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Robust Stabilization of Linear Delay Systems by Retarded Feedback*

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Abstract: This paper is devoted to investigation of robust stabilization of time-invariant linear delay systems. The design method for the guaranteed robust stabilization controllers is given for the systems with perturbation matrices satisfying matching conditions, the existence conditions and design method of robust stabilizing controllers of systems with perturbation matrices not satisfying the matching conditions are also obtained, based on an asymptotic stability theorem first established in the paper for delay systems. A simple retarded feedback stabilization scheme for non-delay linear sytems is especially proposed in the paper. Two examples are given at the end of the paper to illustrate the design method of the controllers developed in the paper.

Key words: delay system; retarded feedback; robustness; guaranteed stabilization

1 Introduction

Consider the time-invariant multidelay linear system

$$\dot{x}(t) = (A_0 + \Delta A_0)x(t) + \sum_{i=1}^{m}(A_i + \Delta A_i)x(t - \tau_i) + Bu(t), \tag{1}$$

where $x \in \mathrm{R}^n, u \in \mathrm{R}^r, A_i, \Delta A_i \in \mathrm{R}^{n\times n}, B \in \mathrm{R}^{n\times r}, 0 \leqslant \tau_i \leqslant \tau$.

The robust satbilization of the systems such as (1) have been investigated in a lot of literature. A common method is to employ the optimal linear regulator for the system $\dot{x}(t) = A_0(t)x(t) + Bu(t)$ to stablize system (1). In the results obtained with this method, $\sum(A_i + \Delta A_i)x(t - \tau_i)$ was treated as a perturbation, thus $A_i, \Delta A_i$ was desired to be small[1~3]. In such treatment, the action of the delayed signals was ignored. Especially, the results were not suitable for the case $A_0 = 0$. Thus such a treatment is not satisfactory. Besides, the "perturbation terms" such as $\sum(A_i + \Delta A_i)x(t - \tau_i)$ were supposed to satisfy the matching conditions in some previous papers, e.g. [1,4], so that guaranteed satbilizing controllers can be designed for the systems. It shoud be pointed out that such a treatment is not natural.

In this paper, a new criterion for the asymptotic stability of time-invariant linear delay systems is first established by constructing suitable Lyapunov functional, this theorem is suitable for the case $A_0 = 0$. With this new criterion, the robust stabilization of system (1)

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is investigated. The matching conditions for the terms $A_1, A_2, \cdots, A_m$ are eliminated in Section 5, thus the results obtained in this paper are suitable for the case $A_0 = 0$.

2 Notations and Lemmas

In this paper, for a vector x, $\| x \|$ denotes the Euclidean norm of x, for a matrix A, $\| x \|$ denotes the 2-norm of A, for a symmetric matrix P, $\lambda_m(P)$ denotes the minimal eigenvalue of P, if P is positive definite (positive semidefinite), we denote $P > 0 (P \geq 0)$. For a matrix $P > 0$, denote the positive definite square root of it by $\sqrt{P}$.

Lemma 1[5] For vectors $X, Y \in R^n$, real number $r > 0, X^TY + Y^TX \leq rX^TX + \frac{1}{r}Y^TY$

Lemma 2[6] If a vector function $x(\cdot): [t_0 - r, +\infty) \to R^n$ satisfies

$$\| x(t) \| \leq \alpha |x_t| + h (t \geq t_0), \; \| x(t) \| \leq \tilde{\varphi}(t) (t_0 - \tau \leq t \leq t_0)$$

where $0 \leq \alpha = \text{const.} < 1$, then there is a constant number $\alpha: 0 < \alpha < \alpha_0 = -\frac{1}{\tau}\ln a$ such that $\| x(t) \| \leq |\tilde{\varphi}| e^{-\alpha(t-t_0)} + h/(1-a)$.

For $x(\cdot): \mathrm{R}^+ \to \mathrm{R}^n, x_t \in C([-\tau, 0], \mathrm{R}^n)$ is defined as $x_t(\theta) = x(t+\theta), \theta \in [-\tau, 0]$.

Lemma 3[8] If $f: [k, +\infty) \to \mathrm{R}^+ = [0, +\infty)$ is uniformly continuous, and $f \in L^1[k, +\infty)$, then $f(t) \to 0 (\text{as } t \to +\infty)$.

3 Stability Theorem

Consider a time-invariant multidelay linear system

$$\dot{x}(t) = A_0 x(t) + \sum_{i=1}^{m} A_i x(t - \tau_i), \tag{2}$$

where $x \in \mathrm{R}^n, A_i \in \mathrm{R}^{n \times n}, 0 \leq \tau_i \leq \tau, i = 1, 2, \cdots, m$.

Theorem 1 If $A = \sum_{i=0}^{m} A_i$ is a stable matrix and

$$\sum_{i=1}^{m} \tau_i \| A_i \| < 1, \tag{3}$$

$$\sum_{i=1}^{m} \tau_i \| A^TA + A_i^T P^2 A_i \| < \lambda_m(Q), \tag{4}$$

where Q is a given positive definite matrix, P is the positive definite solution to the Lyapunov matrix equation

$$A^TP + PA = -Q, \tag{5}$$

then the zero solution of system (2) is uniformly asymptotically stable in large.

Proof Define a D- operator as

$$D(x_t) = x(t) + \sum_{i=1}^{m} A_i \int_{t-\tau_i}^{t} x(s) \mathrm{d}s, \tag{6}$$

then the system (2) can be rewritten as a neutral system

$$\frac{\mathrm{d}}{\mathrm{d}t} D x_t = A x(t). \tag{7}$$

Define a Lyapunov functional

$$V(x_t) = D^{\mathrm{T}}(x_t)PD(x_t) + \sum_{i=1}^{m} \int_{t-\tau_i}^{t} \int_{s}^{t} [PA_i x(u)]^{\mathrm{T}}[PA_i x(u)]\mathrm{d}u\mathrm{d}s, \tag{8}$$

then for $(\sigma,\psi) \in R \times C([-\tau,0],\mathrm{R}^n)$, along the solution $x(t) = x(\sigma,\psi)(t)$, we have

$$\dot{V}(x_t) = 2x^{\mathrm{T}}(t)A^{\mathrm{T}}Px(t) + \sum_{i=1}^{m} \int_{t-\tau_i}^{t} 2[Ax(t)^{\mathrm{T}}[PA_i x(s)]\mathrm{d}s$$

$$+ \sum_{i=1}^{m} \tau_i x^{\mathrm{T}}(t)A_i^{\mathrm{T}}P^2A_i x(t) - \sum_{i=1}^{m} \int_{t-\tau_i}^{t} [PA_i x(s)]^{\mathrm{T}}[PA_i x(s)]\mathrm{d}s. \tag{9}$$

By Lemma 1 we have $2[Ax(t)]^{\mathrm{T}}PA_i x(s) \leqslant x^{\mathrm{T}}(t)A^{\mathrm{T}}Ax(t) + [PA_i x(s)]^{\mathrm{T}}[PA_i x(s)]$, then it follows from (9) that

$$\dot{V}(x_t) \leqslant x^{\mathrm{T}}(t)(A^{\mathrm{T}}P + PA + \sum_{i=1}^{m} \tau_i A^{\mathrm{T}}A + \sum_{i=1}^{m} \tau_i A_i^{\mathrm{T}}P^2A_i)x(t)$$

$$= - x^{\mathrm{T}}(t)[Q - \sum_{i=1}^{m} \tau_i (A^{\mathrm{T}}A + A_i^{\mathrm{T}}P^2A_i)]x(t). \tag{10}$$

By (4), there is a constant $\beta > 0$ such that

$$\dot{V}(x_t) \leqslant - \beta \| x(t) \|^2, \tag{11}$$

integrating the two sides of the inequality from σ to t we obtain

$$\beta \int_{\sigma}^{t} \| x(s) \|^2 \mathrm{d}s \leqslant V(\psi) - \dot{V}(x_t) \leqslant V(\psi), \quad t > \sigma, \tag{12}$$

this shows $\| x(\cdot) \|^2 \in L^1[\sigma, +\infty)$.

Besides, from (8) we have $V(x_t) \leqslant M_1 |x_t|^2$, where

$$M_1 = [(1 + \sum_{i=1}^{m} \tau_i \| A_i \|)^2 + \frac{1}{2} \sum_{i=1}^{m} \tau_i^2 \| A_i \|^2 \| P \|] \| P \|,$$

thus $V(\psi) \leqslant M_1 |\psi|^2$. Since $V(t)$ is monotone decreasing, $V(x_t) \leqslant V(\psi)$, thus

$$\| x(t) \| - (\sum_{i=1}^{m} \tau_i \| A_i \|)|x_t| \leqslant \| D \| \leqslant \sqrt{\lambda_m^{-1}(P)V(\psi)} \leqslant \sqrt{\lambda_m^{-1}(P)M_1} |\psi|, \tag{13}$$

by the assumption (3) and Lemma 2 we get

$$\| x(t) \| \leqslant M|\psi| \, (M = \text{const.}). \tag{14}$$

(14) shows that $x = 0$ is uniformly stable and $\| x(t) \|$ is uniformly bounded, then by (2), $\| \dot{x}(t) \|$ is bounded, this implies that $\| x(t) \|^2$ is uniformly continuous. By Lemma 3, $\| x(t) \|^2 \to 0 (t \to \infty)$. The proof is complete.

Remark 1 One can rewritten the second term of the right side of (9) as $\sum_{i=1}^{m} \int_{t-\tau_i}^{t} 2[\sqrt{P}Ax(t)]^{\mathrm{T}}[\sqrt{P}A_i x(s)]\mathrm{d}s$, then one will obtain $\dot{V}(x_t) \leqslant - x^{\mathrm{T}}(t)[Q - \sum_{i=1}^{m} \tau_i (A^{\mathrm{T}}PA + A_i^{\mathrm{T}}PA_i)]x(t)$, by Lemma 1. In this case, the matrix P in the second term of (8) is replaced by $\sqrt{\mathrm{P}}$. Thus the condition (4) can be replaced by

$$\sum_{i=1}^{m} \tau_i \| A^{\mathrm{T}}PA + A_i^{\mathrm{T}}PA_i \| < \lambda_m(Q). \tag{15}$$

Remark 2 The assumption on the coefficient matrices of the system in Theorem 1 is that $A = \sum_{i=0}^{m} A_i$ is stable. In this assumption, $\{A_0, A_1, \cdots, A_m\}$ is symmetric. In such an assumption, the delayed information is used sufficiently. Thus Theorem 1 is suitable for the completely retarded system

$$\dot{x}(t) = \sum A_i x(t - \tau_i). \tag{16}$$

4 Robust Stabilization (I): The Matching Case

Suppose that the uncertain matrices $\Delta A_0, \Delta A_1, \cdots, \Delta A_m$ satisfy the following matching condition and boundedness conditon:

$$\Delta A_i = BE_i, \quad \| E_i \| \leqslant e_i, \quad i = 0, 1, 2, \cdots, m. \tag{17}$$

Introduce the retarded state feedback

$$u(t) = K_0 x(t) + \sum_{i=1}^{m} K_i x(t - \tau_i). \tag{18}$$

Denote

$$A = \sum_{i=0}^{m} A_i, \quad K = \sum_{i=0}^{m} K_i, \quad E = \sum_{i=0}^{m} E_i, \quad \hat{A}_i = A_i + BK_i, \quad \hat{A} = \sum_{i=0}^{m} \hat{A}_i = A + BK, \tag{19}$$

then the closed-loop system of (1) can be written as

$$\dot{x}(t) = (\hat{A}_0 + BE_0) x(t) + \sum_{i=1}^{m} (\hat{A}_i + BE_i) x(t - \tau_i). \tag{20}$$

Theorem 2 If the pair (A, B) is controllable, $R \in \mathrm{R}^{r \times r}, Q \in \mathrm{R}^{n \times n}$ are positive definite weighted matrices, satisfying

$$Q > \left(\sum_{i=0}^{m} e_i\right)^2 \| R \| I, \tag{21}$$

where P is the positive definite solution of the Riccati matrix equation

$$A^{\mathrm{T}} P + PA - PBR^{-1} B^{\mathrm{T}} P = - Q, \tag{22}$$

and the delays satisfy

$$\sum_{i=1}^{m} \tau_i (\| \hat{A}_i \| + \| B \| e_i) < 1, \tag{23}$$

$$\sum_{i=1}^{m} \tau_i \left[\left(\| \hat{A} \| + \| B \| \left(\sum_{i=0}^{m} e_i \right) \right)^2 + \left(\| \hat{A}_i \| + \| B \| e_i \right)^2 \| P \|^2 \right]$$

$$\leqslant \lambda_m(Q) - \left(\sum_{i=0}^{m} e_i \right)^2 \| R \|, \tag{24}$$

then system (1) is stabilized completely by feedback (18) provided $K_0, K_1, \cdots, K_m$ satisfy

$$K = - R^{-1} B^{\mathrm{T}} P. \tag{25}$$

Proof We use Theorem 1 for (20). To prove the asymptotic stability of the zero solution of (20), we need prove that the coefficient matrix $\sum_{i=0}^{m} (\hat{A}_i + BE_i) = \hat{A} + BE = A +$

$BK + BE$ is stable and the condition corresponding to (4) is satisfied. By Lemma 1,

$$E^TB^TP + PBE = (\sqrt{R}E)^T\sqrt{R^{-1}}B^TP + (PB^T\sqrt{R^{-1}})^T\sqrt{R}E$$
$$\leqslant E^TRE + PBR^{-1}B^TP, \tag{26}$$

it follows from (22), (26) and (21) that

$$\begin{aligned} -\tilde{Q} &\triangleq (A + BK + BE)^TP + P(A + BK + BE) \\ &= A^TP + PA - 2PBR^{-1}B^TP + E^TB^TP + PBE \\ &\leqslant A^TP + PA - 2PBR^{-1}B^TP + E^TRE + PBR^{-1}B^TP \\ &= -Q + E^TRE \leqslant -Q + (\sum_{i=0}^{m} e_i)^2 \| R \| I < 0, \end{aligned} \tag{27}$$

thus $\mathrm{Re}\lambda(A + BK + BE) < 0$, $A + BK + BE$ is stable.

It remains to show the left term of (24) is less than $\lambda_m(\tilde{Q})$. By (27),

$$-\tilde{Q} \leqslant -Q + (\sum_{i=0}^{m} e_i)^2 \| R \| I, \quad \tilde{Q} \geq Q - (\sum_{i=0}^{m} e_i)^2 \| R \| I,$$

thus we have

$$\lambda_m(Q) - (\sum_{i=0}^{m} e_i)^2 \| R \| < \lambda_m(\tilde{Q}).$$

So the proof is complete.

Remark 3 The matrix Q in (21) (22) can be taken as

$$Q = [\varepsilon + (\sum_{i=0}^{m} e_i)^2 \| R \|]I, \ (\varepsilon > 0). \tag{28}$$

Remark 4 $K_0, K_1, \cdots, K_m$ can be chosen arbitrarily provided that (25) is satisfied. To have a larger range for the delays, one can choose $K_0, K_1, \cdots, K_m$ such that $\| \hat{A}_1 \|$, $\| \hat{A}_2 \|$, $\cdots$, $\| \hat{A}_m, \|$ become minimal. Thus we choose $K_0, K_1, \cdots, K_m$ by

$$\| \hat{A}_i \| = \| A_i + BK_i \| = \min\{ \| A_i + BK \| \mid K \in R^{r\times n}\}, \quad i = 1,2,\cdots,m, \tag{29}$$

$$K_0 = K - \sum_{i=1}^{m} K_i. \tag{30}$$

If rank $B = r$, then K_i can be represented as[11]

$$K_i = -[B]^+ A_i = -(B^TB)^{-1}B^TA_i, \quad i = 1,2,\cdots,m. \tag{31}$$

Remark 5 Theorem 1 and Theorem 2 are suitable for the case $A_0 = 0$. In such criteria as in these theorems, the condition on the upper bound of the delays can not be cancelled. For example, by Theorem 1 of [12], the zero solution of the system $\dot{x}(t) = x(t) - 2x(t - \tau)$ $(x \in R^1)$ is asymptotically satble if and only if $0 \leqslant r < \frac{\pi}{3\sqrt{3}}$. Thus if τ is larger than $\frac{\pi}{3\sqrt{3}}$, the zero solution of the system is not asymptotically stable. Thus, in the above theorems, a type of condition such as (3)(4) or (23)(24) is necessary. Of course, the estimation in (3)(4) or (23)(24) still can be improved on further.

5 Robust Stabilization (Ⅱ): The unmatching Case

Suppose that the uncertain matrices satisfy the boundedness condition

$$\| \Delta A_i \| \leqslant \rho_i, i = 1,2,\cdots,m, \quad \| \Delta A \| = \| \sum_{i=0}^{m} \Delta A_i \| \leqslant \rho, \tag{32}$$

With Theorem 1, by a similar argument to that in the proof of Theorem 2 with BE replaced by ΔA, we get

Theorem 3 If (A,B) is controllable, R,Q are given positive definite weighted matrices, P is the positive definite solution of the equation (20), ρ satisfies

$$0 \leqslant \rho < \lambda_m(Q + PBR^{-1}B^{T}P)/(2\| P \|), \tag{33}$$

and the delays $\tau_1,\tau_2,\cdots,\tau_m$ satisfy

$$\sum_{i=1}^{m} \tau_i(\| \hat{A}_i \| + \rho_i) < 1, \tag{34}$$

$$\sum_{i=1}^{m} \tau_i[(\| \hat{A} \| + \rho)^2 + (\| \hat{A}_i \| + \rho_i)^2 \| P \|^2] < \lambda_m(Q + PBR^{-1}B^{T}P) - 2\rho \| P \|, \tag{35}$$

and $K_0,K_1,\cdots,K_m$ are chosen such that (25) is satisfied, then system (1) is stabilized by the feedback law (18).

6 Stabilization of Linear Systems without Delay by Retarded Feedback

Consider linear system

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{36}$$

employ the delayed feedback

$$u(t) = Kx(t-\tau), \tag{37}$$

where τ is to be determined below.

The employment of the delayed feedback is based on the following consideration:

1) In any practical problem, certain time is needed for the signal to be transmitted and processed, thus there exists time delay in any practical feedback in fact[13]. A most typical example is the aircraft control system.

2) In some control systems, time delay is introduced artificially for certain purpose. A typical example is the house temperature control system[4].

3) The time delay provides people with certain time to deal with signal if the delay is larger than the time for the signal to be transimitted.

Substitute the feedback law (37) into (36) we obtain the closed-loop system

$$\dot{x}(t) = Ax(t) + BKx(t-\tau). \tag{38}$$

By Theorem 1 one easily obtains the following

Theorem 4 If (A,B) is controllable, R,Q are given positive definite weighted matrices, P is the positive definite solution of the matrix equation (20), then (36) is stabilized by the feedback law $u(t) = -R^{-1}B^{T}Px(t-\tau)$ if τ is chosen such that

$$\tau \| BK \| < 1 \tag{39}$$

$$\tau(\hat{A}^{T}P\hat{A} + K^{T}B^{T}PBK) < Q + PBR^{-1}B^{T}P, \tag{40}$$

where $K = - R^{-1}B^{T}P, \hat{A} = A + BK = A - R^{-1}B^{T}P$.

Based on this theorem, a stabilization scheme for (36) is given below:

Assumption: (A,B) is controllable,

step 1 solve the Riccati matrix equation (20);

step 2 determine the range for delay τ ;

step 3 compute $K = - R^{-1}B^{T}P$ and set $u(t) = Kx(t - \tau)$.

In this scheme, the assumption is the most basic condition for the system and compution is not much, which is just the ordinary compution. Comparsed with the result of [15], the condition of the present paper is simpler, the result is suitable for more systems and the scheme is simpler.

Remark 6 A remark on the optimal linear regulator of the time-invariant linear systems.

If we assume that there is no delay in the feedback, the closed-loop system we obtain is $\dot{x}(t) = (A - BR^{-1}B^{T}P)x(t)$ by employing the feedback $u(t) = - R^{-1}B^{T}Px(t)$. In fact, there is always certain time delay τ in the feedback, thus the feedback in fact is $u(t) = - R^{-1}B^{T}Px(t - \tau)$, instead of $u(t) = - R^{-1}B^{T}Px(t)$, the closed-loop system is (38). As the delay being considered, the feedback $u(t) = - R^{-1}B^{T}Px(t - \tau)$ is not optimal (see[14], p. 109).

Surely, Theorem 4 in this paper shows that: the "optimal regulators" without time delays being considered can truely stabilize the systems provided the delays are in the admissible range. The conditions on the range of the delays, e. g. (3)(4)or (23)(24), should be checked by the engineers in practical problems.

7 Design Examples

Example 1 Consider the second order system

$$\dot{x}(t) = (\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + \Delta A)x(t - 0.01) + \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}u(t), \tag{41}$$

where ΔA is bounded: $\| \Delta A \| \leq \rho = 0.15$.

The methods of [1~4] are not suitable for the design of controller of this system because this system is a completely retarded system and ΔA is not assumed to satisfy the matching condition.

Here a robust controller of (41) is designed by virtue of Theorem 3 of this paper. Choose $R = I_2$,

$$Q = \begin{bmatrix} 1.75 & 3.00 \\ 3.00 & 7.75 \end{bmatrix}.$$

the solution of (20) then is $P = I_2$. One can easily compute $\| \hat{A}_1 \| = \| \hat{A} \| = 9.2749$, $\| P \| = 1, \lambda_m(Q + PBR^{-1}B^{T}P) = 2.1477$, the conditions (34) and (35) are satisfied. By Theorem 3, the system (41) is stabilized by the feedback

$$u(t) = -R^{-1}B^{T}Px(t-0.01) = -\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} x(t-0.01). \tag{42}$$

Example 2 Consider the second order system

$$\dot{x}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} u(t), \tag{43}$$

here a retarded feedback is designed to stabilize the system by the method given by Theorem 4.

Choose the weighted matrices $R = I_2, Q = (1+\sqrt{2})I_2$,solve the equation (20) we get $P = 2.9566I_2$,thus

$$K = -R^{-1}B^{T}P = \begin{bmatrix} 0 & -2.9566 \\ -2.9566 & 0 \end{bmatrix},$$

$$\hat{A}^{T}P\hat{A} + K^{T}B^{T}PBK = \begin{bmatrix} 37.1638 & 0 \\ 0 & 37.1638 \end{bmatrix},$$

$$Q + PBR^{-1}B^{T}P = \begin{bmatrix} 11.1557 & 0 \\ 0 & 11.1557 \end{bmatrix}.$$

From (39) and (40) we get the estimation: $\tau < 0.3382$, $\tau < 0.3001$, respectively, Thus we get an admissible range for τ: $\tau < 0.3001$. Now we choose $\tau = 0.3$, then we obtain a stabilizing retarded feedback for (43)

$$u(t) = -\begin{bmatrix} 0 & 2.9566 \\ 2.9566 & 0 \end{bmatrix} x(t-0.3). \tag{44}$$

8 Conclusion

A stability theorem, i. e Theorem 1, is established and the robust stabiliation by retarded feedback of time-invariant linear delay systems is investigated based on the stability theorem. Comparsed with the results of [1～3] and [11], the results of this paper has a feature that the design of the controllers makes used of the delayed signal thus the results are suitable for the case $A_0 = 0$. A retarded stabilizing scheme for the linear systems without time delay is also given in this paper, which is meaningful for the design of practical control systems. Besides, the design method is simpler than that given by [15].

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线性时滞系统滞后反馈鲁棒镇定

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摘要：本文研究时不变线性时滞系统的鲁棒镇定问题.通过建立时滞系统的一个渐近稳定性定理，对摄动矩阵满足匹配条件和不满足匹配条件的情况分别给出了完全鲁棒镇定控制器的设计方法与鲁棒镇定控制器的存在性充分条件和设计方法；文中尤其提出了非滞后线性系统的一种简单的滞后反馈镇定方案.文末用例子示范了本文的设计方法.

关键词：时滞系统；滞后反馈；鲁棒性；完全镇定

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