_0 \\
&= [A(Q+Q_2) + qQ](BGC)^{\mathrm{T}} + BGC[qQ + (Q+Q_2)A^{\mathrm{T}}] + BGC(Q+Q_2)(BGC)^{\mathrm{T}} \\
&\quad + A(Q+Q_2)A^{\mathrm{T}} + q(AQ + QA^{\mathrm{T}} + Q_1) + Q_0 \\
&= \{[A(Q+Q_2) + qQ](Q+Q_2)^{-1/2} + BGC(Q+Q_2)^{-1/2}\} \\
&\quad \cdot \{[A(Q+Q_2) + qQ](Q+Q_2)^{-1/2} + BGC(Q+Q_2)^{-1/2}\}^{\mathrm{T}} + A(Q+Q_2)A^{\mathrm{T}}
\end{aligned}$$

$$+ q(AQ + QA^{\mathrm{T}} + Q_1) + Q_0 - [A(Q + Q_2) + qQ](Q + Q_2)^{-1}[qQ + (Q + Q_2)A^{\mathrm{T}}],$$

i. e.

$$\begin{aligned}
&\{[A(Q + Q_2) + qQ](Q + Q_2)^{-1/2} + BGC(Q + Q_2)^{1/2}\} \\
&\quad \cdot \{[A(Q + Q_2) + qQ](Q + Q_2)^{-1/2} + BGC(Q + Q_2)^{1/2}\}^{\mathrm{T}}. \\
&= [A(Q + Q_2) + qQ](Q + Q_2)^{-1}[qQ + (Q + Q_2)A^{\mathrm{T}}] - A(Q + Q_2)A^{\mathrm{T}} \\
&\quad - q(AQ + QA^{\mathrm{T}} + Q_1) - Q_0 \\
&= P.
\end{aligned}$$

Since $P > 0$, so above equality holds if and only if[6]

$$[A(Q + Q_2) + qQ](Q + Q_2)^{-1/2} + BGC(Q + Q_2)^{1/2} = TV, \tag{3.4}$$

where $V \in \mathrm{R}^{(n_x+n_z)\times(n_x+n_z)}$ is some orthogonal matrix. (3. 4) is equivalent to equation

$$BGC = TV(Q + Q_2)^{-1/2} - [A(Q + Q_2) + qQ](Q + Q_2)^{-1}, \tag{3.5}$$

which is consistent iff

$$(I - BB^+)\{TV(Q + Q_2)^{-1/2} - [A(Q + Q_2) + qQ](Q + Q_2)^{-1}\} = 0, \tag{3.6a}$$

and $$\{TV(Q + Q_2)^{-1/2} - [A(Q + Q_2) + qQ](Q + Q_2)^{-1}\}(I - C^+ C) = 0, \tag{3.6b}$$

condition (3. 6) is equivalent to

$$(I - BB^+)TV = (I - BB^+)[A(Q + Q_2) + qQ](Q + Q_2)^{-1/2}], \tag{3.7a}$$

and $$(I - C^+ C)(Q + Q_2)^{-1/2}V = (I - C^+ C)(Q + Q_2)^{-1}[qQ + (Q + Q_2)A^{\mathrm{T}}]T^{-1}. \tag{3.7b}$$

i. e.

$$\begin{bmatrix} (I - BB^+)T \\ (I - C^+ C)(Q + Q_2)^{-1/2} \end{bmatrix} V = \begin{bmatrix} (I - BB^+)[A(Q + Q_2) + qQ](Q + Q_2)^{-1/2} \\ (I - C^+ C)(Q + Q_2)^{-1}[qQ + (Q + Q_2)A^{\mathrm{T}}]T^{-1} \end{bmatrix}, \tag{3.8}$$

or,

1) $$\begin{aligned}
&(I - BB^-)P(I - BB^+) \\
&= (I - BB^+)[A(Q + Q_2) + qQ](Q + Q_2)^{-1} \\
&\quad \cdot [qQ + (Q + Q_2)A^{\mathrm{T}}(I - BB^+),
\end{aligned} \tag{3.9a}$$

2) $$\begin{aligned}
&(I - C^+ C)(Q + Q_2)^{-1}(I - C^+ C) \\
&= (I - C^+ C)(Q + Q_2)^{-1}[qQ + (Q + Q_2)A^{\mathrm{T}}]P^{-1} \\
&\quad \cdot [A(Q + Q_2) + qQ](Q + Q_2)^{-1}(I - C^+ C).
\end{aligned} \tag{3.9b}$$

Equation (3. 9a) and (3. 9b) are equivalent to (3. 3b) and (3. 3c) respectively.

Theorem 3. 3 Assume Q is Ω -assignable, then the dynamic controllers expressed by

$$\begin{aligned}
G = B^+ &\left[TV_M \begin{bmatrix} I_{r_M} & 0 \\ 0 & U \end{bmatrix} V_N^{\mathrm{T}}(Q + Q_2)^{-1/2} - [A(Q + Q_2) + qQ](Q + Q_2)^{-1} \right] \\
&+ Z - B^+ BZCC^-
\end{aligned} \tag{3.10}$$

can place poles of closed-loop perturbed system (2. 3) in a given circular region $\Omega(q, r)$, where $U \in \mathrm{R}^{(n_x+n_z-r_M)\times(n_x+n_z-r_M)}$ is arbitrary orthogonal, $Z \in \mathrm{R}^{(n_u+n_z)\times(n_y-n_z)}$ is an arbitrary matrix.

Proof Since Q is assignable from equation (3.5) we can know that the solution of equation (3.1) is

$$G = B^{+} \{TV(Q+Q_2)^{-1/2} - [A(Q+Q_2)+qQ](Q+Q_2)^{-1}\} + Z - B^{+}BZCC^{+}. \tag{3.10}$$

From equation (3.8) all the orthogonal matrix V can expressed by[6]

$$V = VM\begin{bmatrix} I_{r_M} & 0 \\ 0 & U \end{bmatrix} V_N^T.$$

Substituting V into the expression of G yields (3.10), then $\Lambda(A_c + \Delta A) \subset \Omega(q,r)$ according to Theorem 3.1.

Remark The terms in Z do not influence the closed-loop pole locations, but they affect the control energy, and they supply the design freedom to satisfy other performance contraints.

Algorithm

Step 1 Select a positive definite matrix W, then construct a Ω-assignable Q-matrix by Theorem 3.2;

Step 2 To solve dynamic control laws from expresssion (3.10).

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不确定系统的动态输出反馈圆形区域极点配置

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摘要：本文对一类不确定线性系统研究了实现鲁棒圆形区域极点配置的动态输出反馈控制律设计. 通过一个修正的 Lyapunov 方程，给出了鲁棒控制器的存在条件. 该修正 Lyapunov 方程的逆解就是期望的动态输出反馈控制器的一般解.

关键词：不确定线性系统；鲁棒极点配置；动态输出反馈

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朱纪洪 1968年生于镇江. 1990年获江苏工学院电气技术专业学士学位,同年免试进入南京理工大学攻读自动控制理论及应用专业硕士学位. 于1995年6月获博士学位,同时留校任教,现为南京航空航天大学博士后. 主要研究领域:方差配置控制与滤波,鲁棒控制,H_∞控制,飞机大迎角过失速非线性控制

郭 治 见本刊1996年第2期第174页.

反馈控制理论的新进展——推荐新书

"Minimum Entropy H_∞ Control"和"Feedback Control Theory"

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1 前 言

反馈控制系统设计中的鲁棒性问题的研究,近十几年来有了迅速的发展. 鲁棒性理论已经从早期的鲁棒性分析,这种比较粗糙的和较少结构性以及缺少参数不确定的描述朝着更加清晰的不确定性表达方式发展,至今所得到的包括可作为分析与设计工具的各种不确定性形式的鲁棒控制理论的研究结果,它允许在系统性能与鲁棒性两者之间折衷,以达到回路成形的目的. 所采用的方法在低频和高频分别极小化灵敏度和补灵敏度,这样有可能达到所规定的性能与关于未建模动力学的鲁棒性等目标.

从八十年代开始迅速发展的H_∞控制理论,它克服了一直困扰着二次性能准则中关于鲁棒性的度量问题. 在H_∞控制理论中,系统被描述为具有L_2(有界功率)输入和输出的Hardy空间H_∞的元素,其诱导范数就是H_∞范数,它定义为系统传递矩阵的最大奇异值函数关于$\omega\in[-\infty,\infty]$的上确界. 相对于LQG二次性能度量(等价于$H_2$范数)而言,$H_\infty$性能度量是对应于最坏情况下的频率衰减,即如果$G(s)$是从输入至有关变量的性能的传递矩阵,则$H_\infty$设计就是要求选择控制器使$\|G\|_\infty := \sup_\omega \bar{\sigma}[G(j\omega)]$达到极小. 由于$H_\infty$技术允许精确的回路成形与动态加权的不确定性表征,并相继出现混合灵敏度以及混合H_2/H_∞优化等方法,使得H_∞综合技术得到迅速发展与应用. 尽管如此,仍然存在着保守性. 为了改善在出现结构摄动情况下的保守性. 又产生了结构奇异值方法,这样使得反馈控制理论在近十几年来进入一个新的发展时期. 继八十年代有关的两本重要专著:M. Vidyasagar的"Control System Synthesis: A Factorization Approach"(1984年)和B. A Francis的"A Course in H_∞ Control Theory"(1988年)之后,九十年代又有两本重要专著,分别简介如下.

2 推荐两本新书

1) "Minimum Entropy H_∞ Control" by D. Mastafa and K. Glover, Springer-Verlag, 1990.

该书在H_∞设计技术中,首先引入一个性能泛函

$$I(H;r;\infty) := \lim_{s_0\to\infty} I(H;r;s_0)$$

称为传递函数$H(\cdot)$在无穷远点的熵,其中

$$I(H;r;s_0) := \int_{-\infty}^{\infty} \ln\left|\det(I - r^{-2}H^*(j\omega)H(j\omega))\right|\left[\frac{s_0^2}{s_0^2+\omega^2}\right]d\omega.$$

选择$I(\cdot)$作为极小化泛函是自然的,这是因为它施行了$H^*(j\omega)H(j\omega)\leqslant r^2 I, \forall\ \omega$,(即$\|H_\infty\|\leqslant r$)与极

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