第14卷第1期　　控制理论与应用　　Vol. 14, No. 1
1997年2月　　CONTROL THEORY AND APPLICATIONS　　Feb., 1997

Uncertain Parameter Bounds for Pole Location in Specified Regions *

CHEN Wenhua

(Department of Automatic Control, Nanjing University of Aeronautics & Astronautics · Nanjing, 210016, PRC)

Abstract: The problem of uncertain parameter bound estimation for pole location in specified regions is considered in this paper. This problem includes two subproblems —— pole robustness analysis and parameterization of the controllers assigning the poles of the closed-loop systems within prescribed regions. Based on generalized Lyapunov theory and eigenvalue decomposition, a novel approach to this problem is proposed. The yielded bound is not necessarily symmetric and many existing uncertain parameter bounds are contained in it.

Key words: robustness; pole assignment; Lyapunov theory

1 Introduction

The performance requirements are always expressed as the closed-loop pole locations in system design. Hence one effective approach to the performance robustness is to investigate the robustness of the pole-placement in the specified regions. In this regard, Juang[5] obtains the symmetric upper-bound for element perturbations in the system matrix. Rachit[6] presents the similar results when the specified region is a circle.

In control system design, for special plants and/or in special circumstances, some requirements must be satisfied at first, such as stability, overshoot, no matter what control scheme is used. Then other requirements can be taken into account. One approach to this issue is to characterize the set of all controllers which ensure the closed-loop systems satisfy these requirements. Then one can choose one controller from this set based on other requirements, such as by optimization of some index[1,5]. This problem also arises in fault torelent control[2].

This paper considers the above two problems in a unified framework. These two problems can be included in the problem of estimation of uncertain parameter bounds for pole location in a specified region. A novel approach to the above problem is developed based on the eigenvalue decomposition technique and generalized Lyapunov theory. An efficient algorithm for estimation of the uncertain parameter set is proposed.

2 Sufficient Condition for Poles within Specified Regions

Consider uncertain linear systems

* This project is supported by National Aeronautical Foundation of China.

Manuscript received Oct. 31, 1995, revised Jun. 17, 1996.

$$\dot{X}(t)=AX(t)=(A_o+\sum_{i=1}^{m}k(i)E(i))X(t) \tag{1}$$

where $E(i)$ is the constant matrix which represents the structure of uncertain parameter $k(i)$, $i=1,\cdots,m$, and A_o an $n\times n$ real matrix which is called the nominal system.

The problem considered in this paper is to estimate the uncertain parameters bound such that the system poles are within a prescribed region. The following two problems are included in this problem.

i) Pole robustness analysis. Consider the uncertain system $\dot{X}=(A_o+\Delta A)X$. If the uncertain parameters in ΔA are independent or linear dependent, this system can be written as the system (1). The pole robustness analysis is to estimate the bounds of the uncertain parameters such that the system poles are within the prescribed region.

ii) Characterization of controllers for systems with known parameters. For the known system $\dot{X}=A_oX+BU$ and $Y=CX$, find a set of the feedback controllers $U=KY$ such that the poles of the closed-loop system A_o+BKC are within a prescribed region. The closed-loop system also can be written as the equation (1) where the open loop system A_o and the controller parameters K are considered as the nominal systems and uncertain parameters, respectively. Therefore this problem also can be considered as to estimate the bounds of the uncertain parameters (controller parameters) such that the system poles are within the prescribed region. However, in this case the eigenvalues of A_o may be not within the prescribed region.

Lemma 1[5] The eigenvalues of the real matrix A are within the region H in Fig. 1 if and only if for any positive definite matrix N, there exists a positive definite Hermitian P such that

$$e^{j\theta}(A-\alpha I)^*P+P(A-\alpha I)e^{-j\theta}=-N \tag{2}$$

where $(\cdot)^*$ denotes the complex conjugate transpose matrix of $(\cdot)$.

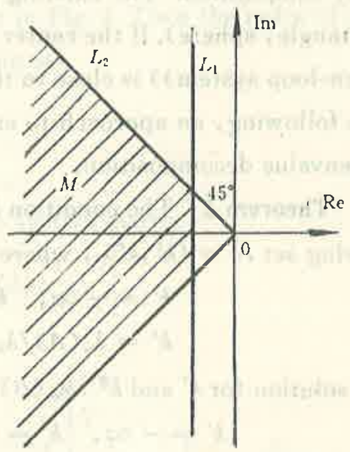

Fig. 1 Prescribed region of poles

Theorem 1 The poles of the system (1) are within the region H if there exists a positive definite Hermitian matrix P

$$\lambda_M(\sum_{i=1}^{m}k(i)P(i))<\lambda_m(Q) \tag{3}$$

where

$$Q=-(e^{j\theta}(A_o-\alpha I)^*P+P(A_o-\alpha I)e^{-j\theta}), \tag{4}$$

$$P(i)=e^{j\theta}E(i)^*P+PE(i)e^{-j\theta},\quad \text{for } i=1,\cdots,m. \tag{5}$$

Proof From Lemma 1, all poles of the system (1) are within the region H if there exists a positive definite Hermitian matrix P such that

$$e^{j\theta}(A_o+\sum_{i=1}^{m}k(i)E(i)-\alpha I)^*P+P(A_o+\sum_{i=1}^{m}k(i)E(i)-\alpha I)e^{-j\theta}<0 \tag{6}$$

or
$$\lambda_M(\sum_{i=1}^{m} k(i)P(i) - Q) < 0 \tag{7}$$
where Q and $P(i)$ are defined in (4) and (5). Since (3) implies (7), hence the result. Q. E. D.

Remark Q is not necessary positive definite. For example, if all eigenvalues of A do not locate in the region H, obviusly Q given by (4) is not positive definite.

Let
$$U = \{K : \lambda_M(\sum_{i=1}^{m} k(i)P(i)) < \lambda_m(Q), K \in \mathbb{R}^m\} \tag{8}$$
where $K = [k(1), k(2), \cdots, k(m)]^{\mathrm{T}}$. If K belongs to U, then the eigenvalues of matrix A are within the region H.

Many existing bounds in robust stability analysis are subsets of this set. When the specified region is the left-half plane and the poles of the nominal system are within this specified region, the problem considered in this paper is converted to the problem of robust stability analysis. The bounds given in [8,9] are obtained based on the following condition
$$\delta_M(\sum_{i=1}^{m} k(i)P(i)) < \lambda_m(Q)$$
where $\delta_M(\cdot)$ denotes the maximum singular value of the matrix $(\cdot)$. This implies the condition (7). Hence those bounds are subsets of the set U.

3 Estimation of Uncertain Parameter Bounds

It is very difficult to estimate the set U directly since the geometry of the set U may be very complicated. The existing approach is to imbed some geometric figure (such as hyperrectangle, sphere). If the center of the figure (i.e., the parameters of the nominal system (or open-loop system)) is close to the actual bound, the bound yielded is very conservative[7]. In the following, an approach to estimate the uncertain parameter bounds is proposed based on eigenvalue decomposition.

Theorem 2 The condition $\lambda_M(kB) < \lambda_m(A)$ is satisfied if and only if k belongs to the following set $N = (k', k'')$, where for $\lambda_m(A) \leqslant 0$,
$$k' = -\infty, \quad k'' = \lambda_m(A)/\lambda_m(B), \quad \lambda_M(B) \geqslant \lambda_m(B) > 0,$$
$$k' = \lambda_m(A)/\lambda_M(B), \quad k'' = \infty, \quad \lambda_m(B) \leqslant \lambda_M(B) < 0,$$
no solution for k' and k'', $\lambda_m(B) \leqslant 0 \leqslant \lambda_M(B)$; and for $\lambda_m(A) > 0$,
$$k' = -\infty, \quad k'' = \infty, \quad \lambda_M(B) = \lambda_M(B) = 0,$$
$$k' = -\infty, \quad k'' = \lambda_m(A)/\lambda_M(B), \quad \lambda_m(B) \leqslant 0, \quad \lambda_M(B) > 0,$$
$$k' = \lambda_m(A)/\lambda_m(B), \quad k'' = \infty, \quad \lambda_m(B) < 0, \quad \lambda_M(B) \leqslant 0,$$
$$k' = \lambda_m(A)/\lambda_m(B), \quad k'' = \lambda_m(A)/\lambda_M(B), \quad \lambda_m(B) < 0 < \lambda_M(B).$$

Proof It directly follows from calculating the maximum eigenvalue of the matrix kB. Q. E. D.

On the basis of Theorem 2 and in the view of that the set U defined by (8) is convex, we construct an algorithm for estimation of uncertain parameter bounds, and the bound yielded is as close to the set U as possible.

Let $b(i) = k(i)/k$, for $i = 1, \cdots, m$. Then the set U can be written as

$$U = \{K : \lambda_M(k\sum_{i=1}^{m} b(i)P(i)) < \lambda_m(Q), K = k[b(1),\cdots,b(m)]^{\mathrm{T}}, K \in \mathbb{R}^m\}.$$

Let

$$a = \lambda_m(Q), \quad d = \lambda_M(\sum_{i=1}^{m} b(i)P(i)) \text{ and } c = \lambda_m(\sum_{i=1}^{m} b(i)P(i)).$$

For a given vector $B^{(j)}$, it follows from Theorem 2 that K belongs to U' if and only if K belongs to

$$S(j) = \mathrm{cov}\{K'^{(j)}, K''^{(j)}\} = \mathrm{cov}\{k'B^{(j)}, k''B^{(j)}\}$$

where

$$B^{(j)} = [b^{(j)}(1), b^{(j)}(2), \cdots, b^{(j)}(m)]^{\mathrm{T}}$$

and k' and k'' are given in Theorem 2 by replacing $\lambda_m(A)$, $\lambda_M(B)$ and $\lambda_m(B)$ by a, d and c respectively.

In fact, k' and k'' are two points on the boundary of the set U along the direction defined by $B^{(j)}$ (where the open set U is extended to a minimum closed set. For precise mathematical manipulation, please refer to [4]). For different vectors $B^{(j)}$, different convex subsets are obtained. Let $j = 1$ to l, we obtain $S^{(1)}, \cdots, S^{(l)}$. Obviously, the convex hull

$$S = \mathrm{cov}\{K'^{(1)}, K''^{(1)}, K'^{(2)}, K''^{(2)}, \cdots, K'^{(l)}, K''^{(l)}\} \tag{9}$$

is a subset of U because U is a convex set and the convex hull of the union of any collection of subsets of U is also a subset of U. By increasing l, the convex hull S is as close to the set U as possible.

Corollary 1 Let M denote the specified region which is the common region of w open domain H_i, $i = 1, \cdots, w$, with respective L_i, α_i, v_i defined as in Fig. 1. Then the poles of system (1) are within the region M if K belongs to the following set

$$S = \bigcap_{i=1}^{w} S_i$$

where S_i is defined as (9) corresponding to H_i, $i = 1, \cdots, w$.

Example Consider a simple uncertain system with the system matrix

$$A = \begin{vmatrix} -1 & 1 \\ -3 & 0 \end{vmatrix} + k(1)\begin{vmatrix} 0 & -1 \\ 0 & 1 \end{vmatrix} + k(2)\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}.$$

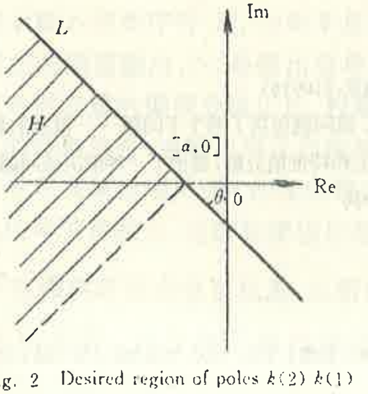

Fig. 2 Desired region of poles $k(2)$ $k(1)$

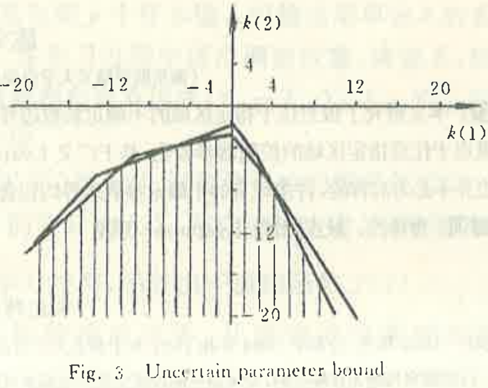

Fig. 3 Uncertain parameter bound

Due to the performance requirements, the poles of the system are desired within the region M depicted in Fig. 2. The region M is the common region of the open domain H_i, $i = 1, 2$, corresponding to L_i, respectively, and with $\alpha_1=-1$, $\theta_1=0$; $\alpha_2=0$, $\theta_2=\pi/4$. We want to estimate the bound of uncertain parameter $k(1),k(2)$ which ensure the poles of the system are within this specified region. The eigenvalues of the nominal system matrix are $(-1+\mathrm{j}\sqrt{11})/2$, $(-1-\mathrm{j}\sqrt{11})/2$, which are not within the desired region M.

Let $P=I$, $b(i)=k(i)/k$, $i=1,2$, and

$$B=\left\{\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix},\begin{pmatrix}1\\1\end{pmatrix},\begin{pmatrix}1\\-1\end{pmatrix},\begin{pmatrix}1\\-10\end{pmatrix},\begin{pmatrix}1\\0.5\end{pmatrix},\begin{pmatrix}1\\0.7\end{pmatrix},\begin{pmatrix}1\\-5\end{pmatrix}\right\}.$$

The yielded bound is depicted in Fig. 3. Obviously, any other existing method can not yield such large bounds.

References

1 Ackermann, J.. Parameter space design of robust control systems. IEEE Trans. Automat. Contr., 1980, AC-25:1058—1072

2 Gao, Z. Q. and Antsakis, P. J.. Pseudo-inverse method for reconfigurable control with guaranteed stability. Proc. of 11th IFAC World Congress, 1990, 293—298

3 Chen Wen Hua and Gu Xing Yuan. Characterization of robust controllers for linear systems with structured uncertainties: a computer aided approach. 1st IFAC Symposium on Design Methods of Control Systems, Zurich, 1991

4 Chen Wen Hua and Gu Xing Yuan. Measure of robustness for linear continuous times systems. 1st European Control Conference, France, 1991

5 Juang, Y. T., et. al.. Robustness of pole assignment in a specified region. IEEE Trans. Automat. Contr., 1989, AC-34:758—760

6 Rachit, A.. Robustness of pole assignment in a specified region of perturbed systems. Int. J. System Science, 1990, 21:579—585

7 Siljak, D. D.. Parameter space methods for robust control design, a guided tour. IEEE Trans. Automat. Contr., 1989, AC-34:674—688

8 Yedavalli, R. K.. Improvement measures of stability robustness for linear state space models. ibid., 1985, 30:577—579

9 Zhou, K. and Kharnogekar, P. P.. Robust stability bounds for linear state space model with structured uncertainty. Ibid., 1987, 32:621—623

极点位于指定区域的不确定参数边界

陈文华

(南京航空航天大学自动控制系·南京,210016)

摘要：本文研究了极点位于指定区域的不确定参数边界估计问题. 该问题包括了两个子问题——极点的鲁棒性分析和配置极点于任意指定区域的控制器参数化. 基于广义 Lyapunov 理论和特征值分解,提出了一种解决该问题的新方法. 得到的边界不必为对称的,许多现有的不确定参数边界均包含在该边界中.

关键词：鲁棒性；极点配置；Lyapunov 理论

本文作者简介

陈文华 1965 年生. 分别于 1988 年和 1991 年于东北大学自动控制系获硕士和博士学位. 1993 年聘为南京航空航天大学自动控制系副教授. 目前研究兴趣为鲁棒控制、变增益控制及航空航天航海系统控制.